2019

AP® CollegeBoard

AP® Physics C: Electricity and Magnetism

Scoring Guidelines Set 2

General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

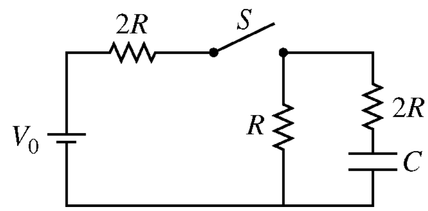
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 1

15 points



The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0, a capacitor of capacitance C, and a switch S. The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0, R, C, and fundamental constants, as appropriate.

(a) LO CNV-7.B.a, SP 5.A, 5.E
2 points

Derive an expression for the steady-state current supplied by the battery.

For using Ohm's law		1 point
$I = \dfrac{V}{R_{eff}} = \dfrac{V_0}{(2R + R)}$		
For correct substitution leading to correct answer		1 point
$I = \dfrac{V_0}{3R}$		

(b) LO CNV-7.B.b, SP 5.A, 5.E
2 points

Derive an expression for the charge on the capacitor.

For using the equation relating stored charge to capacitance		1 point
$q = CV = CV_C = CV_R$		
For determining V_R and substituting into the above equation		1 point
$V_R = IR = \left(\dfrac{V_0}{3R}\right)R = \dfrac{1}{3}V_0$		
$q = CV_R = C\left(\dfrac{1}{3}V_0\right) = \dfrac{1}{3}CV_0$		

Question 1 (continued)

(c) LO CNV-7.B.b, SP 5.A, 5.E
2 points

Derive an expression for the energy stored in the capacitor.

For any correct equation for energy stored in a capacitor		1 point
$U = \dfrac{q^2}{2C}$		
Substitute charge and/or voltage from part (b)		1 point
$U = \dfrac{(CV_0/3)^2}{2C} = \dfrac{1}{18}CV_0^2$		

Now the switch is opened at time $t = 0$.

(d) LO CNV-7.D.a, SP 5.A, 5.E
2 points

Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time t after the switch is opened.

For any correct voltage loop equation		1 point
$V_C - V_R - V_{2R} = 0 \therefore V_C = V_R + V_{2R}$		
$\dfrac{q(t)}{C} = I(R + 2R)$		
<u>Note:</u> Any correct loop equation for when the switch is open earns the point		
For substituting $-dq/dt$ or dq/dt for the current, consistent with loop equation		1 point
$\dfrac{q(t)}{C} = -3R\dfrac{dq}{dt}$		

Question 1 (continued)

(e)

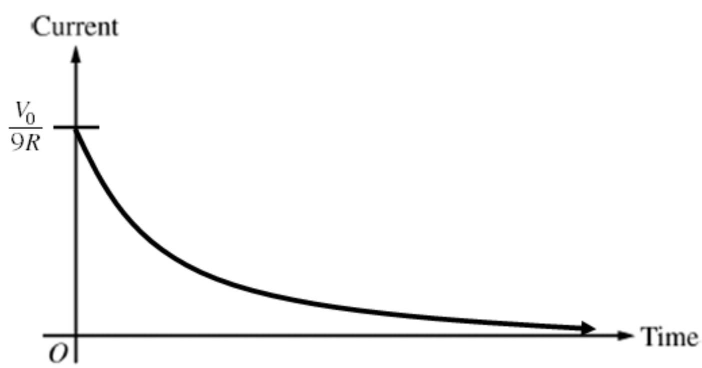
i. LO CNV-7.B.a, SP 5.A, 5.E
2 points

Calculate the current in resistor R immediately after the switch is opened.

For using a voltage consistent with part (b) in $I = \dfrac{V}{R}$		1 point
For substituting the correct resistance ($3R$) into Ohm's law		1 point
$I = \dfrac{V_0/3}{3R} = \dfrac{V_0}{9R}$		

ii. LO CNV-7.E.b, SP 3.C
3 points

On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



For a curve that is concave up throughout graph		1 point
For having the horizontal axis as an asymptote		1 point
For labeling the maximum current consistent with part (e)(i)		1 point

Question 1 (continued)

(f) LO CNV-7.G.a, SP 7.A, 7.C
2 points

Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c)?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

For selecting "Equal to"		1 point
For a correct justification invoking conservation of energy		1 point
Example: After the switch is opened, the capacitor will discharge all of its stored energy and charge. Assuming no energy is lost in the wire, then the only parts of the circuit that will dissipate this energy are the two resistors in series with the capacitor.		
Alternate Solution		*Alternate Points*
For selecting "Less than"		*1 point*
For a correct justification invoking conservation of energy		*1 point*
Example: After the switch is opened, the capacitor will discharge all of its stored energy and charge. Additionally there is energy lost due to resistance in the wire and/or energy loss in the capacitor.		

