2021

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Sample Student Responses and Scoring Commentary

Inside:

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The Response…				
Score of 1 **Report on Existing Knowledge**	**Score of 2** **Report on Existing Knowledge with Simplistic Use of a Research Method**	**Score of 3** **Ineffectual Argument for a New Understanding**	**Score of 4** **Well-Supported, Articulate Argument Conveying a New Understanding**	**Score of 5** **Rich Analysis of a New Understanding Addressing a Gap in the Research Base**
Presents an overly broad topic of inquiry.	Presents a topic of inquiry with narrowing scope or focus, that is NOT carried through either in the method or in the overall line of reasoning.	Carries the focus or scope of a topic of inquiry through the method **AND** overall line of reasoning, even though the focus or scope might still be narrowing.	Focuses a topic of inquiry with clear and narrow parameters, which are addressed through the method and the conclusion.	Focuses a topic of inquiry with clear and narrow parameters, which are addressed through the method and the conclusion.
Situates a topic of inquiry within a single perspective derived from scholarly works **OR** through a variety of perspectives derived from mostly non-scholarly works.	Situates a topic of inquiry within a single perspective derived from scholarly works **OR** through a variety of perspectives derived from mostly non-scholarly works.	Situates a topic of inquiry within relevant scholarly works of varying perspectives, although connections to some works may be unclear.	Explicitly connects a topic of inquiry to relevant scholarly works of varying perspectives **AND** logically explains how the topic of inquiry addresses a gap.	Explicitly connects a topic of inquiry to relevant scholarly works of varying perspectives **AND** logically explains how the topic of inquiry addresses a gap.
Describes a search and report process.	Describes a nonreplicable research method **OR** provides an oversimplified description of a method, with questionable alignment to the purpose of the inquiry.	Describes a reasonably replicable research method, with questionable alignment to the purpose of the inquiry.	Logically defends the alignment of a detailed, replicable research method to the purpose of the inquiry.	Logically defends the alignment of a detailed, replicable research method to the purpose of the inquiry.
Summarizes or reports existing knowledge in the field of understanding pertaining to the topic of inquiry.	Summarizes or reports existing knowledge in the field of understanding pertaining to the topic of inquiry.	Conveys a new understanding or conclusion, with an underdeveloped line of reasoning **OR** insufficient evidence.	Supports a new understanding or conclusion through a logically organized line of reasoning **AND** sufficient evidence. The limitations and/or implications, if present, of the new understanding or conclusion are oversimplified.	Justifies a new understanding or conclusion through a logical progression of inquiry choices, sufficient evidence, explanation of the limitations of the conclusion, and an explanation of the implications to the community of practice.
Generally communicates the student's ideas, although errors in grammar, discipline-specific style, and organization distract or confuse the reader.	Generally communicates the student's ideas, although errors in grammar, discipline-specific style, and organization distract or confuse the reader.	Competently communicates the student's ideas, although there may be some errors in grammar, discipline-specific style, and organization.	Competently communicates the student's ideas, although there may be some errors in grammar, discipline-specific style, and organization.	Enhances the communication of the student's ideas through organization, use of design elements, conventions of grammar, style, mechanics, and word precision, with few to no errors.
Cites **AND/OR** attributes sources (in bibliography/ works cited and/or in-text), with multiple errors and/or an inconsistent use of a discipline-specific style.	Cites **AND/OR** attributes sources (in bibliography/ works cited and/or in-text), with multiple errors and/or an inconsistent use of a discipline-specific style.	Cites **AND** attributes sources, using a discipline-specific style (in both bibliography/works cited **AND** in-text), with few errors or inconsistencies.	Cites **AND** attributes sources, with a consistent use of an appropriate discipline-specific style (in both bibliography/works cited **AND** in-text), with few to no errors.	Cites **AND** attributes sources, with a consistent use of an appropriate discipline-specific style (in both bibliography/works cited **AND** in-text), with few to no errors.

