AP® Physics C: Mechanics
Free-Response Questions
Set 2
### Constants and Conversion Factors

<table>
<thead>
<tr>
<th>Constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton mass, ( m_p )</td>
<td>( 1.67 \times 10^{-27} ) kg</td>
</tr>
<tr>
<td>Neutron mass, ( m_n )</td>
<td>( 1.67 \times 10^{-27} ) kg</td>
</tr>
<tr>
<td>Electron mass, ( m_e )</td>
<td>( 9.11 \times 10^{-31} ) kg</td>
</tr>
<tr>
<td>Avogadro’s number, ( N_0 )</td>
<td>( 6.02 \times 10^{23} ) mol(^{-1} )</td>
</tr>
<tr>
<td>Universal gas constant, ( R )</td>
<td>( 8.31 ) J/(mol·K)</td>
</tr>
<tr>
<td>Boltzmann’s constant, ( k_B )</td>
<td>( 13.8 \times 10^{-23} ) J/K</td>
</tr>
<tr>
<td>Electron charge magnitude, ( e )</td>
<td>( 1.60 \times 10^{-19} ) C</td>
</tr>
<tr>
<td>1 electron volt, ( 1 ) eV</td>
<td>( 1.60 \times 10^{-19} ) J</td>
</tr>
<tr>
<td>Speed of light, ( c )</td>
<td>( 3.00 \times 10^8 ) m/s</td>
</tr>
<tr>
<td>Universal gravitational constant, ( G )</td>
<td>( 6.67 \times 10^{-11} ) (N·m(^2))/kg(^2)</td>
</tr>
<tr>
<td>Acceleration due to gravity at Earth’s surface, ( g )</td>
<td>( 9.8 ) m/s(^2)</td>
</tr>
<tr>
<td>1 unified atomic mass unit, ( u )</td>
<td>( 1.66 \times 10^{-27} ) kg</td>
</tr>
<tr>
<td>Planck’s constant, ( h )</td>
<td>( 6.63 \times 10^{-34} ) J·s</td>
</tr>
<tr>
<td>( hc )</td>
<td>( 1.99 \times 10^{-25} ) J·m</td>
</tr>
<tr>
<td>Vacuum permittivity, ( \varepsilon_0 )</td>
<td>( 8.85 \times 10^{-12} ) C(^2)/(N·m(^2))</td>
</tr>
<tr>
<td>Vacuum permeability, ( \mu_0 )</td>
<td>( 4\pi \times 10^{-7} ) (T·m)/A</td>
</tr>
<tr>
<td>Magnetic constant, ( k' )</td>
<td>( 4\pi \times 10^{-7} ) (T·m)/A</td>
</tr>
<tr>
<td>1 atmosphere pressure, ( 1 ) atm</td>
<td>( 1.0 \times 10^5 ) N/m(^2)</td>
</tr>
</tbody>
</table>

### Prefixes

<table>
<thead>
<tr>
<th>Factor</th>
<th>Prefix</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^9 )</td>
<td>giga</td>
<td>G</td>
</tr>
<tr>
<td>( 10^6 )</td>
<td>mega</td>
<td>M</td>
</tr>
<tr>
<td>( 10^3 )</td>
<td>kilo</td>
<td>k</td>
</tr>
<tr>
<td>( 10^{-2} )</td>
<td>centi</td>
<td>c</td>
</tr>
<tr>
<td>( 10^{-3} )</td>
<td>milli</td>
<td>m</td>
</tr>
<tr>
<td>( 10^{-6} )</td>
<td>micro</td>
<td>µ</td>
</tr>
<tr>
<td>( 10^{-9} )</td>
<td>nano</td>
<td>n</td>
</tr>
<tr>
<td>( 10^{-12} )</td>
<td>pico</td>
<td>p</td>
</tr>
</tbody>
</table>

### Values of Trigonometric Functions for Common Angles

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>0°</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin ( \theta )</td>
<td>0</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{\sqrt{2}}{2} )</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>1</td>
</tr>
<tr>
<td>cos ( \theta )</td>
<td>1</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>( \frac{\sqrt{2}}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>0</td>
</tr>
<tr>
<td>tan ( \theta )</td>
<td>0</td>
<td>( \frac{\sqrt{3}}{3} )</td>
<td>( \frac{\sqrt{2}}{2} )</td>
<td>( \frac{1}{\sqrt{3}} )</td>
<td>( \infty )</td>
</tr>
</tbody>
</table>

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.