Question 1 (continued)

Learning Objectives

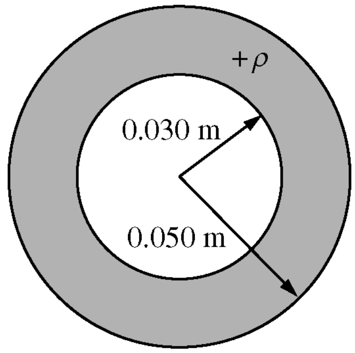
CNV-7.B.a: Calculate the potential difference across a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.
CNV-7.B.b: Calculate the stored charge on a capacitor in a circuit arrangement containing capacitors, resistors, and an energy source under steady-state conditions.
CNV-7.D.a: Derive expressions using calculus to describe the time dependence of the stored charge or potential difference across the capacitor, or the current or potential difference across the resistor in an RC circuit when charging or discharging a capacitor.
CNV-7.E.b: Describe the behavior of the voltage or current behavior over time for a circuit that contains resistors and capacitors in a multi-loop arrangement.
CNV-7.G.a: Describe the energy transfer in charging or discharging a capacitor in an RC circuit.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.

Question 2

15 points



A nonconducting hollow sphere of inner radius 0.030 m and outer radius 0.050 m carries a positive volume charge density ρ, as shown in the figure above. The charge density ρ of the sphere is given as a function of the distance r from the center of the sphere, in meters, by the following.

$$r < 0.030 \text{ m}: \ \rho = 0$$

$$0.030 \text{ m} < r < 0.050 \text{ m}: \ \rho = b/r \text{, where } b = 1.6 \times 10^{-6} \text{ C/m}^2$$

$$r > 0.050 \text{ m}: \ \rho = 0$$

(a) LO CNV-2.D.a, SP 6.B, 6.C
3 points

Calculate the total charge of the sphere.

For indicating the need to integrate the expression for charge density to determine the total charge on the sphere		1 point
$Q = \int \rho dV$		
For proper substitutions into the integration		1 point
$Q = \int \frac{(1.6 \times 10^{-6})}{r}(4\pi r^2)dr$		
For using the proper limits of integration		1 point
$Q = 4\pi(1.6 \times 10^{-6}) \int_{r=0.03}^{r=0.05} r dr = 4\pi(1.6 \times 10^{-6})\left[\frac{r^2}{2}\right]_{r=0.03}^{r=0.05}$		
$Q = (2\pi)(1.6 \times 10^{-6})(0.05^2 - 0.03^2) = 1.61 \times 10^{-8} \text{ C}$		

Question 2 (continued)

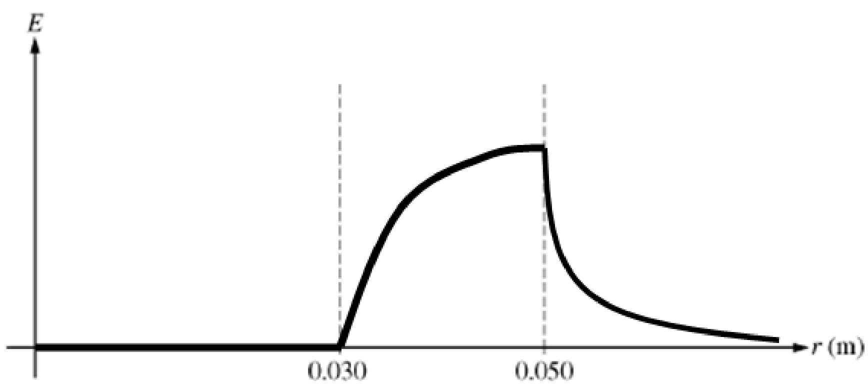
(b) LO CNV-2.D.a, SP 6.C
3 points

Using Gauss's law, calculate the magnitude of the electric field E at the outer surface of the sphere.