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Academic Paper

Overview

This performance task was intended to assess students' ability to conduct scholarly and responsible research and articulate an evidence-based argument that clearly communicates the conclusion, solution, or answer to their stated research question. More specifically, this performance task was intended to assess students' ability to:

- Generate a focused research question that is situated within or connected to a larger scholarly context or community;
- Explore relationships between and among multiple works representing multiple perspectives within the scholarly literature related to the topic of inquiry;
  Articulate what approach, method, or process they have chosen to use to address their research question, why they have chosen that approach to answering their question, and how they employed it;
- Develop and present their own argument, conclusion, or new understanding while acknowledging its limitations and discussing implications;
- Support their conclusion through the compilation, use, and synthesis of relevant and significant evidence generated by their research;
- Use organizational and design elements to effectively convey the paper's message;
- Consistently and accurately cite, attribute, and integrate the knowledge and work of others, while distinguishing between the student's voice and that of others;
- Generate a paper in which word choice and syntax enhance communication by adhering to established conventions of grammar, usage, and mechanics.

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Investigating the Hardness of Water on the Rate of Cooling and its Relation to the Mpemba Effect

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Abstract

A solution of cold water is always expected to freeze faster than a solution of hot water, however, the Mpemba Effect is a rare occurrence in which the opposite occurs. This paper examines potential mechanisms to explain the Mpemba Effect, as well as investigates whether the hardness of water influences the occurrence of the Mpemba Effect. It is found that the hardness of water has an effect on the rate of cooling: as the hardness of the solution increases, so does the rate of cooling. These findings also suggest that increasing the hardness of water will increase the chances of the Mpemba Effect occurring.

Literature Review

The Mpemba Effect is a counterintuitive phenomenon in which hot water freezes faster than cold water. Despite the seemingly backwards definitions, it has been scientifically observed under several experiments, and these experiments will be further discussed. The discovery of the Mpemba Effect dates all the way to famed Greek scientist Aristotle, who asserted in his treatise, *Meteorology*, “The fact that the water has previously been warmed contributes to its freezing quickly: for so it cools sooner. Hence many people, when they want to cool hot water quickly, begin by putting it in the sun” (Aristotle 350 B.C.E). 16th century English philosopher Francis Bacon later wrote in 1620 that “water a little warmed is more easily frozen than that which is quite cold” in his philosophical work, The Novum Organum (Bacon 1902), and Descartes wrote in 1637 about this phenomenon in *Les Meteores* “...we can also see by experiment that water which has been kept hot for a long time freezes faster than any other sort” (Descartes 1965).

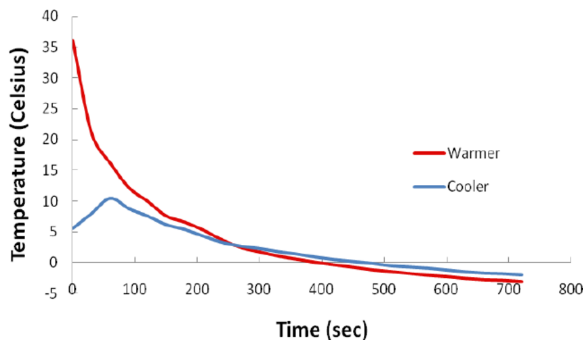

Figure 1. This graph represents an initially warmer solution of water cooling faster than an initially cooler solution of water. Source: Andrew Wang et al.

The idea of the Mpemba Effect became modernized when secondary school student Erasto Mpemba discovered the effect by accident when making ice-cream in 1963. Mpemba was puzzled by the discovery and sought the help of Physics professor, Dennis Osborne. Together Mpmba and Osborne tested the effect and published their results in their paper, “Cool?” (Mpemba and Osborne 1969). Since then there have been numerous scientific investigations and controversy over the cause of the effect, which to this day, still cannot be explained. As Bora Zivkocis, an editor at Scientific American magazine, explains in the 2012 publication, *The Best Science Writing Online:* “Later experiments on the effect have been far less conclusive than Mpemba and Osborne’s.

