2021



AP[°] Statistics Sample Student Responses

and Scoring Commentary

Inside:

Free Response Question 4

- **☑** Scoring Guideline
- ☑ Student Samples
- **☑** Scoring Commentary

© 2021 College Board. College Board, Advanced Placement, AP, AP Central, and the acorn logo are registered trademarks of College Board. Visit College Board on the web: collegeboard.org. AP Central is the official online home for the AP Program: apcentral.collegeboard.org.

Question 4: Focus on Inference

General Scoring Notes

- This question is scored in four sections. Each section is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The first section includes statements of the null and alternative hypotheses and identification of the appropriate hypothesis test in part (a). The second section includes verifying the conditions for the test identified in part (a) and calculating the value of the test statistic and the corresponding *p*-value. The third section includes the conclusion for the test identified in part (a). The response is then categorized based on the scores assigned to each section and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each section of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

	Model Solution	Scoring	
(a) Section 1	Let p represent the proportion of all customers of the pet supply company who would place an order within 30 days after receiving an e-mail with a coupon for \$10 off the next purchase.	 Essentially correct (E) if the response satisfies the following three components: States the correct equality for the null hypothesis for a proportion (e.g., p = 0.40) 	
	The null hypothesis is $H_0: p = 0.40$, and the alternative hypothesis is $H_a: p > 0.40$.	AND the correct direction of the one-sided alternative hypothesis for a proportion (e.g., p > 0.40)	
	An appropriate test is a one-sample <i>z</i> -test for a population proportion.	2. Provides sufficient context for the parameter by including reference to the <i>population</i> proportion AND the sampling units (customers) AND the response variable (placing an order after receiving a coupon)	
		 Identifies a one-sample <i>z</i>-test for a population proportion by name (e.g., "one-proportion <i>z</i>-test" but <u>not</u> merely "one-sample <i>z</i>-test") or by formula 	
		Partially correct (P) if the response does not meet the criteria for E but satisfies either component 1 and/or component 3.Incorrect (I) if the response does not meet the criteria for E or P.	

Additional Notes:

- The elements of component 2 do not have to be satisfied with the statement of the hypotheses. They may be satisfied by work presented anywhere in the response, most likely by the statement of the conclusion.
- If the statement of the hypotheses refers to population proportion and the conclusion refers to sample proportion (or vice versa), then the population aspect of component 2 is not satisfied.

- A response that states the null hypothesis as $H_0: p \le 0.40$ may satisfy component 1.
- To satisfy component 1, the hypotheses must be stated in terms of a proportion. If a symbol other than p or π is used to denote the proportion, it must be clearly defined as a proportion (but does not need to reflect the context of customers who would place an order within 30 days after receiving a coupon) in order for the response to satisfy component 1. It is acceptable to use " p_0 " to denote the proportion.
- A response that states the hypotheses in words (e.g., "the null hypothesis is that the proportion is 0.40, and the alternative hypothesis is that the proportion is greater than 0.40") may satisfy component 1. Neither context nor the concept of the *population* is required to satisfy component 1.
- A response that states the hypotheses in words (e.g., "the null hypothesis is that the proportion of all customers who would place an order within 30 days after receiving a coupon is equal to 0.40, and the alternative hypothesis is that the proportion is greater than 0.40") may satisfy component 1 and component 2.
- If the response clearly refers to the *sample* proportion instead of the *population* proportion using words or a symbol (e.g., \hat{p}), then component 2 is not satisfied unless the symbol used is defined as the *population* proportion.
- A response may satisfy the population aspect of component 2 by doing the following:
 - o referring to population in the statement of the conclusion of the inferential procedure
 - \circ using notation such as p, p_0 , or π when defining the hypothesis statements.
- A response may satisfy the sampling units aspect of component 2 by referring to "people who place an order" or similar statement.
- If the response identifies the correct test by name, but also states an incorrect formula, then component 3 is not satisfied.
- If the response identifies the test by formula using a *t*-percentile instead of a *z*-percentile, then component 3 is not satisfied.