For correctly evaluating the surface integral in Gauss's law		1 point
$\oint E \bullet dA = E\left(4\pi r^2\right)$		
For correctly substituting the answer from part (a) and correct radius into above equation		1 point
$\dfrac{Q_{enc}}{\varepsilon_0} = E\left(4\pi r^2\right) \therefore E = \dfrac{Q_{enc}}{4\pi\varepsilon_0 r^2} = \dfrac{\left(1.61 \times 10^{-8}\ \text{C}\right)}{(4\pi)\left(8.85 \times 10^{-12}\right)(0.05)^2}$		
For an answer consistent with part (a) with correct units		1 point
$E = 5.79 \times 10^4\ \text{N/C}$		

(c) LO CNV-2.C, SP 3.C
3 points

On the axes below, sketch the magnitude of the electric field E as a function of distance r from the center of the sphere.



For clearly showing a graph with a value of $E = 0$ for $r < 0.030$ m		1 point
For a continuous graph that starts at zero, is concave down, and increases in value from $r = 0.030$ to $r = 0.050$		1 point
For a continuous graph that decreases asymptotically toward the horizontal axis for $r > 0.050$ m		1 point

Question 2 (continued)

(d) LO CNV-1.G.a, SP 6.B, 6.C
2 points

Calculate the electric potential V at the outer surface of the sphere. Assume the electric potential to be zero at infinity.

For substituting the total charge from part (a) into a correct expression for electric potential		1 point
For substituting $r = 0.05$ m into a correct expression for electric potential		1 point
$V_R = \dfrac{Q_{tot}}{4\pi\varepsilon_0 r} = \dfrac{\left(9\times10^9\right)\left(1.61\times10^{-8}\ \text{C}\right)}{(0.05\ \text{m})} = 2900\ \text{V}$		
Alternate Solution		*Alternate Points*
For substituting the total charge from part (a) into an integration for electric potential		*1 point*
$\Delta V_R = V_R - V_\infty = V_R = -\int E dr = -\int \dfrac{Q_{enc}}{4\pi\varepsilon_0 r^2}dr = -\int \dfrac{\left(9\times10^9\right)\left(1.61\times10^{-8}\ \text{C}\right)}{r^2}dr$		
For integrating with correct limits of integration		*1 point*
$V_R = -\displaystyle\int_{r=\infty}^{r=0.05\ \text{m}} \dfrac{145}{r^2}dr = -145\left[-\dfrac{1}{r}\right]_{r=\infty}^{r=0.05\ \text{m}} = 145\left(\dfrac{1}{(0.05\ \text{m})} - \dfrac{1}{\infty}\right) = 2900\ \text{V}$		

A proton is released from rest at the outer surface of the sphere at time $t = 0$ s.

(e)

i. LO ACT-1.D, SP 6.B, 6.C
2 points

Calculate the magnitude of the initial acceleration of the proton.

For using a correct expression of Newton's second law in terms of the electric field		1 point
$F = ma \therefore qE = ma \therefore a = \dfrac{qE}{m}$		
For correctly substituting into equation above		1 point
$a = \dfrac{qE}{m} = \dfrac{\left(1.6\times10^{-19}\ \text{C}\right)\left(5.79\times10^{4}\ \text{N/C}\right)}{1.67\times10^{-27}\ \text{kg}} = 5.55\times10^{12}\ \text{m/s}^2$		

Question 2 (continued)

(e) continued

ii. LO CNV-1.E, SP 6.B, 6.C
2 points

Calculate the speed of the proton after a long time.

For a correct expression of kinetic energy in terms of the electric potential difference		1 point
$-q\Delta V = \frac{1}{2}mv^2$		
For correctly substituting into equation above		1 point
$v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{-2\left(1.6\times10^{-19}\ \text{C}\right)\left(0-2900\ \text{V}\right)}{\left(1.67\times10^{-27}\ \text{kg}\right)}} = 7.45\times10^{5}\ \text{m/s}$		

Learning Objectives

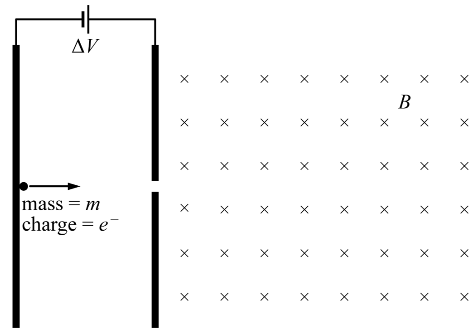
ACT-1.D: Determine the motion of a charged object of specified charge and mass under the influence of an electrostatic force.
CNV-1.E: Calculate the work done or changes in kinetic energy (or changes in speed) of a charged particle when it is moved through some known potential difference.
CNV-1.G.a: Use the general relationship between electric field and electric potential to calculate the relationships between the magnitude of electric field or the potential difference as a function of position.
CNV-2.C: State and use Gauss's law in integral form to derive unknown electric fields for planar, spherical, or cylindrically symmetrical charge distributions.
CNV-2.D.a: Using appropriate mathematics (which may involve calculus), calculate the total charge contained in lines, surfaces, or volumes when given a linear-charge density, a surface-charge density, or a volume-charge density of the charge configuration.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