II. The direction of current is the direction in which positive charges would drift.

III. The electric potential is zero at an infinite distance from an isolated point charge.

IV. All batteries and meters are ideal unless otherwise stated.

V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.
### MECHANICS

\[
\begin{align*}
    v_x &= v_{x0} + a_x t \\
    x &= x_0 + v_{x0} t + \frac{1}{2} a_x t^2 \\
    v_x^2 &= v_{x0}^2 + 2a_x (x - x_0) \\
    \ddot{a} &= \frac{\ddot{F}}{m} = \frac{\ddot{F}_{\text{net}}}{m} \\
    \ddot{F} &= \frac{d\dot{p}}{dt} \\
    \ddot{J} &= \int \ddot{F} \cdot d\ddot{r} = \Delta \ddot{p} \\
    \ddot{p} &= m\ddot{v} \\
    |\ddot{F}_f| &\leq \mu |\ddot{F}_N| \\
    \Delta E &= W = \int \ddot{F} \cdot d\ddot{r} \\
    K &= \frac{1}{2}mv^2 \\
    P &= \frac{dE}{dt} \\
    P &= \ddot{F} \cdot \ddot{v} \\
    \Delta U_g &= mg\Delta h \\
    a_c &= \frac{v_x^2}{r} = \omega^2 r \\
    \ddot{r} &= \ddot{r} \times \ddot{F} \\
    \ddot{r} &= \frac{\ddot{r}_{\text{net}}}{I} \\
    I &= \int r^2 dm = \sum mr^2 \\
    x &= x_{\text{max}} \cos(\omega t + \phi) \\
    T &= \frac{2\pi}{\omega} = \frac{1}{\dot{f}} \\
    T_s &= 2\pi \sqrt{\frac{m}{k}} \\
    T_p &= 2\pi \sqrt{\frac{E}{g}} \\
    x_{cm} &= \frac{\sum m_i x_i}{\sum m_i} \\
    v &= r\omega \\
    \ddot{L} &= \ddot{r} \times \ddot{p} = I\ddot{\omega} \\
    K &= \frac{1}{2}I\omega^2 \\
    \omega &= \omega_0 + \alpha t \\
    \theta &= \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2
\end{align*}
\]

### ELECTRICITY AND MAGNETISM

\[
\begin{align*}
    \frac{|\ddot{F}_E|}{4\pi\varepsilon_0} &= \frac{q_1q_2}{r^2} \\
    \ddot{E} &= \frac{\ddot{F}_E}{q} \\
    \oint \ddot{E} \cdot d\ddot{A} &= \frac{Q}{\varepsilon_0} \\
    E_x &= -\frac{dV}{dx} \\
    \Delta V &= -\int \ddot{E} \cdot d\ddot{r} \\
    V &= \frac{1}{4\pi\varepsilon_0} \sum \frac{q_i}{r_i} \\
    U_E &= qV = \frac{1}{4\pi\varepsilon_0} \frac{q_1q_2}{r} \\
    \frac{1}{C_s} &= \sum \frac{1}{C_i} \\
    I &= \frac{dQ}{dt} \\
    U_C &= \frac{1}{2}Q\Delta V = \frac{1}{2} C(\Delta V)^2 \\
    d\ddot{B} &= \frac{\mu_0}{4\pi} \frac{I d\ddot{r} \times \ddot{\hat{r}}}{r^2} \\
    R &= \frac{\rho\ell}{A} \\
    \ddot{E} &= \rho\ddot{J} \\
    I &= Ne\nu_d A \\
    \Phi_B &= \int \ddot{B} \cdot d\ddot{A} \\
    I &= \frac{\Delta V}{R} \\
    \Phi_B &= \int \ddot{B} \cdot d\ddot{A} \\
    \frac{1}{R_p} &= \sum \frac{1}{R_i} \\
    U_L &= \frac{1}{2} LI^2 \\
    P &= I\Delta V
\end{align*}
\]
GEOMETRY AND TRIGONOMETRY

