# AP Physics l: Algebra-Based Sample Student Responses and Scoring Commentary 

## Inside:

Free Response Question 5
$\checkmark$ Scoring Guideline
$\checkmark$ Student Samples
$\checkmark$ Scoring Commentary
(a) i. For correct expressions for the torques from the weight of each object

The torques are $m_{0} g\left(2 r_{0}\right)$ for object 1 and $\left(1.5 m_{0} g\right) r_{0}$ for object 2
For indicating that the two torques are exerted in opposite directions
$\tau_{\text {net }}=m_{0} g\left(2 r_{0}\right)-\left(1.5 m_{0} g\right) r_{0}$
For the derivation of a correct answer of $0.5 m_{0} g r_{0}$
Example response for part (a)(i)
$\tau_{\text {net }}=\tau_{1}-\tau_{2}$
$\tau_{\text {net }}=m_{0} g\left(2 r_{0}\right)-\left(1.5 m_{0} g\right) r_{0}$
$\tau_{\text {net }}=0.5 m_{0} g r_{0}$
ii. For an explanation that object 1 exerts a larger torque than object $2 \quad 1$ point

Example response for part (a)(ii)
Object 1 is twice as far from the axle as object 2, while object 2 has only 1.5 times the weight of object 1. So, object 1 exerts a larger torque.

Total for part (a) 4 points
(b) Correct answer: "Opposite directions"

Scoring note: If the wrong answer is selected, the response is not graded.
For a correct answer and a correct explanation

## Example response for part (b)

The objects exerted torques in opposite directions, with object 1 exerting a larger torque, so object 1 determines the net torque direction. With the torque from object 1 removed, the net torque and angular acceleration switch direction (becoming clockwise) to the torque from object 2. The angular velocity does not change direction immediately and is still counterclockwise.
(c) For a linear graph between 0 and $t_{C}$, with an initial angular velocity of zero and nonzero slope 1 point

Scoring note: The slope can be positive or negative.
For a change in the sign of the slope at $t=t_{C}$
1 point
AND
no discontinuity.
Example response for part (c)


Total for part (c) 2 points

## Total for question 57 points

## Begin your response to QUESTION 5 on this page.


5. (7 points, suggested time 13 minutes)

Two pulleys with different radii are attached to each other so that they rotate together about a horizontal axle through their common center. There is negligible friction in the axle. Object 1 hangs from a light string wrapped around the larger pulley, while object 2 hangs from another light string wrapped around the smaller pulley, as shown in the figure above.
$m_{0}$ is the mass of object 1 .
$1.5 m_{0}$ is the mass of object 2 .
$r_{0}$ is the radius of the smaller pulley.
$2 r_{0}$ is the radius of the larger pulley.
(a) At time $t=0$, the pulleys are released from rest and the objects begin to accelerate.
i. Derive an expression for the magnitude of the net torque exerted on the objects-pulleys system about the axle after the pulleys are released. Express your answer in terms of $m_{0}, r_{0}$, and physical constants, as appropriate.
$\tau=F_{h} \cdot r$

$$
\begin{aligned}
& \sum \tau=\tau_{\mathrm{Fg}_{1}-}-\tau_{\mathrm{Fg}}^{2} \\
& \Sigma \tau=m_{0} g\left(2 r_{0}\right)-1.5 m_{0} g\left(r_{0}\right) \\
& \Sigma \tau=2 m_{0} g r_{0}-1.5 m_{0} g r_{0}
\end{aligned}
$$

$$
\xi \tau=\frac{1}{2} m_{0} g r_{0}
$$

ii. Object 1 accelerates downward after the pulleys are released. Briefly explain why.

$$
\begin{aligned}
& \text { Object } 1 \text { has a greater radius than object } 2 \text {, } \\
& \text { and while object } 2 \text { has a greater mass. } \\
& \text { object } 1^{\prime} \text { 's radius is greater than object } 2^{\prime} \text { 's } \\
& \text { mass, so object } 1 \text { has a greater torque, } \\
& \text { so it accelerates downward. }
\end{aligned}
$$

## P1 Q5 A p2

## Continue your response to QUESTION 5 on this page.

(b) At a later time $t=t_{C}$, the string of object 1 is cut while the objects are still moving and the pulley is still rotating. Immediately after the string is cut, how do the directions of the angular velocity and angular acceleration of the pulley compare to each other?

X Same direction _ Opposite directions
Briefly explain your reasoning.
After the string is cut the pulley will begin to rotate it the opposite direction towards object 2 because of $1 \mathrm{H}^{\prime}$ 's mass, so both the angular velocity and acceleration would go in the same direction.
(c) On the axes below, sketch a graph of the angular velocity $\omega$ of the system consisting of the two pulleys as a function of time $t$. Include the entire time interval shown. The pulleys are released at $t=0$, and the string is cut at $t=t_{C}$.


## P1 Q5 B p1

Begin your response to QUESTION 5 on this page.

