
AP[®] Statistics

Sample Student Responses and Scoring Commentary

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Free Response Question 1

- Scoring Guideline**
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Question 1: Focus on Exploring Data**4 points****General Scoring Notes**

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

	Model Solution	Scoring
(a)	<p>The five-number summary of the distribution of length of stay is:</p> <p>Minimum = 5 days Lower quartile (Q_1) = 6 days Median = 7 days Upper quartile (Q_3) = 8 days Maximum = 21 days</p>	<p>Essentially correct (E) if the response provides correct values for ALL FIVE of the summary statistics with labels (minimum, lower quartile, median, upper quartile, and maximum).</p> <p>Partially correct (P) if the response provides correct values for only THREE or FOUR of the summary statistics with labels.</p> <p>Incorrect (I) if the response does not meet the criteria for E or P.</p>

Additional Notes:

- Any discussion of the mean, IQR, or the standard deviation of length of stay should be ignored in scoring.
- Inclusion or omission of units of measurement (days) has no bearing on scoring.
- If the response includes exactly 5 unlabeled numbers expressed together as a vertical or horizontal list, interpret the numbers as being labeled as the minimum, lower quartile, median, upper quartile, and maximum, respectively.
- A response that includes only five numbers that are correct values for the five-number summary without providing a complete set of labels or not putting them in an ordered list may be scored P.

Model Solution	Scoring
<p>(b) (i) The patients who stayed for 12 days and 21 days are considered outliers using method A. An outlier using method A is a value greater than $1.5 \times \text{IQR}$ above the third quartile (Q_3) or more than $1.5 \times \text{IQR}$ below the first quartile (Q_1). Because $Q_1 - 1.5 \times \text{IQR} = 6 - 1.5(8 - 6) = 3$, then any values below 3 are considered outliers. There are no such values. Because $Q_3 + 1.5 \times \text{IQR} = 8 + 1.5(8 - 6) = 11$, then any values above 11 are considered outliers.</p> <p>(ii) The patient who stayed for 21 days is the only outlier using method B. An outlier using method B is a value located 2 or more standard deviations above, or below, the mean. Because $\text{Mean} \pm 2 \times \text{SD} = 7.42 \pm 2(2.37)$, then any value that is outside of the interval (2.68, 12.16) is considered an outlier.</p>	<p>Essentially correct (E) if the response satisfies the following four components:</p> <ol style="list-style-type: none"> 1. Correctly identifies the two outliers in part (b-i) as the patients who stayed for 12 days and 21 days 2. Provides a justification for part (b-i) by calculating the lower and upper outlier criteria for the $1.5 \times \text{IQR}$ rule (e.g., “using method A, an outlier is any value below 3 days or above 11 days”) 3. Correctly identifies the one outlier in part (b-ii) as the patient who stayed for 21 days 4. Provides a justification for part (b-ii) by calculating the lower and upper outlier criteria for the 2 standard deviations rule (e.g., “using method B, an outlier is any value below 2.68 days or above 12.16 days”) <p>Partially correct (P) if the response satisfies only two or three of the four components.</p> <p>Incorrect (I) if the response does not meet the criteria for E or P.</p>

Additional Notes:

- A response for part (b-ii) that manually computes the standard deviation as 2.374 and then uses it to construct an interval of (2.672, 12.168) satisfies component 4.
- Component 1 and component 2 are satisfied if the response to part (b-i) uses correct calculations with incorrect values of summary statistics reported in the response to part (a).

Model Solution	Scoring
<p>(c) Quartiles and the IQR are less sensitive to extreme values in strongly skewed distributions than the mean and standard deviation. Relative to the quartiles, the mean is pulled more toward the extreme values in the longer tail of a strongly skewed distribution.</p> <p>For a distribution that is strongly skewed to the right, the sample mean will be pulled more toward the extreme values in the longer right tail of the distribution than the sample median, and the ratio of the standard deviation to the IQR will tend to be larger than that for more nearly symmetric distributions. As a result, this pulls the value of the outlier criterion for method B, $\text{Mean} + 2 \times \text{SD}$, more toward the extreme values in the right tail of the distribution than the outlier criterion for method A, $Q_3 + 1.5 \times \text{IQR}$. This decreases the ability of method B to identify outliers relative to method A, which means that method A may identify more outliers than method B for a distribution that is strongly skewed to the right.</p>	<p>Essentially correct (E) if the response satisfies the following two components:</p> <ol style="list-style-type: none">1. Indicates that the mean is pulled more toward the extreme values in the longer right tail for a strongly right-skewed distribution than the quartiles (or median) OR indicates that the ratio of the standard deviation to the IQR tends to be larger for strongly skewed distributions than for more nearly symmetric distributions2. Provides an explanation that links effects of skewness on an increased ability of method A to detect outliers relative to method B (e.g., “the larger shift in the mean relative to the shift in the median (or quartiles) has a greater effect on decreasing the ability of method B to detect outliers compared to method A” OR “the larger increase in the standard deviation, relative to the IQR, results in a greater increase in the range of non-outlier values for method B compared to method A”) <p>Partially correct (P) if the response satisfies only one of the two components.</p> <p>Incorrect (I) if the response does not meet the criteria for E or P.</p>