Confidence Interval Approach:

- If a one-sample *z*-interval for a population proportion is identified correctly by name (e.g., "one-proportion *z*-interval" but <u>not</u> merely "one-sample *z*-interval") or by formula, then component 3 is satisfied.
- If a response uses a one-sample *z*-interval for a population proportion, then component 2 is satisfied if the response indicates that it is a confidence interval for the proportion of all customers who would place an order within 30 days after receiving a coupon, even if the hypotheses are not stated.

	Model Solution	Scoring	
(a) Section 2	The independent observations condition for performing the one-sample <i>z</i> -test for a population proportion is satisfied because the data were obtained from a random sample of 90 customers who placed an order in the past year and, because sampling of customers is done without replacement, it is assumed that this large online company has more than 10(90) = 900 customers. The sample size is large enough to support an assumption that the sampling distribution of \hat{p} is approximately normal because (90)(0.4) = 36 and $(90)(1 - 0.4) = 54$ are both at least 10. The value of the sample proportion is $\hat{p} = \frac{38}{90} \approx 0.422$ and the value of the test	 Essentially correct (E) if the response satisfies the following four components: 1. Checks the independence condition by referring to the random selection of 90 customers AND indicating that the company is assumed to have at least 900 customers (i.e., 90 ≤ 0.10N) 2. Checks that the sample size is large enough to support the assumption that the sampling distribution of p̂ is approximately normal by verifying that (90)(0.4) and (90)(1 – 0.4) are both at least 10 (or 5) 3. Correctly reports the value of the z-statistic 4. Correctly reports the p-value, consistent with the reported test statistic and stated alternative hypothesis 	
	statistic is $z = \frac{\frac{38}{90} - 0.40}{\sqrt{\frac{(0.40)(0.60)}{90}}} \approx 0.430$. The corresponding <i>p</i> -value is $P(z > 0.430) \approx 0.333$.	Partially correct (P) if the response satisfies only two or three of the four components.Incorrect (I) if the response does not meet the criteria for E or P.	
Addition	al Natas.		

Additional Notes:

- In order to satisfy the reference to the random selection of 90 customers in component 1 it is minimally acceptable to state "random sample check" or "SRS check." However, component 1 is not satisfied if the response implies that random *assignment* was used or only states "random check."
- In order to satisfy component 2, the response must include actual values of the observed successes and failures, or values for the expected successes and failures, or formulas for the expected number of successes and failures with values inserted AND the response must make a comparison of the two values with some standard criterion, such as 5 or 10. If expressions such as (90)(0.4) and (90)(1 0.4) are used, simplification is not required.
 - Examples of acceptable quantities (comparisons must still be made):
 - 38 and 52 (observed counts)
 - 36 and 54 (expected counts under the null hypothesis)
 - (90)(0.4) and (90)(1-0.4)
 - (90)(0.4222) and (90)(1-0.422)
 - Unless values of all parameters are explicitly defined in the response, the following quantities are unacceptable:
 - 90p, 90(1-p), np, n(1-p)
 - $90\hat{p}, 90(1-\hat{p}), n\hat{p}, n(1-\hat{p})$
 - n(0.4), n(0.6), n(0.4222), n(1 0.4222)

- When computing the test statistic, using a \hat{p} of 0.42 in the numerator results in a test statistic equal to 0.39 with a *p*-value of 0.36. These values satisfy components 3 and 4.
- If the response uses \hat{p} in the null standard error formula to calculate the *z*-statistic, component 3 may be satisfied.
- A response that reports the correct value for the *z*-statistic but contains errors in supporting work may still satisfy component 3.
- If the response satisfies component 4, any supporting work for the *p*-value may be treated as extraneous.
- If the response compares the value of the test statistic to a critical value instead of reporting a *p*-value, then the critical value (1.645), or a critical value consistent with the stated alternative hypothesis, satisfies component 4.
- If a two-tailed alternative hypothesis is stated or the direction of the stated one-tailed alternative hypothesis is incorrect, then the *p*-value must be consistent with the stated alternative hypothesis to satisfy component 4.
- If the response omits identifying the hypotheses, the correct one-sided alternative hypothesis is assumed when scoring component 4.
- If an incorrect alternative hypothesis is stated, then the *p*-value must be consistent with the stated alternative hypothesis to satisfy component 4.

