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General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student’s solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as “derive” and “calculate” on the exams, and what is expected for each, see “The Free-Response Sections — Student Presentation” in the AP Physics: Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description or “Terms Defined” in the AP Physics 1: Algebra-Based Course and Exam Description and the AP Physics 2: Algebra-Based Course and Exam Description.

5. The scoring guidelines typically show numerical results using the value \( g = 9.8 \text{ m/s}^2 \), but the use of 10 \text{ m/s}^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.
A very long, thin, nonconducting cylinder of length \( L \) is centered on the \( y \)-axis, as shown above. The cylinder has a uniform linear charge density \(+\lambda\). Point \( P \) is located on the \( y \)-axis at \( y = c \), where \( L \gg c \).

(a)

i. LO CNV-3.B.a, SP 7.A
1 point

On the figure shown below, draw an arrow to indicate the direction of the electric field at point \( P \) due to the long cylinder. The arrow should start on and point away from the dot.

![Diagram showing the direction of the electric field at point P](image)

For drawing an arrow at point \( P \) that points upward | 1 point

ii. LO CNV-2.F, SP 7.C
1 point

Describe the shape and location of a Gaussian surface that can be used to determine the electric field at point \( P \) due to the long cylinder.

For describing a Gaussian surface that could be used to determine the electric field at point \( P \) | 1 point

Example: Drawing a cylinder that is coaxial with the thin cylinder and whose surface contains point \( P \) can be used to determine the electric field at point \( P \).

Note: Credit is earned if the student draws the correct surface on the figure.
Question 1 (continued)

(a) continued


3 points

Use your Gaussian surface to derive an expression for the magnitude of the electric field at point P. Express your answer in terms of $\lambda$, $c$, $L$, and physical constants, as appropriate.

<table>
<thead>
<tr>
<th>For using Gauss’s law to determine the electric field at point $P$</th>
<th>1 point</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{q_{enc}}{\varepsilon_0} = \int E \cdot dA \implies \frac{Q}{\varepsilon_0} = EA$</td>
<td></td>
</tr>
</tbody>
</table>

For correctly substituting for the charge into the equation above 1 point

For correctly substituting for the area or into the equation above 1 point

| $\frac{\lambda L}{\varepsilon_0} = E (2\pi c L) \implies E = \frac{\lambda}{2\pi \varepsilon_0 c} = \frac{2k\lambda}{c}$ | |

(b) LO ACT-1.D, SP 3.C

2 points

A proton is released from rest at point P. On the axes below, sketch the velocity $v$ as a function of position $y$ and the acceleration $a$ as a function of position $y$ for the proton.

For a concave down graph for $v$ as a function of position $x$ that does not start at the origin 1 point

For a concave up graph for $a$ as a function of position $x$ that has an asymptote at the horizontal axis 1 point
The original cylinder is now replaced with a much shorter thin, nonconducting cylinder with the same uniform linear charge density $+\lambda$, as shown in the figure below. The length of the cylinder to the right of the $y$-axis is $a$, and the length of the cylinder to the left of the $y$-axis is $b$, where $a < b$.

(c) LO CNV-3.B.a, SP 7.A
2 points

On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the shorter cylinder. The arrow should start on and point away from the dot.

For drawing an arrow at point $P$ that points to the right | 1 point
For drawing an arrow at point $P$ that points up and to the right | 1 point

(d)
   i. and ii.   LO CNV-2.F, SP 7.A
   1 point

Is there a single Gaussian surface that can be used with Gauss’s law to derive an expression for the electric field at point $P$?

____ Yes      ____ No

If your answer to part (d)(i) is yes, explain how you can use Gauss’s law to derive an expression for the field at point $P$. If your answer to part (d)(i) is no, explain why Gauss’s law cannot be applied to derive an expression for the electric field in this case.

For selecting “No” with a valid explanation | 1 point

Claim: Select “No.”
Evidence: The length of the cylinder is not much greater than the distance from the cylinder to point $P$ and the charge distribution is asymmetric.
Reasoning: Therefore, cannot use the approximation of the constant magnitude of electric field over a cylindrical surface.
A student in class argues that using the integral shown below might be a useful approach for determining the electric field at point P.

\[ E = \int \frac{1}{4\pi\varepsilon_0} \frac{1}{r^2} \, dq \]

The student uses this approach and writes the following two integrals for the magnitude of the horizontal and vertical components of the electric field at point P.

Horizontal component:
\[
|E_x| = \frac{\lambda}{4\pi\varepsilon_0} \int_{-b}^{a} \frac{x}{\left(c^2 + x^2\right)^{3/2}} \, dx
\]

Vertical component:
\[
|E_y| = \frac{\lambda}{4\pi\varepsilon_0} \int_{-b}^{a} \frac{y}{\left(c^2 + x^2\right)^{3/2}} \, dy
\]

One of the two expressions above is not correct. Which expression is not correct?