Some have seen similar results, while others have observed no effect at all...A number of physicists seem skeptical that the effect truly exists" (Zivkovic 2012). The difficulty to replicate the effect, along with the fact that there are no experimentally proven theories to explain the effect, contribute to the great amount of controversy surrounding the effect. However, there are several theoretical explanations to the underlying effects which will be now be discussed:

1. Supercooling "is the process of chilling a liquid below its freezing point, without it becoming solid" (ScienceDaily). J. D.Brownridge, with the Department of Physics at Binghamton University, asserts in his experiment published in the American Journal of Physics that in order for the Mpemba Effect to occur, "the cold water must supercool to a temperature significantly lower than the temperature to which the hot water supercools" (Brownridge 2011). His experiment "observed hot water freezing before cold water 28 times in 28 attempts under the conditions described here" (Brownridge 2011). Supercooling is often used as a proposed mechanism because, as David Auerbach, American writer and Yale graduate, explains, "The hot water supercools, but only slightly, before spontaneously freezing. Superficially it looks completely frozen. The cold water supercools to a lower local temperature than the hot before it spontaneously freezes" (Auerbach 1998).
2. Natural convection is "fundamentally gravity at work—water becomes lighter and rises when heated and then becomes heavier and falls when cooled" (Ingersoll 2016). Convection enables the solution of water to be non-uniform, and because density increases with cooler temperatures, the surface of the water will be warmer at the top. When the warmer water has cooled to the same temperature as the cooler water, the surface of the water will be warmer, and its rate of cooling will be faster (Jeng 1998). Vynnycky, Professor of Applied Mathematics at University of Limerick and Kimura, JSPS research fellow at Tohoku University, experimentally tested this mechanism and found that "whereas natural convection gives the correct general timescale for freezing, supercooling adjusts the actual time required" (Vynnycky and Kimura 2015).
3. Evaporation may contribute to the Mpemba Effect as well, as the warmer solution of water may lose significant amounts of water to evaporation, and the reduced mass will cause the water to cool and freeze faster (Jeng 1998). G. S. Kell with the Division of Chemistry National Research Council of Canada produced his own experiment and described that if faster cooling is solely due to

evaporation, then hot water would lose enough mass to cool faster (Kell 1969). Evaporation is most likely not the single explanation for the Mpemba Effect because Osborne found in his experiments that the amount of water was not sufficient enough for the water to cool faster (Mpemba and Osborne 1969).

4. The surrounding of the container may also influence the Mpemba Effect, as the warmer water may change the environment around it and affect the cooling process (Jeng 2006). As Balazovi, affiliated with the University of Constantinus the Philosopher in Nitra, and Tomasik, affiliated with Matej Bel University, explain, “If the containers are cooled in a freezer with ice coating, the container with hot water can melt ice under itself” which would eliminate the heat faster (Balážovič and Tomášik 2012). However, this incident would only occur if the experiment is being conducted in a freezer with layers of frost.

Note that all proposed solutions would not be the sole reason for the Mpemba Effect to occur, and it could even be a contribution of all factors that lead to its existence. As Vynnycky and Jimura acknowledge, “in combination, the results suggest that, whereas natural convection gives the correct general timescale for freezing, supercooling adjusts the actual time required” (Vynnycky and Kimura 2015). Therefore, there is no one cohesive explanation for the Mpemba Effect, further raising the question as to how much one mechanism may contribute to the Mpemba Effect.

There are several studies worth mentioning that contributed major points of understanding to the topic of inquiry. In 2016, Burridge and Linden of the Imperial College London attempted to reproduce the Mpemba Effect under carefully controlled conditions, but were unable to find any meaningful evidence of its existence. The team started with plotting past experiments' times measuring the Mpemba Effect for a solution to cool to 0°C as well as variations in those times. Through careful examination they found that “majority of the data reported lie below the ‘Mpemba effect line and hence the Mpemba effect was clearly not observed in these cases” (Burridge and Linden 2016). In their own experiment, Burridge and Linden designed an experiment meant to mimic Mpemba and Osborne's experiment, in which three samples of water were placed in glass beakers, boiled, and left to cool at varying times to obtain three different initial temperatures. They were then placed in a freezer and the temperature of each sample was recorded at differing times. Despite their best efforts, they “were not able to make observations of any physical effects which could reasonably be described as the Mpemba effect” (Burridge and Linden 2016).

