AP Statistics

Free-Response Questions

Formulas begin on page 3. Questions begin on page 6. Tables begin on page 13.

Formulas

(I) Descriptive Statistics

$$\overline{x} = \frac{\sum x_i}{n}$$

$$s_x = \sqrt{\frac{1}{n-1} \sum \left(x_i - \overline{x}\right)^2}$$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}}$$

$$\hat{y} = b_0 + b_1 x$$

$$b_1 = \frac{\sum (x_i - \overline{x})(y_i - \overline{y})}{\sum (x_i - \overline{x})^2}$$

$$b_0 = \overline{y} - b_1 \overline{x}$$

$$r = \frac{1}{n-1} \sum \left(\frac{x_i - \overline{x}}{s_x} \right) \left(\frac{y_i - \overline{y}}{s_y} \right)$$

$$b_1 = r \frac{s_y}{s_x}$$

$$s_{b_1} = \frac{\sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n - 2}}}{\sqrt{\sum (x_i - \overline{x})^2}}$$

(II) Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$E(X) = \mu_{\mathcal{X}} = \sum x_i p_i$$

$$Var(X) = \sigma_x^2 = \sum (x_i - \mu_x)^2 p_i$$

If X has a binomial distribution with parameters n and p, then:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n - k}$$

$$\mu_{\chi} = np$$

$$\sigma_{\chi} = \sqrt{np(1-p)}$$

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

If \overline{x} is the mean of a random sample of size n from an infinite population with mean μ and standard deviation σ , then:

$$\mu_{\overline{x}} = \mu$$

$$\sigma_{\overline{\chi}} = \frac{\sigma}{\sqrt{n}}$$

(III) Inferential Statistics

 $Standardized \ test \ statistic: \frac{statistic - parameter}{standard \ deviation \ of \ statistic}$

Confidence interval: statistic ± (critical value) • (standard deviation of statistic)

Single-Sample

Statistic	Standard Deviation of Statistic
Sample Mean	$\frac{\sigma}{\sqrt{n}}$
Sample Proportion	$\sqrt{\frac{p(1-p)}{n}}$

Two-Sample

Statistic	Standard Deviation of Statistic
Difference of sample means	$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
	Special case when $\sigma_1 = \sigma_2$ $\sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$
Difference of sample proportions	$\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$ Special case when $p_1 = p_2$
	$\sqrt{p(1-p)}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$

Chi-square test statistic =
$$\sum \frac{(observed - expected)^2}{expected}$$

STATISTICS SECTION II Part A

Questions 1-5

Spend about 65 minutes on this part of the exam. Percent of Section II score—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

- 1. Researchers studying a pack of gray wolves in North America collected data on the length *x*, in meters, from nose to tip of tail, and the weight *y*, in kilograms, of the wolves. A scatterplot of weight versus length revealed a relationship between the two variables described as positive, linear, and strong.
 - (a) For the situation described above, explain what is meant by each of the following words.
 - (i) Positive:
 - (ii) Linear:
 - (iii) Strong:

The data collected from the wolves were used to create the least-squares equation $\hat{y} = -16.46 + 35.02x$.

- (b) Interpret the meaning of the slope of the least-squares regression line in context.
- (c) One wolf in the pack with a length of 1.4 meters had a residual of -9.67 kilograms. What was the weight of the wolf?

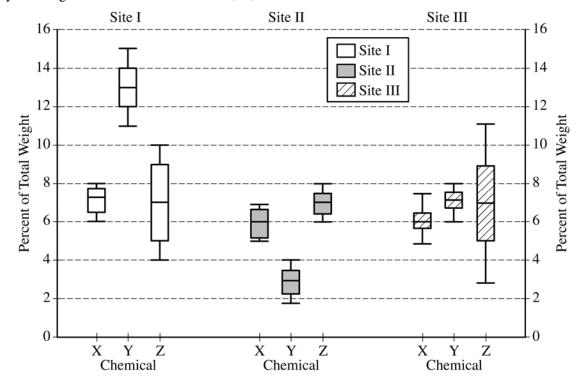
- 2. The manager of a local fast-food restaurant is concerned about customers who ask for a water cup when placing an order but fill the cup with a soft drink from the beverage fountain instead of filling the cup with water. The manager selected a random sample of 80 customers who asked for a water cup when placing an order and found that 23 of those customers filled the cup with a soft drink from the beverage fountain.
 - (a) Construct and interpret a 95 percent confidence interval for the proportion of all customers who, having asked for a water cup when placing an order, will fill the cup with a soft drink from the beverage fountain.
 - (b) The manager estimates that each customer who asks for a water cup but fills it with a soft drink costs the restaurant \$0.25. Suppose that in the month of June 3,000 customers ask for a water cup when placing an order. Use the confidence interval constructed in part (a) to give an interval estimate for the cost to the restaurant for the month of June from the customers who ask for a water cup but fill the cup with a soft drink.