____ Horizontal component  ____ Vertical component

For correctly selecting “Vertical component”  1 point
### Question 1 (continued)

(e) continued

ii. LO CNV-3.A, SP 7.D

4 points

Identify two mistakes in the incorrect expression, and explain how to correct the mistakes.

<table>
<thead>
<tr>
<th>Mistake Description</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>For indicating the integral is not along the length of the cylinder</td>
<td>1 point</td>
</tr>
<tr>
<td>For an appropriate correction</td>
<td>1 point</td>
</tr>
<tr>
<td>Claim: Change $dy$ to $dx$.</td>
<td></td>
</tr>
<tr>
<td>Evidence: The integral is not along the cylinder.</td>
<td></td>
</tr>
<tr>
<td>Reasoning: This change will make the integral valid.</td>
<td></td>
</tr>
<tr>
<td>For indicating the power on the denominator term for the vertical component is incorrect</td>
<td>1 point</td>
</tr>
<tr>
<td>For an appropriate correction</td>
<td>1 point</td>
</tr>
<tr>
<td>Claim: The power on the term in the denominator should be $3/2$.</td>
<td></td>
</tr>
<tr>
<td>Evidence: The units of the integrand are not valid.</td>
<td></td>
</tr>
<tr>
<td>Reasoning: This change will make the integrand valid.</td>
<td></td>
</tr>
</tbody>
</table>

### Learning Objectives

**ACT-1.D:** Determine the motion of a charged object of specified charge and mass under the influence of an electrostatic force.

**CNV-2.C:** State and use Gauss’s law in integral form to derive unknown electric fields for planar, spherical, or cylindrically symmetrical charge distributions.

**CNV-2.F:** Describe the general features of an unknown charge distribution, given other features of the system.

**CNV-3.A:** Derive expressions for the electric field of specified charge distributions using integration and the principle of superposition. Examples of such charge distributions include a uniformly charged wire, a thin ring of charge (along the axis of the ring), and a semicircular or part of a semicircular arc.

**CNV-3.B.a:** Identify and qualitatively describe situations in which the direction and magnitude of the electric field can be deduced from symmetry considerations and understanding the general behavior of certain charge distributions.

### Science Practices

**3.C:** Sketch a graph that shows a functional relationship between two quantities.

**5.A:** Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.

**5.E:** Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.

**7.A:** Make a scientific claim.

**7.C:** Support a claim with evidence from physical representations.

**7.D:** Provide reasoning to justify a claim using physical principles or laws.
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

1. A very long, thin, nonconducting cylinder of length $L$ is centered on the $y$-axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point $P$ is located on the $y$-axis at $y = c$, where $L \gg c$.

(a) i. On the figure shown below, draw an arrow to indicate the direction of the electric field at point $P$ due to the long cylinder. The arrow should start on and point away from the dot.

ii. Describe the shape and location of a Gaussian surface that can be used to determine the electric field at point $P$ due to the long cylinder.

A cylindrical Gaussian surface with its center aligned with the long cylinder can be used to determine the electric field.

iii. Use your Gaussian surface to derive an expression for the magnitude of the electric field at point $P$. Express your answer in terms of $\lambda$, $c$, $L$, and physical constants, as appropriate.

\[
\mathbf{E} \cdot \Delta \mathbf{S} = \frac{\text{qenc}}{\varepsilon_0}
\]

\[
E (2\pi r L) = \frac{\lambda c}{\varepsilon_0}
\]

\[
E (c) \approx \frac{\lambda c}{2\pi c \varepsilon_0}
\]
(b) A proton is released from rest at point P. On the axes below, sketch the velocity $v$ as a function of position $y$ and the acceleration $a$ as a function of position $y$ for the proton.

The original cylinder is now replaced with a much shorter thin, nonconducting cylinder with the same uniform linear charge density $+\lambda$, as shown in the figure below. The length of the cylinder to the right of the $y$-axis is $a$, and the length of the cylinder to the left of the $y$-axis is $b$, where $a < b$.

(c) On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the shorter cylinder. The arrow should start on and point away from the dot.

Question 1 continues on the next page.
(d)  
i. Is there a single Gaussian surface that can be used with Gauss's law to derive an expression for the electric field at point P?  
   ___ Yes   ___ No  

ii. If your answer to part (d)(i) is yes, explain how you can use Gauss's law to derive an expression for the field at point P. If your answer to part (d)(i) is no, explain why Gauss's law cannot be applied to derive an expression for the electric field in this case.

Note: This figure is shown again for reference.

A student in class argues that using the integral shown below might be a useful approach for determining the electric field at point P.

\[ E = \int \frac{1}{4\pi\varepsilon_0} \frac{1}{r^2} \, dq \]

The student uses this approach and writes the following two integrals for the magnitude of the horizontal and vertical components of the electric field at point P.