In another experiment, Lasanta and colleagues experimented with the Mpemba Effect in granular fluids in 2017. Granular matter refers to “systems with a large number of hard objects (grains) of mesoscopic size ranging from millimeters to meters" (Dufty 2007). They found that the Mpemba Effect is indeed present in granular fluids under certain conditions that are uniformly heated and a particle velocity distribution that significantly deviates from the Maxwell-Boltzmann Distribution (Lasanta et al. 2017). This Distribution is defined as the particular "probability distribution used for describing the speeds of various particles within a stationary container at a specific temperature" (DeepAI 2019).

Zhiyue Lu of the University of North Carolina at Chapel Hill and Oren Raz of the Weizmann Institute of Science demonstrated in 2017 the Markovian Mpemba Effect, which "occurs when the distance from equilibrium of the initially hot system is larger than that of the initially cold system, but becomes smaller after a long enough time" (Lu et al. 2017). In this study, Lu and Raz analytically obtained a sufficient condition for the Markovian Mpemba Effect and in addition, predicted an inverse effect in heating where a cold system could heat up faster than a warm system.

Most recently published in the scientific journal, *Nature,* in August 2020, Kumar and Bechhoefer of Simon Fraser University were able to develop a means for demonstrating the Mpemba Effect in a controlled environment. Kumar and Hechhoefer built a colloidal system to experimentally test the Mpemba Effect, which is a "type of mixture in which one part is dispersed constantly throughout another" (Milani and Golkar 2019). The team was able to create a "special combination of experimental parameters...which correspond to exponentially faster cooling" (Kumar and Bechhoefer 2020). This experiment was performed in a highly controlled setting that involved dropping thousands of small glass beads into the beaker using optical tweezers. Zhiyue Lu, as mentioned in the previous study even asserted about the experiment, "This is the first time that an experiment can be claimed as a clean, perfectly controlled experiment that demonstrates this effect" (Conover 2020).

Transitioning from previous works in the field related to the Mpemba Effect to the current inquiry of research, the colligative properties of solutions and mixtures will now be discussed. It is important to discuss properties of mixtures because the research that follows will be using solutions that do not solely contain distilled water but also minerals dissolved in water. Colligative properties are properties of a solution that differ from those of pure solvent and depend on the number of solute molecules, not the nature of the molecules. Such properties may include osmotic pressure, lower vapor pressure, lower melting temperature, and higher boiling temperature (Hammel 1976). In this research, the colligative properties of solutions may influence the time it takes for the solution to cool. A study that investigated the effect of sucrose concentration on equilibrium properties (such as cooling times) found that "Solutes (sucrose) decreased the ice melting temperature," suggesting that colligative properties do indeed affect the process of cooling (Braga and Cunha 2005).

Hardness of water simply refers to the amount of dissolved magnesium and calcium in the water. Harder water has higher amounts of these minerals (USGS). As most research in the field has related to pure solutions of water, the topic of inquiry in this paper will be aiming to study what effect the hardness of water, specifically the amount of dissolved calcium and magnesium, may have on the cooling process and whether this will have any contribution to the Mpemba Effect. It is hypothesized that the harder the water, the greater effect it will have on the occurrence of the Mpemba Effect.

To be more precise when defining the Mpemba Effect in context of the following research, the effect will be described in terms of having three temperatures, $T_h > T_w > T_c$, in which the time it takes a solution of temperature T_h to cool from its initial temperature to 0°C, the freezing point of water, is shorter than the time it takes a separate solution at temperature T_w to also cool to 0°C. T_c represents the temperature of the bath water in which the solution will be placed to freeze.

Since there is currently no accepted theory as to specifically why the Mpemba Effect occurs, it is hoped that the subsequent research will contribute to the understanding through analyzing whether the chemical composition of a solution of hard water has any effect on the Mpemba Effect. This consideration leads to the following question: Does the hardness of water affect the cooling process and the occurrence of the Mpemba Effect and how might this contribute to the field of knowledge about the Mpemba Effect? This inquiry will also answer the broader question of the effect of different solute concentrations, specifically calcium and magnesium, on the rate of cooling.