- 3. A grocery store purchases melons from two distributors, J and K. Distributor J provides melons from organic farms. The distribution of the diameters of the melons from Distributor J is approximately normal with mean 133 millimeters (mm) and standard deviation 5 mm.
 - (a) For a melon selected at random from Distributor J, what is the probability that the melon will have a diameter greater than 137 mm?

Distributor K provides melons from nonorganic farms. The probability is 0.8413 that a melon selected at random from Distributor K will have a diameter greater than 137 mm. For all the melons at the grocery store, 70 percent of the melons are provided by Distributor J and 30 percent are provided by Distributor K.

- (b) For a melon selected at random from the grocery store, what is the probability that the melon will have a diameter greater than 137 mm?
- (c) Given that a melon selected at random from the grocery store has a diameter greater than 137 mm, what is the probability that the melon will be from Distributor J?

4. The chemicals in clay used to make pottery can differ depending on the geographical region where the clay originated. Sometimes, archaeologists use a chemical analysis of clay to help identify where a piece of pottery originated. Such an analysis measures the amount of a chemical in the clay as a percent of the total weight of the piece of pottery. The boxplots below summarize analyses done for three chemicals—X, Y, and Z—on pieces of pottery that originated at one of three sites: I, II, or III.

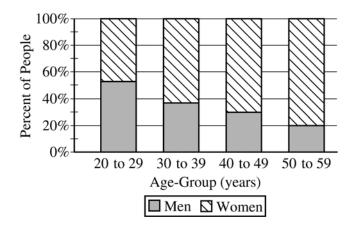


- (a) For chemical Z, describe how the percents found in the pieces of pottery are similar and how they differ among the three sites.
- (b) Consider a piece of pottery known to have originated at one of the three sites, but the actual site is not known.
 - (i) Suppose an analysis of the clay reveals that the sum of the percents of the three chemicals X, Y, and Z is 20.5%. Based on the boxplots, which site—I, II, or III—is the most likely site where the piece of pottery originated? Justify your choice.
 - (ii) Suppose only one chemical could be analyzed in the piece of pottery. Which chemical—X, Y, or Z—would be the most useful in identifying the site where the piece of pottery originated? Justify your choice.

5. The table and the bar chart below summarize the age at diagnosis, in years, for a random sample of 207 men and women currently being treated for schizophrenia.

Age-Group (years)

	20 to 29	30 to 39	40 to 49	50 to 59	Total
Women	46	40	21	12	119
Men	53	23	9	3	88
Total	99	63	30	15	207



Do the data provide convincing statistical evidence of an association between age-group and gender in the diagnosis of schizophrenia?

STATISTICS SECTION II Part B

Ouestion 6

Spend about 25 minutes on this part of the exam. Percent of Section II score—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. Consider an experiment in which two men and two women will be randomly assigned to either a treatment group or a control group in such a way that each group has two people. The people are identified as Man 1, Man 2, Woman 1, and Woman 2. The six possible arrangements are shown below.

Arrangement A		
Treatment	Control	
Man 1	Woman 1	
Man 2	Woman 2	

Arrangement B		
Treatment	Control	
Man 1 Woman 1	Man 2 Woman 2	

Arrangement C		
Treatment	Control	
Man 1 Woman 2	Man 2 Woman 1	

Arrangement D		
Treatment	Control	
Woman 1	Man 1	
Woman 2	Man 2	

Arrangement E		
Treatment Control		
Man 2 Woman 2	Man 1 Woman 1	

Arrangement F		
Treatment	Control	
Man 2 Woman 1	Man 1 Woman 2	

Two possible methods of assignment are being considered: the sequential coin flip method, as described in part (a), and the chip method, as described in part (b). For each method, the order of the assignment will be Man 1, Man 2, Woman 1, Woman 2.