Confidence Interval Approach:

- If either a one-sided 95 percent confidence interval is correctly calculated as (0.337, 1), or a two-sided 90 percent confidence interval is correctly calculated as (0.337, 0.508), then component 3 is satisfied.
- If only the lower end of a confidence interval is used to reach a conclusion, then component 4 is satisfied.
- Application of a confidence interval approach must be consistent with the stated alternative to satisfy component 4. A two-sided 95 percent confidence interval is (0.320, 0.524), and a lower one-sided 95 percent confidence interval is (0, 0.508).

	Model Solution	Scoring	
(a) Section 3	Because the <i>p</i> -value is greater than $\alpha = 0.05$, the null hypothesis should not be rejected. The results from this study do not provide convincing statistical evidence that the manager's belief is correct. That is, there is not convincing statistical evidence that more than 40 percent of all customers of the pet supply company would place an order within 30 days after receiving an e-mail with a coupon for \$10 off the next purchase.	 Essentially correct (E) if the response satisfies the following two components: 1. Provides a correct justification of the conclusion based on whether the <i>p</i>-value is less than α = 0.05 (or a comparison of the value of the test statistic to an appropriate critical value, e.g., z < -1.645) 2. States a correct conclusion consistent with the stated alternative hypothesis OR states a conclusion that answers the inference question (e.g., states the conclusion in terms of the manager's belief) 	
		Partially correct (P) if the response satisfies only one of the two components.	
		Incorrect (I) if the response does not meet the criteria for E or P.	

- Although including proper context (the concept of population proportion and referencing the response variable) is important in stating the conclusion, context displayed in stating the conclusion is considered in scoring component 2 of Section 1.
- The response need not make an explicit decision about the null hypothesis (reject H₀ or fail to reject H₀) in order to satisfy component 1. However, if an incorrect decision is stated, then component 1 is not satisfied.
- If the conclusion and justification are consistent with an incorrect *p*-value (or an incorrect value of the test statistic, or an incorrect confidence interval), the response may satisfy component 1 and component 2.
- If the response omits hypotheses, assume the correct alternative hypothesis, $H_a: p > 0.40$, was provided when scoring component 1 and component 2.
- If the conclusion includes a definitive statement (e.g., "this proves that we do not have enough evidence to claim..." or "there is no evidence..."), then component 2 is not satisfied.
- If the response includes a statement that is equivalent to accepting the null hypothesis (e.g., "we conclude that the proportion of customers who will place an order is 0.40"), then component 2 is not satisfied.
- If the response includes an incorrect interpretation of the *p*-value, then the score for Section 3 is lowered one level (that is, from E to P or from P to I).
- The clarity and quality of the statement of the conclusion and the statement of the justification may be used in a holistic approach to decide whether to score up or down (e.g., raising a score of 2.5 to 3 or reducing a score of 2.5 to 2).

Confidence Interval Approach:

• If the alternative hypothesis is specified correctly as $H_a: p > 0.40$, then component 1 is satisfied if the justification is based on whether 0.40 is below the lower end of the confidence interval. If the alternative hypothesis is stated in the wrong direction, then component 1 is satisfied if the justification is based on whether 0.40 is above the upper end of the confidence interval.

- If no alternative hypothesis is specified in the response, then assume the correct alternative hypothesis is provided when scoring component 2.
- If an incorrect two-sided alternative hypothesis is specified, then component 2 is satisfied if the justification is based on whether 0.40 is included in the confidence interval.
- If the response includes an incorrect interpretation of the confidence interval, then the score for Section 3 is lowered one level (that is, from E to P or from P to I).

	Model Solution	Scoring
(b) Section 4	Because the null hypothesis was not rejected in part (a), a Type II error could have been made. A Type II error occurs when the null hypothesis is false and is not rejected. In this case, a Type II error is made by failing to reject the null hypothesis that 40 percent (or less) of all customers of the pet supply company would place an order within 30 days after receiving an e-mail with a coupon for \$10 off the next purchase, when in fact, more than 40 percent would do so. Consequently, the manager may decide not to use the coupon promotion when it actually would result in more than 40 percent of their customers making a purchase within 30 days.	 Essentially correct (E) if the response satisfies the following two components: States that a Type II error could have been made Provides a reasonable interpretation of the consequence of the stated error AND uses sufficient context, by including at minimum "those who would place an order" or "coupon" or "sales" (e.g., indicating the manager may not use the coupon promotion when it would actually lead to more than 40% of customers placing an order) Partially correct (P) if the response satisfies only one of the two components.
		Incorrect (I) if the response does not meet the criteria for E or P.
Addition	al Notes:	