Method

The Mpemba Effect is an extremely rare occurrence and is considerably harder to reproduce than Mpemba and Osborne's paper would imply because the Mpemba Effect only occurs under extremely specific experimental circumstances (Zivkovic 2012). Therefore, the intentions of this study are not to experimentally observe the effect, but rather to gather further insight into the cooling process and the factors that potentially contribute to inducing the Mpemba Effect.

This study will employ a true experimental method measuring the cooling rates of solutions of varying levels of hardness by manipulating only one single variable per experiment with quantitative results. An experimental method is preferred for this type of study as opposed to other methods such as correlational or comparative as only one variable is manipulated and the effects are analyzed.

The method used in this study is meant to be carried out similarly to the experiments of Mpemba and Osborne. Burridge and Linden also designed an experiment to mimic Mpemba and Osborne's (Burridge and Linden 2016). In Mpemba and Osborne's study, the team tested 70g of water of recently boiled water in 100g beakers frozen in the icebox of a domestic refrigerator and created temperature curves based off of different initial temperatures (Mbempa and Osborne 1969). The beakers were placed on a sheet of polystyrene foam to provide thermal insulation.

The variable that will differ from Mpemba's experiment in this study is the changing concentrations of dissolved calcium (Ca) within the solution. This study will contain 3 sets of different experiments. In the first set of experiments, the initial temperatures will stay constant and the hardness of the water will change; this is meant to investigate any effect hardness of water has on the cooling process. In the second and third set of experiments, the hardness of the water will stay constant and the initial temperatures will vary; these experiments are meant to investigate any occurrence of the Mpemba Effect using concentrations similar to that of tap water.

Calcium carbonate appears as a white, odorless powder or colorless crystals, commonly referred to as limestone or chalk (PubChem). Because calcium carbonate is highly insoluble, laboratory grade calcium nitrate powder, a highly soluble substance, will be used instead and mixed with solutions of distilled water to create the range of different concentrations of calcium ions. A 1.0 M stock solution of calcium nitrate was created by dissolving 164.088 grams of solid calcium nitrate (calcium nitrate's molar mass is 164.088 g/mol) into 1 L of water. This stock solution will be used to create different concentrations of calcium in the experiments.

The time for the solutions to freeze will be quantified as the time it takes for each solution to reach 0°C. Time for phase changes from liquid to solid will not be measured so that supercooling will not be involved in this process, similar to the experiments conducted by Burridge and Linden. Temperatures will be taken using the Vernier Stainless Steel Temperature probe, "a rugged, general-purpose temperature sensor that can be used in organic liquids, salt solutions, acids, and bases" (Vernier). The thermometer connects to LoggerPro, a data-collection and analysis software, to create temperature curves.

For the second and third parts of the experiment, the hardness level of water will remain consistent. Each part of the experiment will be carried out three times to ensure accuracy.

Part 1 of Experimental Process:

1. Fill four 100 mL Pyrex beakers with 10mL solutions of varying concentrations. The first beaker will contain only distilled water as a control. Create a 0.5M, 0.75M, and 1.0M solution mixing distilled water and 1.0M $Ca(NO_3)_2$ stock solution.
2. Place beakers on a sheet of polystyrene foam and place in the freezer, noting the initial room temperature of the solution.
3. Use a Vernier Thermometer connected to LoggerPro to collect data. Continue data collection until all solutions have reached 0°C.

Part 2 and 3 of Experimental Process:

1. Fill three 100mL Pyrex beakers with 10mL solutions (distilled water for Part 2 and 1.0M $Ca(NO_3)_2$ for Part 3) of varying initial temperatures.
2. Heat the first beaker to an initial temperature of 45℃ using a hot plate

covered with parafilm to minimize evaporation.

3. Heat subsequent beakers to initial temperatures of 35℃ and 25℃.
4. Place beakers on a sheet of polystyrene foam and place in the freezer
5. Use a Vernier Thermometer connected to LoggerPro to collect data. Continue data collection until all solutions have reached 0°C.