- (a) For the sequential coin flip method, a fair coin is flipped until one group has two people. An outcome of tails assigns the person to the treatment group, and an outcome of heads assigns the person to the control group. As soon as one group has two people, the remaining people are automatically assigned to the other group.
 - (i) Complete the table below by calculating the probability of each arrangement occurring if the sequential coin flip method is used.

Arrangement	A	В	С	D	E	F
Probability						

(ii) For the sequential coin flip method, what is the probability that Man 1 and Man 2 are assigned to the same group?

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The six arrangements are repeated below.

Arrangement A		
Treatment	Control	
Man 1	Woman 1	
Man 2	Woman 2	

Arrangement B		
Treatment Control		
Man 1	Man 2	
Woman 1	Woman 2	

Arrangement C								
Treatment	Control							
Man 1	Man 2							
Woman 2	Woman 1							

Arrangement D							
Treatment	Control						
Woman 1	Man 1						
Woman 2	Man 2						

Arrangement E								
Treatment	Control							
Man 2	Man 1							
Woman 2	Woman 1							

Arrangement F								
Treatment	Control							
Man 2	Man 1							
Woman 1	Woman 2							

- (b) For the chip method, two chips are marked "treatment" and two chips are marked "control." Each person selects one chip at random without replacement.
 - (i) Complete the table below by calculating the probability of each arrangement occurring if the chip method is used.

Arrangement	A	В	С	D	E	F
Probability						

- (ii) For the chip method, what is the probability that Man 1 and Man 2 are assigned to the same group?
- (c) Sixteen participants consisting of 10 students and 6 teachers at an elementary school will be used for an experiment to determine lunch preference for the school population of students and teachers. As the participants enter the school cafeteria for lunch, they will be randomly assigned to receive one of two lunches so that 8 will receive a salad, and 8 will receive a grilled cheese sandwich. The students will enter the cafeteria first, and the teachers will enter next. Which method, the sequential coin flip method or the chip method, should be used to assign the treatments? Justify your choice.

STOP

END OF EXAM

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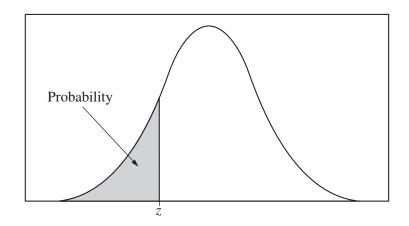


Table entry for z is the probability lying below z.

Table A Standard normal probabilities

Tubic 11	Standar	a normar p	Tobabilitie							
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

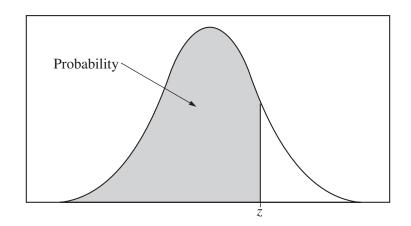


Table entry for z is the probability lying below z.

 $Table\ A\quad ({\it Continued})$

_	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
<i>Z</i>										
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

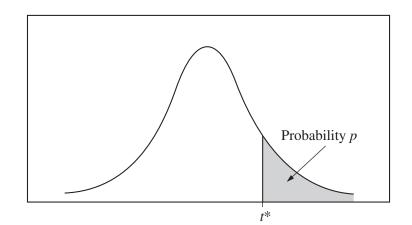


Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^* .

Table B t distribution critical values

Table B	t distri	oution crit	tical values	1									
		Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005	
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6	
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60	
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92	
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610	
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869	
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959	
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408	
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041	
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781	
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587	
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437	
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318	
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221	
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140	
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073	
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015	
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965	
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922	
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883	
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850	
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819	
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792	
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768	
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745	
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725	
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707	
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690	
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674	
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659	
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646	
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551	
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496	
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460	
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416	
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390	
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300	
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291	
•	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%	

Confidence level C

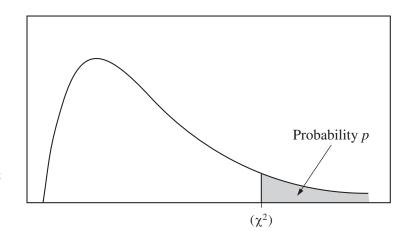


Table entry for p is the point (χ^2) with probability p lying above it.

Table C χ^2 critical values

1 abic C	λ critical	values										
						Tail prol	oability p					
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2