Scoring for Question 1	Score
Complete Response Three parts essentially correct	4
Substantial Response Two parts essentially correct and one part partially correct	3
Developing Response Two parts essentially correct and no part partially correct <i>OR</i> One part essentially correct and one or two parts partially correct <i>OR</i> Three parts partially correct	2
Minimal Response One part essentially correct and no part partially correct <i>OR</i> No part essentially correct and two parts partially correct	1

Begin your response to **QUESTION 1** on this page.

**STATISTICS
SECTION II**

Total Time—1 hour and 30 minutes

6 Questions

Part A

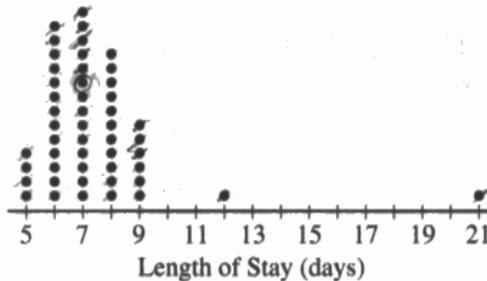
Questions 1-5

Spend about 1 hour and 5 minutes on this part of the exam.

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

- The length of stay in a hospital after receiving a particular treatment is of interest to the patient, the hospital, and insurance providers. Of particular interest are unusually short or long lengths of stay. A random sample of 50 patients who received the treatment was selected, and the length of stay, in number of days, was recorded for each patient. The results are summarized in the following table and are shown in the dotplot.

Length of stay (days)	5	6	7	8	9	12	21
Number of patients	4	13	14	11	6	1	1



- Determine the five-number summary of the distribution of length of stay.

Minimum - 5 days Q_1 - 6 days
 Maximum - 21 days
 Median (Q_2) - 7 days Q_3 - 8 days

Continue your response to **QUESTION 1** on this page.

(b) Consider two rules for identifying outliers, method A and method B. Let method A represent the $1.5 \times \text{IQR}$ rule, and let method B represent the 2 standard deviations rule.

(i) Using method A, determine any data points that are potential outliers in the distribution of length of stay. Justify your answer.

$$\text{IQR} = (8 - 6) = 2 \quad \text{lower } Q_1 - 1.5 \text{IQR} \rightarrow 6 - 1.5(2) = 3 \text{ days}$$

$$\text{upper } Q_3 + 1.5 \text{IQR} \rightarrow 8 + 1.5(2) = 11 \text{ day}$$

Since any data point below 3 days or above 11 days is an outlier the data points at 12 days and 21 days are outliers in distribution of length of stay.

(ii) The mean length of stay for the sample is 7.42 days with a standard deviation of 2.37 days. Using method B, determine any data points that are potential outliers in the distribution of length of stay. Justify your answer.

$$2 \text{ standard deviations away}$$

$$\text{upper } 7.42 + 2(2.37) = 12.16 \text{ days}$$

$$\text{lower } 7.42 - 2(2.37) = 2.68 \text{ days}$$

Since any data point below 2.68 day or above 12.16 days is an outlier the data point at 21 days is definitely an outlier in the distribution of length of stay.

(c) Explain why method A might identify more data points as potential outliers than method B for a distribution that is strongly skewed to the right.

Method A is based of of median and IQR which are resistant to outliers and more resistant to skew than mean or standard deviation. Since mean follows the skew method B will encompass more of the graph in the non-outlier range than method A that will not follow skew.

Begin your response to **QUESTION 1** on this page.