Question 3

15 points



Two plates are set up with a potential difference V between them. A small sphere of mass m and charge $-e$ is placed at the left-hand plate, which has a negative charge, and is allowed to accelerate across the space between the plates and pass through a small opening. After passing through the small opening, the sphere enters a region in which there is a uniform magnetic field of magnitude B directed into the page, as shown above. Ignore gravitational effects. Express all algebraic answers in terms of V, m, e, B, and fundamental constants, as appropriate.

(a)

i. LO CHG-1.A.a, SP 7.A
1 point

What is the initial direction of the force on the sphere as it enters the magnetic field?

____ Into the page ____ Out of the page
____ Toward the top of the page ____ Toward the bottom of the page

For selecting "Toward the bottom of the page"		1 point

ii. LO CHG-1.B, SP 7.A
1 point

Describe the path taken by the sphere after it enters the magnetic field.

For describing a circular path for the sphere consistent with the selection from part (a)(i)		1 point
Example: The sphere will move in a circular path toward the bottom of the page.		

Question 3 (continued)

(b) LO CNV-1.E, SP 5.A, 5.E
2 points

Derive an expression for the speed of the sphere as it passes through the small opening.

For using a valid equation relating potential difference to kinetic energy of the sphere		1 point
$\Delta K = -\Delta U \therefore \frac{1}{2}mv^2 = -q\Delta V$		
$\frac{1}{2}mv^2 = eV$		
For the correct answer with supporting work		1 point
$v = \sqrt{\frac{2eV}{m}}$		

(c) LO CHG-1.C, SP 5.A, 5.E
3 points

Derive an expression for the radius of the path taken by the sphere as it moves through the magnetic field.

For an expression relating the magnetic force to the centripetal force		1 point
$\frac{mv^2}{r} = Bqv$		
For substituting the charge of the sphere into the above equation and solving for r		1 point
$\frac{mv}{r} = Be \therefore r = \frac{mv}{Be}$		
For substituting the answer from part (b)		1 point
$r = \frac{m}{Be}\sqrt{\frac{2eV}{m}} = \sqrt{\frac{2mV}{eB^2}}$		

An experiment is performed in which a beam of electrons is accelerated across the space between the plates and passes through the small opening. After passing through the opening, the electrons travel in a semicircular path and strike the right-hand plate. The potential difference between the plates is varied in regular increments, as shown in the table below. For each potential difference, the magnetic field is varied in order to cause the beam to strike the right-hand plate at a distance of 0.020 m from the opening.

Potential difference (V)	60	70	100	110	120	140
Magnetic field $(\mathrm{T} \times 10^{-3})$	2.62	2.78	3.39	3.54	3.78	3.99

Question 3 (continued)

(d) LO CHG-1.C, SP 4.C
1 point

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the mass-to-charge ratio of an electron.