- If the response to part (a) rejects the null hypothesis, then •
 - Component 1 is satisfied if the response states that a Type I error could have been made. 0
 - Component 2 is satisfied if the response provides a reasonable interpretation of the consequence of a Type I error AND uses sufficient context by including at minimum "those who would place an order" or "coupon" or "sales."
- If the response states that a Type II error could have been made, followed by an incorrect description of a ٠ Type II error (e.g., "did not find convincing evidence that more than 40% of customers will place an order when there actually was evidence of more than 40%"), component 1 is not satisfied.
- The clarity and quality of the statement of the consequence may be used in a holistic approach to decide ٠ whether to score up or down (e.g., raising a score of 2.5 to 3 or reducing a score of 2.5 to 2).

Scoring for Question 4

Each essentially correct (E) pa	rt counts as 1 point, a	and each partially correct	(P)	part counts as $\frac{1}{2}$ point.

	Score
Complete Response	4
Substantial Response	3
Developing Response	2
Minimal Response	1

If a response is between two scores (for example, 2 ½ points), use a holistic approach to decide whether to score up or down, depending on the strength of the response and quality of the communication.

Begin your response to **QUESTION 4** on this page.

4. The manager of a large company that sells pet supplies online wants to increase sales by encouraging repeat purchases. The manager believes that if past customers are offered \$10 off their next purchase, more than 40 percent of them will place an order. To investigate the belief, 90 customers who placed an order in the past year are selected a random. Each of the selected customers is sent an e-mail with a coupon for \$10 off the next purchase if the order is placed within 30 days. Of those who receive the coupon, 38 place an order. (a) Is there convincing statistical evidence, at the significance level of $\alpha = 0.05$, that the manager's belief is correct? Complete the appropriate inference procedure to support your answer. x = 38 2 - prop - 1 sample or = .05 0 = 90State: Ho: P=.40 H. P. 40 P: The true proportion of port customers who place an order after being offered a \$\$ \$10 coupon Plan: randomization : SRS J independence: Assume that the company has at least 900 customers who have placed an order with the past year normality: np 210 n(1-p)210 The distribution st q0(.4)2101 90(.6)2101 approximately normal Do: 1 prop 2 tent prop > . 4 the true propertion of past customers who placed 2=.43 their next purchase is greater than .4 D= .333 8=.422 0= 90 Canclude: Since our p-value (.32) is greater than d(.05), we fail to reject have convincing evidence that. Ho. We do not GO ON TO THE NEXT PAGE. Unauthorized copying or reuse of this page is illegal. Page 8

Continue your response to **QUESTION 4** on this page. (b) Based on your conclusion from part (a), which of the two errors, Type I or Type II, could have been made? Interpret the consequence of the error in context. type 11 error call have been made which is when A you fail to reject the Ho even though the Ha was is correct. If the accured, the manager would decide to not offer the \$10 coupon because it is believed that it increase sales enough when in reality Joesnt me sally sue coupon led to greatter than yose of these being offered the on order, resulting in a missed opportunity for it placing Sales. GO ON TO THE NEXT PAGE Unauthorized copying or reuse of this page is illegal. Page 9

Begin your response to **QUESTION 4** on this page.

- 4. The manager of a large company that sells pet supplies online wants to increase sales by encouraging repeat purchases. The manager believes that if past customers are offered \$10 off their next purchase, more than 40 percent of them will place an order. To investigate the belief, 90 customers who placed an order in the past year are selected at random. Each of the selected customers is sent an e-mail with a coupon for \$10 off the next purchase if the order is placed within 30 days. Of those who receive the coupon, 38 place an order.
 - (a) Is there convincing statistical evidence, at the significance level of $\alpha = 0.05$, that the manager's belief is correct? Complete the appropriate inference procedure to support your answer.