Results

The results across all three experiments are displayed in the following charts. Each chart includes a cooling curve, averaged across each trial, in time (minutes) versus temperature (℃). Experiment 1- in which only the concentration of calcium nitrate was manipulated - contained three trials per variable, while Experiments 2 and 3 - in which only the initial temperature was manipulated - contained two trials per variable. Experiment 2 contained trials of distilled water and Experiment 3 contained 1.0M solutions of calcium nitrate. Alongside each cooling curve is also a linear line of best fit. The r-squared values in the table below suggest a strong goodness-of-fit for each linear regression, as each r-squared value is no less than 0.94. For physical processes, an r-squared value of over 0.90 indicates a strong and accurate measurement.

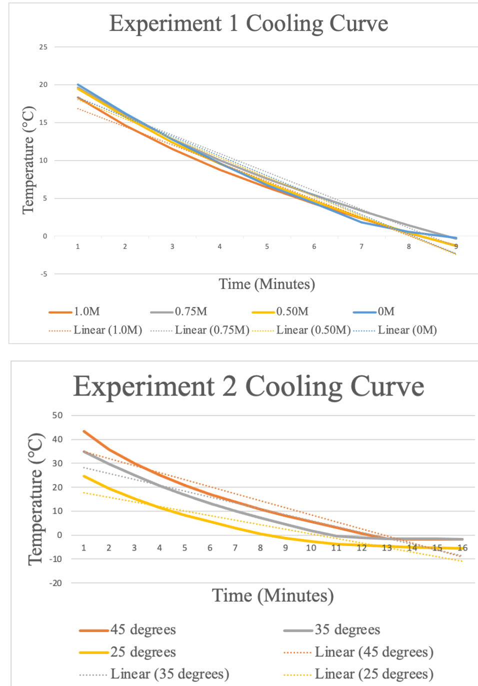

Figure 2. Cooling curves obtained across the first two experiments

A linear regression and Pearson correlation coefficient was obtained for each cooling curve. The tables below give the slope of the linear regression, its r-squared value, and standard error across each trial.

Experiment 1:

Concentration	R-squared Value	Slope	Standard Error
1.0M	0.98342	-2.58444	0.11759
0.75M	0.98532	-2.56278	0.11322
0.50M	0.98600	-2.45500	0.11540
0M	0.96993	-2.39666	0.17198

Experiment 2:

Temperature (℃)	R-squared Value	Slope	Standard Error
45	0.96525	-3.53296	0.20211
35	0.97502	-3.29563	0.16678
25	0.97785	-3.18667	0.18125

Because this study focuses on the rate of cooling rather than the specific times to cool, only the slopes of each cooling curve were further examined. The standard errors of each slope obtained from the linear regression are displayed down below. The first experiment suggests that as concentration increases, the rate of cooling decreases. The second experiment suggests that as initial temperature increases, the rate of cooling increases.

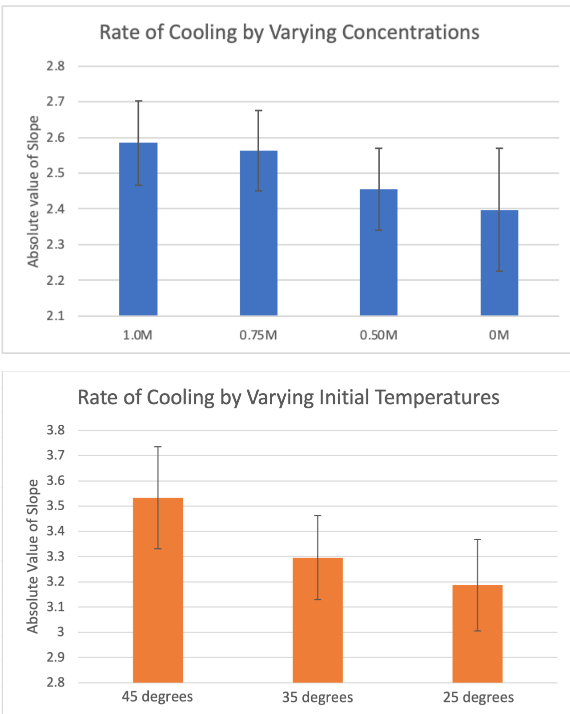

Figure 3. The rate of cooling for Experiment 1 and 2 with standard error bars

The third experiment will be discussed separately due to a different result with the rate of cooling.