Vertical axis:____________
Horizontal axis:____________

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

$$r = \sqrt{\frac{2m_e V}{eB^2}} \therefore B^2 = \frac{2m_e}{er^2}V$$

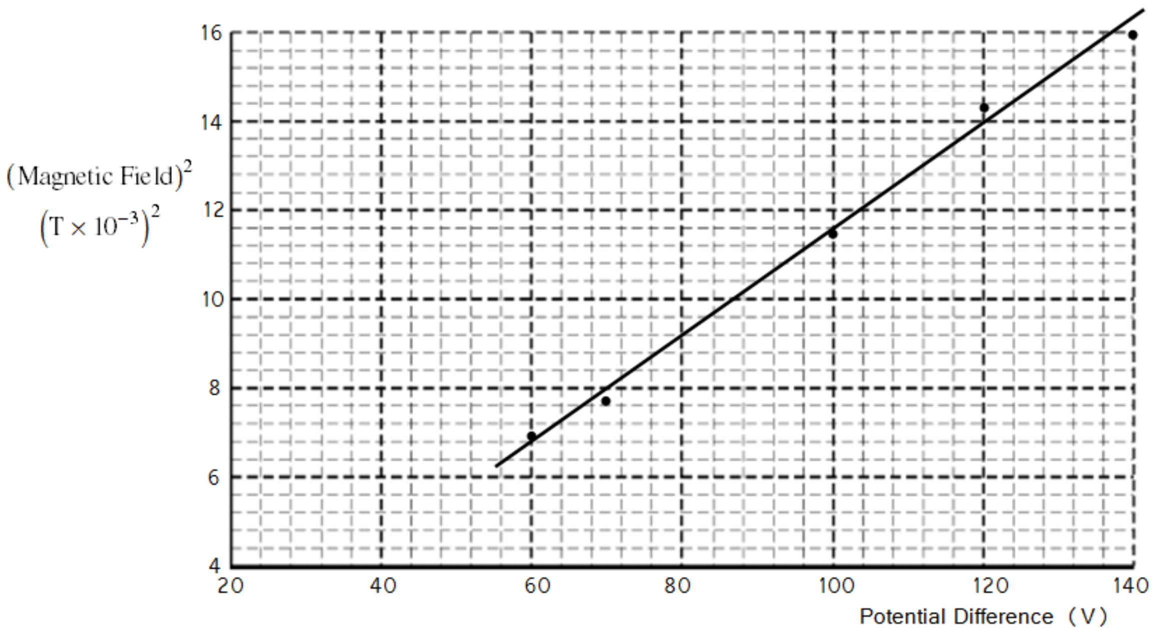
Potential difference (V)	60	70	100	110	120	140
Magnetic field $\left(\text{T} \times 10^{-3}\right)$	2.62	2.78	3.39	3.54	3.78	3.99
$(\text{Magnetic field})^2$ $\left(\text{T} \times 10^{-3}\right)^2$	6.86	7.73	11.5	12.5	14.3	15.9

For indicating axis choices with appropriate quantities to produce a straight line		1 point
Note: There are several possible choices.		

Question 3 (continued)

(e) LO CHG-1.C, SP 3.A, 4.C
4 points

On the graph below, plot the relationship determined in part (d). Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.



For correctly labeling both axes with quantities and units		1 point
For correctly scaling and labeling the axes so that the points use at least half the grid		1 point
For correctly plotting the data		1 point
For drawing a straight line that represents the data		1 point

(f) LO CHG-1.C, SP 4.D, 5.E
3 points

Using the straight line from part (e), determine the mass-to-charge ratio of an electron.

For correctly calculating the slope from the best-fit straight line and not from the data points unless the points fall on the best-fit line		1 point
$\text{slope} = \dfrac{14 - 9.2}{120 - 80} = 0.12\ \dfrac{(\text{T} \times 10^{-3})^2}{V} = 1.2 \times 10^{-7}\ \text{T}^2/\text{V}$		
For a correct expression relating the slope to the mass to charge ratio of an electron		1 point
$B^2 = \dfrac{2m_e}{er^2}V\ \therefore\ \text{slope} = \dfrac{2m_e}{er^2}\ \therefore\ \dfrac{m_e}{e} = \dfrac{r^2 \times \text{slope}}{2}$		
For substituting correct values into the above equation		1 point
$\dfrac{m_e}{e} = \dfrac{(0.01\ \text{m})^2 \times (1.2 \times 10^{-7}\ \text{T}^2/\text{V})}{2} = 6.0 \times 10^{-12}\ \text{kg/C}$		

Question 3 (continued)

Learning Objectives

CHG-1.A.a: Calculate the magnitude and direction of the magnetic force of interaction between a moving charged particle of specified charge and velocity moving in a region of a uniform magnetic field.
CHG-1.B: Describe the path of different moving charged particles (i.e., of different type of charge or mass) in a uniform magnetic field.
CHG-1.C: Derive an expression for the radius of a circular path for a charged particle of specified characteristics moving in a specified magnetic field.
CNV-1.E: Calculate the work done or changes in kinetic energy (or changes in speed) of a charged particle when it is moved through some known potential difference.

Science Practices

3.A: Select and plot appropriate data.
4.C: Linearize data and/or determine a best-fit line or curve.
4.D: Select relevant features of a graph to describe a physical situation or solve problems.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
7.A: Make a scientific claim.