Horizontal component:  
\[ |E_x| = \frac{\lambda}{4\pi\varepsilon_0} \int_b^a \frac{x}{(c^2 + x^2)^{3/2}} \, dx \]

Vertical component:  
\[ |E_y| = \frac{\lambda}{4\pi\varepsilon_0} \int_b^a \frac{y}{(c^2 + x^2)^{3/2}} \, dy \]

(e)  
i. One of the two expressions above is not correct. Which expression is not correct?  
   ___ Horizontal component   ___ Vertical component

ii. Identify two mistakes in the incorrect expression, and explain how to correct the mistakes.

In this expression, the integral should still be with respect to dx since it is a horizontal integral, so he should change it from dy to dx. Also, the denominator of the integrals should be \( c^2 + x^2 \) not \( c^2 + x^4 \).
PHYSICS C: ELECTRICITY AND MAGNETISM
SECTION II
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1. A very long, thin, nonconducting cylinder of length $L$ is centered on the $y$-axis, as shown above. The cylinder has a uniform linear charge density $+\lambda$. Point P is located on the $y$-axis at $y = c$, where $L \gg c$.

(a)

i. On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the long cylinder. The arrow should start on and point away from the dot.

![Diagram]

ii. Describe the shape and location of a Gaussian surface that can be used to determine the electric field at point P due to the long cylinder.

[A large cylinder laid horizontally that wraps around the long thin cylinder with the side going through P.

iii. Use your Gaussian surface to derive an expression for the magnitude of the electric field at point P. Express your answer in terms of $\lambda$, $c$, $L$, and physical constants, as appropriate.

$$E = \frac{\lambda}{2\pi \varepsilon_0 L}$$

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GO ON TO THE NEXT PAGE.
(b) A proton is released from rest at point P. On the axes below, sketch the velocity $v$ as a function of position $y$ and the acceleration $a$ as a function of position $y$ for the proton.

The original cylinder is now replaced with a much shorter thin, nonconducting cylinder with the same uniform linear charge density $+\lambda$, as shown in the figure below. The length of the cylinder to the right of the $y$-axis is $a$, and the length of the cylinder to the left of the $y$-axis is $b$, where $a < b$.

(c) On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the shorter cylinder. The arrow should start on and point away from the dot.

Question 1 continues on the next page.
(d)

i. Is there a single Gaussian surface that can be used with Gauss's law to derive an expression for the electric field at point P?

   ____ Yes   ____ No

ii. If your answer to part (d)(i) is yes, explain how you can use Gauss's law to derive an expression for the field at point P. If your answer to part (d)(i) is no, explain why Gauss's law cannot be applied to derive an expression for the electric field in this case.

   There is no Gaussian surface that can be drawn where the flux is constant throughout.

   \[ \text{Note: This figure is shown again for reference.} \]

A student in class argues that using the integral shown below might be a useful approach for determining the electric field at point P.

\[ E = \int \frac{1}{4\pi\varepsilon_0} \frac{1}{r^2} \, dq \]

The student uses this approach and writes the following two integrals for the magnitude of the horizontal and vertical components of the electric field at point P.

**Horizontal component:**

\[ |E_x| = \frac{\lambda}{4\pi\varepsilon_0} \int_{-b}^{a} \frac{x}{(c^2 + x^2)^{3/2}} \, dx \]

**Vertical component:**

\[ |E_y| = \frac{\lambda}{4\pi\varepsilon_0} \int_{-b}^{a} \frac{y}{(c^2 + x^2)} \, dy \]

(e)

i. One of the two expressions above is not correct. Which expression is not correct?

   ____ Horizontal component   ____ Vertical component

ii. Identify two mistakes in the incorrect expression, and explain how to correct the mistakes.

   1. The student should be integrating in terms of \( x \) not \( y \).

   2. The \( y \) component should be replaced with \( c \).

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GO ON TO THE NEXT PAGE.
PHYSICS C: ELECTRICITY AND MAGNETISM
SECTION II
Time—45 minutes
3 Questions

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1. A very long, thin, nonconducting cylinder of length \( L \) is centered on the \( y \)-axis, as shown above. The cylinder has a uniform linear charge density \( +\lambda \). Point \( P \) is located on the \( y \)-axis at \( y = c \), where \( L \ll c \).

(a)

i. On the figure shown below, draw an arrow to indicate the direction of the electric field at point \( P \) due to the long cylinder. The arrow should start on and point away from the dot.

![Diagram]

ii. Describe the shape and location of a Gaussian surface that can be used to determine the electric field at point \( P \) due to the long cylinder.