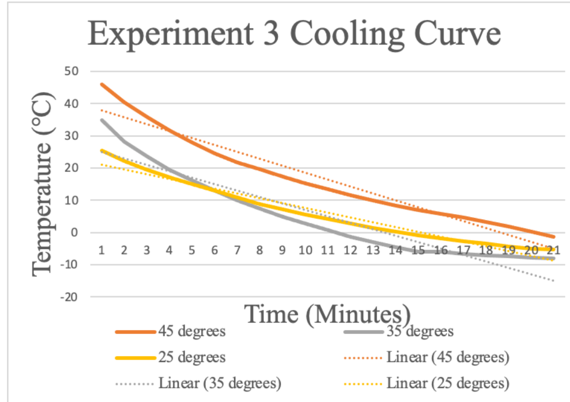

Figure 4. Cooling curves obtained across the first two experiments

Results from the third experiment actually show a display of the Mpemba Effect, as the initially warmer sample of 35℃ cools faster than the initial cooler sample of 25℃ which can also be visually seen in the above graph. This was an unexpected result from the study, however the r-squared values obtained from each linear regression still show a strong goodness-of-fit. The following table and chart show the rate of cooling for the third experiment and also suggests a much higher rate of cooling for the initial 35℃ sample than the other two. The results still suggest statistically significant differences between two different rates of cooling because the standard errors do not overlap with one another.

Experiment 3:

Temperature (℃)	R-squared Value	Slope	Standard Error
45	0.94662	-2.14285	0.11673
35	0.96888	-3.11870	0.17672
25	0.98131	-1.83178	0.07011

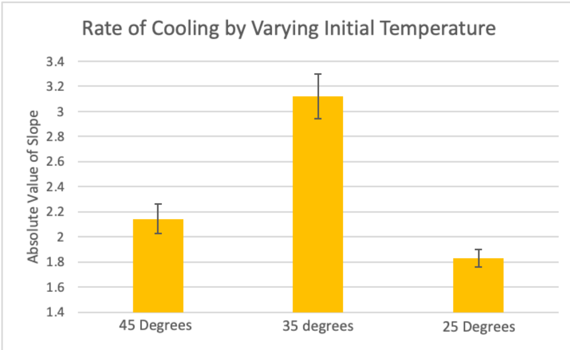

Figure 5. The rate of cooling for experiment 3 with standard error bars

Discussion

The results of this study demonstrate that the hardness of water does indeed affect the rate of cooling. How exactly different concentrations of calcium change the exact rate of cooling are not presented in precise form, but the results of Experiment 1 does show a general trend of increase in the rate of cooling as concentration also increases. This is supported by the r-squared values for each slope among the different concentrations, in which three trials were performed and averaged across into a single linear regression.

The second experiment showed a general trend of increase in the rate of cooling as initial temperature increased. These results were taken from two samples per trial, and therefore are not as strong as the first experiment. The same goes for the third experiment, in which the results are particularly interesting as the Mpemba Effect actually takes place between the 35 and 25 degree samples.

This could be due to a variety of factors, one being that the high concentration of calcium in this experiment (1.0M) has a significant effect on the process of cooling, as the molecules of dissolved calcium and nitrate ions interfere with the cooling process. Furthermore, evaporation might cause small changes in the volume, especially in solutions that were heated to higher initial temperatures. If solutions with higher initial temperatures experienced more evaporation, then the mass of the solution decreased and thus cooled more quickly. This effect was minimized, however, by placing parafilm over the beakers while the solutions were being heated over the hot plate, which prevented some of the liquid from evaporating into gas and into the air. Finally, another possibility of the Mpemba Effect occurring could be explained by convection currents, in which the solution is not uniformly distributed throughout the solution. Therefore, initially warmer samples are warmer at the top of the beaker when it cools to the same temperature as the cooler sample, thus having a higher rate of cooling.