5. (7 points, suggested time 13 minutes)

Two pulleys with different radii are attached to each other so that they rotate together about a horizontal axle through their common center. There is negligible friction in the axle. Object 1 hangs from a light string wrapped around the larger pulley, while object 2 hangs from another light string wrapped around the smaller pulley, as shown in the figure above.
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(a) At time $t=0$, the pulleys are released from rest and the objects begin to accelerate.
i. Derive an expression for the magnitude of the net torque exerted on the objects-pulleys system about the axle after the pulleys are released. Express your answer in terms of $m_{0}, r_{0}$, and physical constants, as appropriate.

$$
\begin{array}{ll}
\varepsilon \tau=\tau_{1}-\tau_{2} \\
\varepsilon \tau=m_{0}\left(2 r_{0}\right)-1.5 \mu_{0}\left(r_{0}\right)
\end{array} \quad \nearrow \quad \varepsilon \tau=2 \mu_{0} r_{0}-1.5 \mu_{0} r_{0}
$$

ii. Object 1 accelerates downward after the pulleys are released. Briefly explain why.

$$
\text { Because } \tau \text { is force porpaliculat at a distance, object } 7 \text { with }
$$

a mass of Mo at a distance of $Z_{0}$. hes more tongue the object 2 With a mass of $1.5_{M_{0}}$ at a distance of $r_{0}$ thins consing the
acelleration.

## P1 Q5 B p2

## Continue your response to QUESTION 5 on this page.

(b) At a later time $t=t_{C}$, the string of object 1 is cut while the objects are still moving and the pulley is still rotating. Immediately after the string is cut, how do the directions of the angular velocity and angular acceleration of the pulley compare to each other?
$\sqrt{ }$ Same direction __Opposite directions
Briefly explain your reasoning.

> after the string is cut both angular velocity and mover acellioation Would change directions together, in the direction of object i's force
(c) On the axes below, sketch a graph of the angular velocity $\omega$ of the system consisting of the two pulleys as a function of time $t$. Include the entire time interval shown. The pulleys are released at $t=0$, and the string is cut at $t=t_{C}$.


## P1 Q5 C pi

## Begin your response to QUESTION 5 on this page.


5. ( 7 points, suggested time 13 minutes)

Two pulleys with different radii are attached to each other so that they rotate together about a horizontal axle through their common center. There is negligible friction in the axle. Object 1 hangs from a light string wrapped around the larger pulley, while object 2 hangs from another light string wrapped around the smaller pulley, as shown in the figure above.
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i. Derive an expression for the magnitude of the net torque exerted on the objects-pulleys system about the axle after the pulleys are released. Express your answer in terms of $m_{0}, r_{0}$, and physical constants, as appropriate.

ii. Object 1 accelerates downward after the pulleys are released. Briefly explain why.


## P1 Q5 C p2

## Continue your response to QUESTION 5 on this page.

(b) At a later time $t=t_{C}$, the string of object 1 is cut while the objects are still moving and the pulley is still rotating. Immediately after the string is cut, how do the directions of the angular velocity and angular acceleration of the pulley compare to each other?

X Same direction _Opposite directions
Briefly explain your reasoning.

(c) On the axes below, sketch a graph of the angular velocity $\omega$ of the system consisting of the two pulleys as a function of time $t$. Include the entire time interval shown. The pulleys are released at $t=0$, and the string is cut at $t=t_{C}$.


## Question 5

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

## Overview

Understandings-Responses were expected to:

- demonstrate an understanding of torque, specifically the net torque exerted on a system, and the effect of force and radius on torque. A mathematical representation was required in addition to a conceptual explanation.
- explain why a change in velocity occurred by using an analysis of the torques exerted on the system.
- express the motion of the pulley both in written and graphical form.

Skills—Responses were expected to:

- demonstrate an ability to create a mathematical representation of the net torque on the system.
- construct explanations and make claims about the motion of the pulley based on the concept of torque.
- make connections between the concepts involved with rotational kinematics and those involved with torque.
- create a graphical representation for the angular velocity with respect to time.


## Sample: P1 Q5 A

## Score: 6

Part (a)(i) earned 3 points. One point was earned for including the correct expressions for both torques. One point was earned for indicating that the two torques are exerted in opposite directions by subtracting the two expressions for the torques. One point was earned for the derivation including a series of steps resulting in the correct answer. Part (a)(ii) earned 1 point. The response explains that object l exerts a larger torque than object 2. Part (b) earned no points because "Same direction" is checked. Part (c) earned 2 points. One point was earned for including a linear graph between 0 and $t_{c}$ with an initial angular velocity of zero and nonzero slope. One point was earned for including a change in the sign of the slope at $t=t_{c}$ with no discontinuity.

## Sample: P1 Q5 B

## Score: 4

Part (a)(i) earned l point for indicating that the two torques are exerted in opposite directions. Note: Although the expressions for the torques are not correct, this point was earned because the response clearly equates their difference to the net torque and it contains the correct numerical coefficients for $m_{0}$ and $r_{0}$. The first and third points were not earned because the expressions for the two torques do not include the acceleration due to gravity and the final answer is not correct. Part (a)(ii) earned 1 point for explaining that object 1 exerts a larger torque than object 2. Part (b) earned no points because "Same direction" is checked. Part (c) earned 2 points. One point was earned for including a linear graph between 0 and $t_{c}$ with an initial angular velocity of zero and nonzero slope. One point was earned for including a change in the sign of the slope at $\mathrm{t}=t_{c}$ with no discontinuity.

## Question 5 (continued)

## Sample: P1 Q5 C

Score: 2
Part (a)(i) earned no points. Part (a)(ii) earned no points because the response does not indicate that object 1 exerts a larger torque than object 2. Part (b) earned no points because "Same direction" is checked. Part (c) earned 2 points. One point was earned for including a linear graph between 0 and $t_{c}$ with an initial angular velocity of zero and nonzero slope. The positive and negative directions for angular motion are not defined in the prompt so the slope can be either positive or negative. One point was earned for including a change in the sign of the slope at $\mathrm{t}=t_{c}$ with no discontinuity.