The Gaussian surface can be take the shape of the cylinder of length \( L \).

iii. Use your Gaussian surface to derive an expression for the magnitude of the electric field at point \( P \). Express your answer in terms of \( \lambda \), \( c \), \( L \), and physical constants, as appropriate.

\[
E_A = \frac{q_{emc}}{\varepsilon_0}
\]

\[
E \left( \frac{\pi c^2 L}{\lambda} \right) = \frac{q_{emc}}{\varepsilon_0}
\]

\[
E = \frac{\lambda L}{\pi c^2 L} = \frac{\lambda}{\pi c^2}
\]

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(b) A proton is released from rest at point P. On the axes below, sketch the velocity $v$ as a function of position $y$ and the acceleration $a$ as a function of position $y$ for the proton.

The original cylinder is now replaced with a much shorter thin, nonconducting cylinder with the same uniform linear charge density $+\lambda$, as shown in the figure below. The length of the cylinder to the right of the $y$-axis is $a$, and the length of the cylinder to the left of the $y$-axis is $b$, where $a < b$.

(c) On the figure shown below, draw an arrow to indicate the direction of the electric field at point P due to the shorter cylinder. The arrow should start on and point away from the dot.

Question 1 continues on the next page.
(d)

i. Is there a single Gaussian surface that can be used with Gauss’s law to derive an expression for the electric field at point P?
   ___ Yes   ___ No

ii. If your answer to part (d)(i) is yes, explain how you can use Gauss’s law to derive an expression for the field at point P. If your answer to part (d)(i) is no, explain why Gauss’s law cannot be applied to derive an expression for the electric field in this case.

   Note: This figure is shown again for reference.

   A student in class argues that using the integral shown below might be a useful approach for determining the electric field at point P.

   \[ E = \int \frac{1}{4\pi e_0} \frac{1}{r^2} \, dq \]

   The student uses this approach and writes the following two integrals for the magnitude of the horizontal and vertical components of the electric field at point P.

   \[ |E_x| = \frac{\lambda}{4\pi e_0} \int_{-b}^{a} \frac{x}{(c^2 + x^2)^{3/2}} \, dx \quad \text{and} \quad |E_y| = \frac{\lambda}{4\pi e_0} \int_{-b}^{a} \frac{y}{(c^2 + x^2)} \, dy \]

(e)

i. One of the two expressions above is not correct. Which expression is not correct?
   ___ Horizontal component   ___ Vertical component

ii. Identify two mistakes in the incorrect expression, and explain how to correct the mistakes.

   \[ \text{It is not supposed to be } (c^2 + x^2)^{3/2} \]
Question 1

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses to this question were expected to demonstrate the following:

- An understanding of the properties of the electric field due to a charge distribution
- The ability to use Gauss’s law
- The ability to identify an appropriate Gaussian surface
- The ability to graphically describe the motion of a charged particle in an electric field
- An understanding of when Gauss’s law is an appropriate approach to solve a problem
- The ability to separate a vector into components
- The ability to carry out integration along a line

Sample: E Q1 A
Score: 15

All parts of this response earned full credit. Part (a)(i) has an arrow pointing upward, so 1 point was earned. Part (a)(ii) describes an appropriate Gaussian surface, so 1 point was earned. Part (a)(iii) correctly substitutes the area and charge into Gauss’s law, so 3 points were earned. Part (b) has a concave down curve for the velocity-position graph that does not start at the origin, and a concave down curve for the acceleration graph with a horizontal axis asymptote, so 2 points were earned. Part (c) has an arrow pointing up and to the right, so 2 points were earned. Part (d) has a correct selection and valid explanation, so 1 point was earned. Part (e) has a correct selection and correctly identifies two mistakes with appropriate corrections, so 5 points were earned.

Sample: E Q1 B
Score: 10

Parts (a)(i), (a)(ii), (c), and (d) earned full credit, 1 point, 1 point, 2 points, and 1 point, respectively. Part (a)(iii) uses Gauss’s law but incorrectly substitutes for the charge and does not substitute for the radius, so 1 point was earned. Part (b) has a concave down curve for the acceleration graph with a horizontal axis asymptote but incorrectly starts the velocity graph at the origin, so 1 point was earned. Part (e) has a correct selection but only correctly identifies one mistake with appropriate correction, so 3 points were earned.

Sample: E Q1 C
Score: 5

Parts (a)(i) and (c) earned full credit, 1 point and 2 points, respectively. Part (a)(ii) describes an appropriate Gaussian surface but does not indicate that it is coaxial with the original cylinder, so no points were earned. Part (a)(iii) uses Gauss’s law and substitutes correctly for the charge but uses an incorrect area, so 2 points were earned. Part (b) has two incorrect graphs, so no points were earned. Part (d) has an incomplete statement, so no points were earned. Part (e) has an incorrect selection and does not correctly identify any mistakes, so no points were earned.