The original hypothesis states that the harder the water, the greater effect it will have on the Mpemba Effect. This study has statistically significant results to support the above hypothesis due to the measured r-squared values across all three experiments. By specifically examining the rate at which the cooling occurs for each experiment, there is evidence that the hardness of water has an effect on the Mpemba Effect.

This study is limited in its ability to be in a highly controlled environment. There are several variables that cannot be controlled such as the distribution of calcium ions within the solution, as well as the distribution of temperature throughout the system. It is impossible to keep these variables in a perfectly controlled environment without extremely high laboratory-grade equipment that a typical high school student does not have access to.

Conclusion

The study does not try to verify whether or not the Mpemba Effect exists, but rather investigates potential reasonings or mechanisms to explain why it occurs through the different experiments with the limited resources available. There were several factors that may have influenced the results including small measurement uncertainties, the random behavior of particles in a solution, and the fluctuation of temperature by the freezer itself. Small measurement uncertainties, such as with the amount of solution in the beaker, are simply just random errors that can slightly vary the data. Similarly, the random behavior of particles is impossible to trace without extremely high-grade laboratory equipment. If the freezer had fluctuating temperatures as well, this would have changed the rate at which each of the substances cooled, which is why several trials occurred for each experiment.

The research concludes that hardness of water does affect the rate of cooling, and thus might also affect the probability of the Mpemba Effect. However, there is not evidence to say with certainty that a harder solution of water increases the chance of the Mpemba Effect occurring.

These results suggest that further trials should be conducted using samples of hard water, or other samples of mixed solutions to further understand what extent these solutes have on the rate of cooling. Although the findings in this paper do suggest that the hardness of water affects the rate of cooling, evaluating more specific parameters, such as the specific rate at which a change in concentration changes the rate in cooling, would provide much greater insight. With this better understanding, scientists may be able to come to a firmer conclusion as to why the Mpemba Effect exists. Kumar and Bechhoefer have already demonstrated that the Mpemba Effect does exist under extremely controlled conditions, and future studies should aim to understand why exactly this occurs. The future field of knowledge might continue to investigate the hardness of water in deeper contexts and its potential relation to the Mpemba Effect.

The Mpemba Effect still remains a puzzle to scientists, and has sparked much discussion and experimentation in the scientific community in pursuit of an explanation. Even as scientists continue to search for an explanation, it can still be used as a simple tool to engage students to be excited about strange phenomena that occur in science using experiments simple enough to be performed at home. For example, all that is needed for an at-home experiment would be cups, water, and a freezer. The Mpemba Effect is a topic that can be explored in many different depths and of all ages, making it such a versatile and interesting subject to be investigated.

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Academic Paper

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Sample: D
Score: 4

This paper earned a score of 4. The topic of the paper is focused and narrow, specifically looking at “whether the hardness of water influences the occurrence of the Mpemba Effect,” seen on page 1. This topic is carried throughout the entire paper. The paper is also situated in multiple perspectives, and the topic is specifically connected to scholarly works. For instance, on page 2, the paper discusses multiple theories to describe the Mpemba effect and how those theories shape the study at hand. A gap is introduced and defended on page 3, stating, “Therefore, there is no one cohesive explanation for the Mpemba Effect, further raising questions as to how much one mechanism may contribute to the Mpemba Effect.” The paper introduces a replicable method that is justified in the literature on page 4. For example, the paper defends one inquiry choice by stating, “It is important to discuss properties of mixtures because the research that follows will be using solutions that do not solely contain distilled water but also minerals dissolved in water.” The methods are also justified by following previous experiments on the topic. For example, the paper states on page 5, “The method used in this study is meant to be carried out similarly to the experiments of Mpemba and Osborne.” The paper did not earn a score of 5 because the discussion of the limitations is more about the limitations of the method and not specifically about limitations on the conclusion of the study (see page 10). There is little connection of the findings back to the body of literature. The paper also did not earn a 5 because while communication is clear, there are inconsistencies in captioning and citations that detract from the overall communication.

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