Ho:
$$p \le .4$$

Ha: $p \ge .4$
Ha: $p \ge 0.4$
Ha: $p \ge 0.$

Continue your response to **QUESTION 4** on this page. (b) Based on your conclusion from part (a), which of the two errors, Type I or Type II, could have been made? Interpret the consequence of the error in context. Il we fail to reject the null hypothesis; then we could have made a Type llerror. we would have incorrectly assumed that free customers would place orders if they were rewarded for coming back, which could have costed the business a lot of money in orderst customers. Eventually, they would start to lose money and possibly go bankerupt, if they didn't introduce a new or other kind of rewards system to keep their customers roming back. GO ON TO THE NEXT PAGE Unauthorized copying or reuse of this page is illegal Page 9

Begin your response to **QUESTION 4** on this page.

- 4. The manager of a large company that sells pet supplies online wants to increase sales by encouraging repeat purchases. The manager believes that if past customers are offered \$10 off their next purchase, more than 40 percent of them will place an order. To investigate the belief, 90 customers who placed an order in the past year are selected at random. Each of the selected customers is sent an e-mail with a coupon for \$10 off the next purchase if the order is placed within 30 days. Of those who receive the coupon, 38 place an order.
 - (a) Is there convincing statistical evidence, at the significance level of $\alpha = 0.05$, that the manager's belief is correct? Complete the appropriate inference procedure to support your answer.

Ho : if given coupon, customer order rate : 40%
Ha ; if given coupon, customer order rate 240%
1 prop Z test
• random sample used
• 40 customers likely & 10% all customers last year
-> assume independent
• 0.4.40: 36 210 0.6.90:54210
nP, ng 200
=> assume normal
$$P(z=0.9303): 0.333$$

 $Z = \frac{38.36}{\sqrt{0.42.0.58}} : 0.4303$
No, there is not convincing statistical
evidence at the significance level of dico.05
associate p-value = 0.333. Therefore we fail to
reject the null hypothesis that when given
a caupon, the customer 6' order rate will
incredede.
Unsumored opping or nume of this page is lingal.
Page B GO ON TO THE NEXT PAGE.

Sample 4C, pg 2 of 2

Continue your response to QUESTION 4 on this page. (b) Based on your conclusion from part (a), which of the two errors, Type I or Type II, could have been made? Interpret the consequence of the error in context. A Type II error could have been made, as we failed to reject the null hypothesis. This could result in as believing that & coupon would had increase the castomers' order rate, when it in fact may. 13 M. GO ON TO THE NEXT PAGE. Unauthorized copying or reuse of this page is illegal. Page 9

Question 4

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The primary goals of this question were to assess a student's ability to (1) identify an appropriate inference procedure to test a claim about a population proportion; (2) identify the appropriate null hypothesis and the appropriate alternative hypothesis; (3) check conditions required for accurate application of the identified inference procedure; (4) compute the value of a test statistic and the corresponding *p*-value; (5) state and justify a conclusion about the claim; and (6) determine whether a Type I or Type II error could have been made and describe a consequence of the identified type of error.

This question primarily assesses skills associated with inference, including skills in skill category 1: Selecting Statistical Methods; skill category 3: Using Probability and Simulation; and skill category 4: Statistical Argumentation. Skills required for responding to this question include (1.B) Identify key and relevant information to answer a question or solve a problem, (1.E) Identify an appropriate inference method for significance tests, (1.F) Identify null and alternative hypotheses, (3.E) Calculate a test statistic and find a *p*-value, provided conditions for inference are met, (4.A) Make an appropriate claim or draw an appropriate conclusion, (4.C) Verify that inference procedures apply in a given situation, and (4.E) Justify a claim using a decision based on significance tests.

This question covers content from Unit 6: Inference for Categorical Data: Proportions of the course framework in the AP Statistics Course and Exam Description. Refer to topics 6.4, 6.5, 6.6, and 6.7, and learning objectives DAT-3.B, UNC-5.A, VAR-6.D, VAR-6.E, VAR-6.F, and VAR-6.G.

Sample: 4A Score: 4

The response earned the following: Section 1 - E; Section 2 - E; Section 3 - E; Section 4 - E.

In section 1, the response satisfies all three components. The response satisfies component 1 by correctly identifying the hypothesis statements. The response satisfies component 2 by stating, "the true proportion of past customers who placed an order after being offered a \$10 coupon." The words "true proportion" specify the population proportion, but the response also does this by referring to p in the hypothesis statements. The response satisfies component 3 because the response identifies the correct hypothesis test by name, "1 prop z test." Section 1 was scored essentially correct (E).