**STATISTICS
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6 Questions

Part A

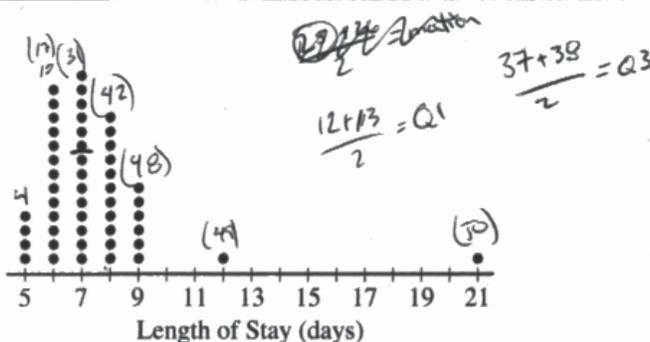
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- The length of stay in a hospital after receiving a particular treatment is of interest to the patient, the hospital, and insurance providers. Of particular interest are unusually short or long lengths of stay. A random sample of 50 patients who received the treatment was selected, and the length of stay, in number of days, was recorded for each patient. The results are summarized in the following table and are shown in the dotplot.

Length of stay (days)	5	6	7	8	9	12	21
Number of patients	4	13	14	11	6	1	1



- Determine the five-number summary of the distribution of length of stay.

minimum = 5 days
 $Q_1 = 6$
~~mean~~ median = 7
 $Q_3 = 9$
 maximum = 21 days

Continue your response to **QUESTION 1** on this page.

- (b) Consider two rules for identifying outliers, method A and method B. Let method A represent the $1.5 \times \text{IQR}$ rule, and let method B represent the 2 standard deviations rule.

- (i) Using method A, determine any data points that are potential outliers in the distribution of length of stay. Justify your answer.

$$\text{IQR} = 2 \text{ days}$$

$$2 \cdot 1.5 = 3$$

$$7 \pm 3 = (4, 10)$$

Outliers $\rightarrow 12, 21$

- (ii) The mean length of stay for the sample is 7.42 days with a standard deviation of 2.37 days. Using method B, determine any data points that are potential outliers in the distribution of length of stay. Justify your answer.

$$\text{standard deviation} = 2.37 \text{ days}$$

$$2.37 \cdot 2 = 4.74 \text{ days}$$

$$7.42 \pm 4.74 = (2.68, 12.16)$$

The only data point that method B describes as an outlier is @ 21 days

- (c) Explain why method A might identify more data points as potential outliers than method B for a distribution that is strongly skewed to the right.

Method A identified more data points as outliers because the mean is skewed towards the right and the standard deviation is larger than it would be in a normally distributed dotplot.

Begin your response to **QUESTION 1** on this page.

**STATISTICS
SECTION II**

Total Time—1 hour and 30 minutes

6 Questions

Part A

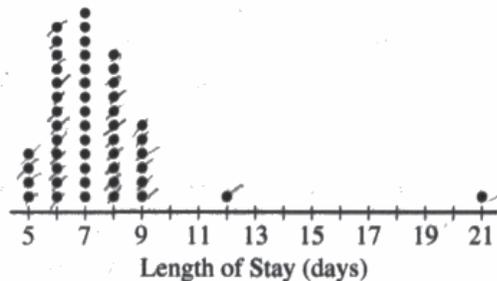
Questions 1-5

Spend about 1 hour and 5 minutes on this part of the exam.

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

- The length of stay in a hospital after receiving a particular treatment is of interest to the patient, the hospital, and insurance providers. Of particular interest are unusually short or long lengths of stay. A random sample of 50 patients who received the treatment was selected, and the length of stay, in number of days, was recorded for each patient. The results are summarized in the following table and are shown in the dotplot.

Length of stay (days)	5	6	7	8	9	12	21
Number of patients	4	13	14	11	6	1	1



- Determine the five-number summary of the distribution of length of stay.

mean: 7.42 days
 min: 5 days
 max: 21 days
 mode: 7 days
 range: 16 days

Continue your response to **QUESTION 1** on this page.

(b) Consider two rules for identifying outliers, method A and method B. Let method A represent the $1.5 \times \text{IQR}$ rule, and let method B represent the 2 standard deviations rule.

(i) Using method A, determine any data points that are potential outliers in the distribution of length of stay. Justify your answer.