Rectangle

\[ A = bh \]

Triangle

\[ A = \frac{1}{2}bh \]

Circle

\[ A = \pi r^2 \]
\[ C = 2\pi r \]
\[ s = r\theta \]

Rectangular Solid

\[ V = \ell \omega h \]

Cylinder

\[ V = \pi r^2 \ell \]
\[ S = 2\pi r \ell + 2\pi r^2 \]

Sphere

\[ V = \frac{4}{3}\pi r^3 \]
\[ S = 4\pi r^2 \]

Right Triangle

\[ a^2 + b^2 = c^2 \]
\[ \sin \theta = \frac{a}{c} \]
\[ \cos \theta = \frac{b}{c} \]
\[ \tan \theta = \frac{a}{b} \]

CALCULUS

\[ \frac{df}{dx} = \frac{df}{du} \frac{du}{dx} \]
\[ \frac{d}{dx} (x^n) = nx^{n-1} \]
\[ \frac{d}{dx} (e^{ax}) = ae^{ax} \]
\[ \frac{d}{dx} (\ln ax) = \frac{1}{x} \]
\[ \frac{d}{dx} [\sin (ax)] = a\cos(ax) \]
\[ \frac{d}{dx} [\cos (ax)] = -a\sin(ax) \]
\[ \int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1 \]
\[ \int e^{ax} \, dx = \frac{1}{a} e^{ax} \]
\[ \int \frac{dx}{x + a} = \ln |x + a| \]
\[ \int \cos (ax) \, dx = \frac{1}{a} \sin (ax) \]
\[ \int \sin (ax) \, dx = -\frac{1}{a} \cos (ax) \]

VECTOR PRODUCTS

\[ \vec{A} \cdot \vec{B} = AB \cos \theta \]
\[ |\vec{A} \times \vec{B}| = AB \sin \theta \]
1. Students design an experiment using blocks of adjustable mass to investigate friction using the setup shown. Block 1 of initial mass 0.44 kg is placed on a rough horizontal surface and connected by a string to block 2 of initial mass 0.20 kg. The string extends over a pulley that has negligible mass and friction.

   (a) Calculate the minimum value of the coefficient of static friction \( \mu_s \) that would keep the two-block system at rest.
The coefficient of friction is such that when block 2 is released from rest, block 1 travels across the surface. The acceleration \( a \) of each block is recorded with motion detectors 1 and 2, as shown in the figure. The data for the motion detectors as functions of time \( t \) are shown on the graphs. For each motion detector, the positive direction is away from the detector.

(b) On the dots below, which represent the blocks, draw and label the forces (not components) that act on each block. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

(c) Calculate the coefficient of kinetic friction \( \mu_k \) between block 1 and the table.
Continue your response to QUESTION 1 on this page.

(d) Careful measurements determine that the coefficient of kinetic friction is larger than the value calculated in part (c). Does the following explanation sufficiently account for the observed discrepancy?

“The horizontal table was not perfectly level before the experiment was conducted. The observed difference in the angle accounts for the difference in the expected and calculated values of \( \mu_k \).”

Yes \( \quad \) No

Justify your answer.

The experiment is moved to a surface with negligible friction and run for eight trials. In each trial, the students vary the masses \( m_1 \) and \( m_2 \) of blocks 1 and 2, respectively, while keeping the total mass \( (m_1 + m_2) = 0.64 \text{ kg} \) constant. The data for the acceleration \( a \) of block 1 as a function of \( m_2 \) are shown on the graph below.