In section 2 the response satisfies all four components. For component 1 the response correctly states "SRS" and "Assume that the company has at least 900 customers." For component 2 the response correctly compares "90(.4)" and "90(.6)" to 10. For components 3 and 4 the response reports the correct values of the *z*-statistic and *p*-value, "z = .43" and "p = .333," respectively. Section 2 was scored essentially correct (E).

In section 3 the response satisfies both components. For component 1 the response correctly compares the *p*-value to α by stating "our *p*-value (.33) is greater than α (.05)." For component 2 the response provides a correct conclusion by stating, "We do not have convincing evidence that the true proportion of past customers who placed an order after being offered \$10 off their next purchase is greater than .4." Section 3 was scored essentially correct (E).

Question 4 (continued)

In section 4 the response satisfies both components. For component 1 the response correctly states a "type II error could have been made." For component 2 the response provides a reasonable interpretation of the consequence of the stated error with sufficient context in two ways. First, the response states, "If this occured, the manager would decide to not offer the \$10 coupon because it is believed that it doesn't increase sales enough when in reality the coupon led to greater than 40% of those being offered it placing an order." Second the response states there was a "missed opportunity for sales," which alone also satisfies component 2. Section 4 was scored essentially correct (E).

Sample: 4B Score: 3

The response earned the following: Section 1 - P; Section 2 - P; Section 3 - E; Section 4 - E.

In section 1 the response satisfies component 1 by specifying correct hypotheses for a proportion. Recall the third Additional Note states $H_0: p \le 0.40$ may satisfy component 1. The response does not satisfy component 2 because there is no reference to the coupon. The response satisfies component 3 because the response correctly names the test by using a *z*-statistic formula " $z = \frac{0.422 - 0.40}{\sqrt{\frac{0.4(1 - .4)}{90}}}$." Section 1 was scored partially correct (P).

In section 2 the response does not satisfy component 1 because the response does not correctly check the independence condition. Simply stating "Random?" with a checkmark is not sufficient for the independence condition. The response satisfies component 2 by correctly checking the sample size condition. The response satisfies component 3 because the response reports the correct value of the *z*-statistic. The response does not satisfy component 4 because the reported *p*-value does not match the *z*-statistic and is not consistent with the stated alternative hypothesis. Section 2 was scored partially correct (P).

In section 3 the response satisfies both components by correctly comparing the *p*-value to α and providing a correct conclusion. Remember that context is not required in the conclusion for section 3. Context was required in section 1. Section 3 was scored essentially correct (E).

In section 4 the response satisfies component 1 by correctly indicating a Type II error could have been made. The response satisfies component 2 by providing a reasonable interpretation of the consequence of the stated error with sufficient context: "We would have incorrectly assumed that few customers would place orders if they were rewarded for coming back, which could have costed the business a lot of money in orders & customers." Section 4 was scored essentially correct (E).

Sample: 4C Score: 2

The response earned the following: Section 1 - P; Section 2 - E; Section 3 - I; Section 4 - P.

In section 1 the response satisfies component 1 by specifying correct hypotheses for a proportion as a "rate" and percent. The response does not satisfy component 2 because there is no explicit reference to the population using words or the notation p for the population proportion. The response does not satisfy component 3 because, although the response correctly names the test, the *z*-statistic formula provided is not correct. Section 1 was scored partially correct (P).

Question 4 (continued)

In section 2 the response satisfies components 1 and 2 by correctly checking the conditions. The response reports the correct values of the *z*-statistic and *p*-value, satisfying components 3 and 4. The incorrect formula was taken into account when scoring component 3 in section 1 and does not affect the scoring of component 3 in section 2. Section 2 was scored essentially correct (E).

In section 3 the response does not compare the *p*-value to α , so the response does not satisfy component 1. The response does not satisfy component 2 because the conclusion is incorrect. Section 3 was scored incorrect (I).

In section 4 the response satisfies component 1 by correctly indicating a Type II error could have been made. The response does not satisfy component 2 because the response does not provide a correct consequence of the error. Section 4 was scored partially correct (P).