$Q_1 = 6$
 $Q_2 = 8$
 $8 - 6 = 2 \leftarrow \text{IQR}$
 median: 7
 $7 - 1.5(2) = 7 - 3 = 4$
 $7 + 1.5(2) = 10$
 $10 < 12$
 upper fence $10 < 21$
 12 and 21 are potential outliers
 no lower outliers

(ii) The mean length of stay for the sample is 7.42 days with a standard deviation of 2.37 days. Using method B, determine any data points that are potential outliers in the distribution of length of stay. Justify your answer.

lower fence: $7.42 - 2(2.37) = 2.68$
 upper fence: $7.42 + 2(2.37) = 12.14$

21 is a potential outlier

(c) Explain why method A might identify more data points as potential outliers than method B for a distribution that is strongly skewed to the right.

Skewed to the right 

If a data set is skewed to the right it will have a small IQR whereas a skewed data set will have a larger standard deviation. The smaller the IQR, the smaller the distance between upper and lower fences leaving more data points to be considered outliers.

large IQR:  ← no outliers
 small IQR:  ← many outliers

Question 1

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The primary goals of this question were to assess a student’s ability to (1) determine values for the five-number summary of data provided in a table and in a dotplot; (2) identify potential outliers using a method based on the five-number summary; (3) identify potential outliers using a method based on the sample mean and standard deviation; and (4) explain why the method based on the five-number summary would tend to identify more potential outliers than the method based on the sample mean and standard deviation for a data sampled from a distribution strongly skewed to the right.

This question primarily assesses skills in skill category 2: Data Analysis. Skills required for responding to this question include (2.C) Calculate summary statistics, relative positions of points within a distribution, correlation, and predicted response, and (4.B) Interpret statistical calculations and findings to assign meaning or assess a claim.

This question covers content from Unit 1: Exploring One-Variable Data of the course framework in the AP Statistics Course and Exam Description. Refer to topic 1.7, and learning objectives UNC-1.I, and UNC-1.K.

Sample: 1A

Score: 4

The response earned the following: Part (a) – E; Part (b) – E; Part (c) – E.

In part (a) the response provides correct values for all five of the summary statistics with labels. Part (a) was scored essentially correct (E).

In parts (b-i) and (b-ii) the response correctly identifies the outliers and provides justification by calculating the upper and lower outlier criteria for each method. Part (b) was scored essentially correct (E).

In part (c) the response states the “mean follows the skew” indicating that the mean is pulled more toward the extreme values and also states that the median is resistant to outliers, satisfying component 1. The response links the effect of skewness on the mean to the increase of the “non-outlier range” for method B as compared to method A, satisfying component 2. Part (c) was scored essentially correct (E).

Sample: 1B

Score: 2

The response earned the following: Part (a) – E; Part (b) – P; Part (c) – I.

In part (a) the response provides correct values for all five of the summary statistics with labels. Part (a) was scored essentially correct (E).

In part (b-i) the response correctly identifies the outliers but does not provide correct justification because the median, rather than the quartiles, is used in the calculation of the upper and lower outlier criteria. Therefore component 1 is satisfied and component 2 is not satisfied. In part (b-ii) the response correctly identifies the outlier and provides justification by calculating the upper and lower outlier criteria. Therefore components 3 and 4 are satisfied. Part (b) was scored partially correct (P).

Question 1 (continued)

In part (c) the response indicates that in a skewed distribution the mean is skewed to the right, and the standard deviation is larger but does not discuss the impact of a skewed distribution on the quartiles, median, or IQR. Therefore component 1 is not satisfied. The response does not link the effects of skewness on the ability of the methods to detect outliers and, therefore, component 2 is not satisfied. Part (c) was scored incorrect (I).

Sample: 1C

Score: 1

The response earned the following: Part (a) – I; Part (b) – P; Part (c) – P.

In part (a) the response provides correct values for only two of the summary statistics with labels, the maximum and the minimum. Note that the value of 7 is labeled as the mode, so no value for the median is provided. Part (a) was scored incorrect (I).

In part (b-i) the response correctly identifies the outliers but does not provide correct justification because the median, rather than the quartiles, is used in the calculation of the upper and lower outlier criteria. Therefore component 1 is satisfied, and component 2 is not satisfied. In part (b-ii) the response correctly identifies the outlier and provides justification by calculating the upper and lower outlier criteria. The response uses a truncated value for the mean, but this does not impact the validity of the justification. Therefore components 3 and 4 are satisfied. Part (b) was scored partially correct (P).

In part (c) the response states “it will have a small IQR whereas a skewed data set will have a larger standard deviation,” satisfying component 1. The response does not provide an explanation that links effects of skewness on an increased ability of method A to detect outliers relative to method B and does not satisfy component 2. Part (c) was scored partially correct (P).