![Graph](image)

(e)

i. Draw a best-fit line for the data points.

ii. Using the straight line, calculate an experimental value for the acceleration due to gravity \( g \).
(f) The students lift the left end of the surface so that the surface is inclined at an angle to the horizontal, and the experiment for $m_2 = 0.20\, \text{kg}$ is repeated. Would the acceleration of the system be greater than, less than, or equal to the acceleration of the system in the original experiment?

_____ Greater than  _____ Less than  _____ Equal to

Justify your claim.
2. Object A is a long, thin, uniform rod of mass $M$ and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(a) Using integral calculus, derive an expression to show that the rotational inertia $I_A$ of object A about the pivot is given by $\frac{4}{3} ML^2$.

Object B of total mass $M$ is formed by attaching two thin, uniform, identical rods of length $L$ at a right angle to each other. Object B is held in place, as shown above. Express your answers in part (b) in terms of $L$.

(b) Determine the following for the given coordinate system shown in the figure.

i. The $x$-coordinate of the center of mass of object B

ii. The $y$-coordinate of the center of mass of object B

Use a pencil or pen with black or dark blue ink only. Do NOT write your name. Do NOT write outside the box.

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Object B has a rotational inertia of $I_B$ about its pivot.

(c) Is the value of $I_B$ greater than, less than, or equal to $I_A$?

_____ Greater than  _____ Less than  _____ Equal to

Justify your answer.

Object B is released from rest and begins to rotate about its pivot.

(d) On the axes below, sketch graphs of the magnitude of the angular acceleration $\alpha$ and the angular speed $\omega$ of object B as functions of time $t$ from the time it is released to the time its center of mass reaches its lowest point.
(e) While object B rotates from the horizontal position down through the angle \( \theta \) shown above, is the magnitude of its angular acceleration increasing, decreasing, or not changing?

____ Increasing  ____ Decreasing  ____ Not changing

Justify your answer.

Object B rotates through the position shown above.

(f) Derive an expression for the angular speed of object B when it is in the position shown above. Express your answer in terms of \( M, L, I_B \), and physical constants, as appropriate.
3. A block of mass $m$ is placed on top of an ideal spring of spring constant $k$. The block is pushed against the spring, compressing the spring a distance $\Delta x$. The block is released from rest, leaves the spring at the position shown in the figure, travels upward, and enters a track with a constant radius of curvature $R$ that has negligible friction. The block enters the track at point A, maintains contact with the track, and exits horizontally at point B, a distance $3R$ above the point the block was released. The block then falls to the ground and lands a horizontal distance $D$ from the end of the track. Express all algebraic answers in terms of $m$, $k$, $\Delta x$, $R$, and physical constants, as appropriate. The size of the block is much smaller than the radius of curvature of the track.

(a) On the dot below, which represents the block, draw and label the forces (not components) that act on the block while still in contact with the track at point B. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

Justify your choice of vectors.

GO ON TO THE NEXT PAGE.
Continue your response to QUESTION 3 on this page.

(b)

i. Derive an expression for the speed $v$ of the block at point B.

ii. Derive an expression for the magnitude of the net force $F$ on the block at point B.

(c) Derive an expression for the minimum value of $\Delta x_{\text{min}}$ required in order for the block to maintain contact with the track through point B.

The procedure is repeated several times with the distance $\Delta x > \Delta x_{\text{min}}$.

(d) Calculate the distance $D$ that the block travels.
(e) The graph below shows the best-fit line drawn by the students through their data of $D$ as a function of $\Delta x$.

\[ D \]
\[ O \]
\[ \Delta x_{\text{min}} \]

\[ D_{\text{MIN}} \]

\[ \Delta x \]

i. Explain why there are no data for section I of the graph.

ii. Explain the reason for the shape and minimum value of section II on the graph.
STOP

END OF EXAM