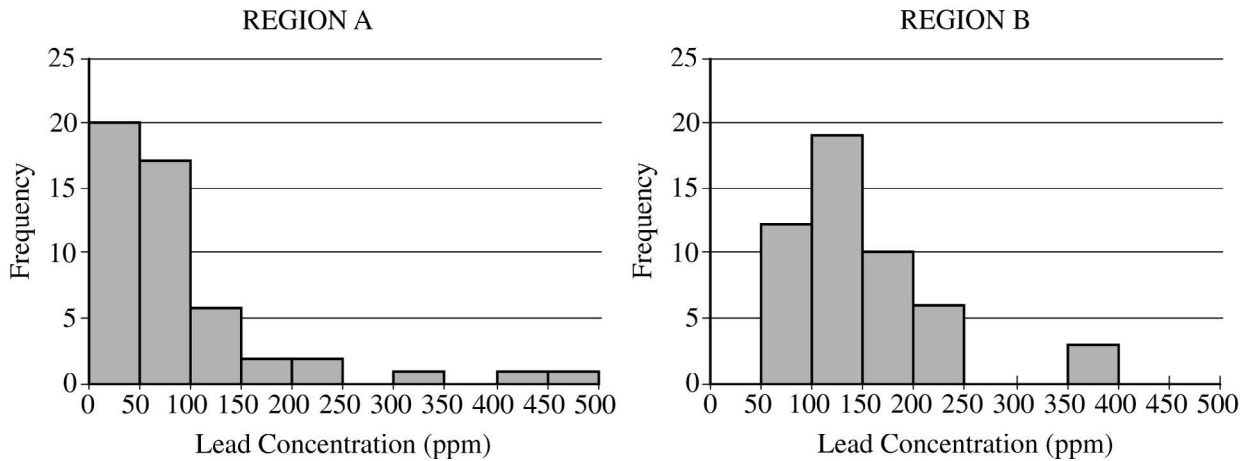




## Question 1: Focus on Exploring Data

1. A geologist studying lead concentration in soil selected random samples of soil from two regions: region A and region B. The following histograms show the distribution of lead concentration, in parts per million (ppm), for the two samples.



- (A) Write a few sentences comparing the distributions of lead concentration in the two samples.
- (B) To investigate whether the mean lead concentration is different in region A than in region B, the geologist conducted the appropriate test. All conditions for inference are met, and the  $p$ -value of the test is 0.007. Based on the  $p$ -value, is there convincing statistical evidence, at a level of significance of  $\alpha = 0.05$ , that there is a difference between the mean lead concentration of region A and the mean lead concentration of region B? Justify your answer.

## Scoring Guidelines for Question 1: Focus on Exploring Data

4 points

### General Scoring Notes

Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct, partially correct, or incorrect. The response is then categorized based on the scores assigned to each letter part and awarded a score between 0 and 4 (see the table at the end of each question).

	Model Solution	Scoring
<p><b>Part (A)</b></p> <p><b>2.A</b></p> <p><b>2.D</b></p> <p><b>UNC-1.H</b></p> <p><b>UNC-1.M</b></p> <p><b>UNC-1.O</b></p>	<p>The distributions of lead concentration for both Region A and Region B are skewed to the right.</p> <p>The median lead concentration of Region B (between 100 ppm and 150 ppm) is greater than the median lead concentration of Region A (between 50 ppm and 100 ppm).</p> <p>The range of lead concentration of Region A (between 400 ppm and 500 ppm) is greater than the range of lead concentration of Region B (between 250 ppm and 350 ppm)</p>	<p><b>Essentially correct (E)</b> if the response addresses the following <b>five</b> components:</p> <ol style="list-style-type: none"> <li>1. Shape (right skewed)</li> <li>2. Center (e.g., correct location of mean and/or median)</li> <li>3. Variability (e.g., correct range and/or IQR)</li> <li>4. Comparison between regions of at least one of the following: shapes, centers, or variability</li> <li>5. Context (lead concentration)</li> </ol> <p><b>Partially correct (P)</b> if the response includes only <b>three</b> or <b>four</b> of the five components listed above.</p> <p><b>Incorrect (I)</b> if the response does not satisfy the criteria for E or P.</p>
<p><b>Part (B)</b></p> <p><b>4.E</b></p> <p><b>DAT-3.H</b></p>	<p>Because the <math>p</math>-value of 0.007 is less than the level of significance of 0.05, the data provide convincing statistical evidence that there is a difference between the mean lead concentration in the two regions.</p>	<p><b>Essentially correct (E)</b> if the response provides <b>both</b> of the following:</p> <ol style="list-style-type: none"> <li>1. Correct conclusion (<i>Yes, the data provide convincing statistical evidence</i>) in context (<i>lead concentration</i>)</li> <li>2. Justification based on comparing the <math>p</math>-value to the significance level, <math>\alpha</math></li> </ol> <p><b>Partially correct (P) Option 1:</b> The response provides <b>both</b> of the following:</p> <ol style="list-style-type: none"> <li>1. Correct conclusion (<i>Yes</i>) but without context</li> <li>2. Justification based on comparing the <math>p</math>-value to the significance level, <math>\alpha</math></li> </ol> <p><b>Partially correct (P) Option 2:</b> The response provides <b>both</b> of the following:</p> <ol style="list-style-type: none"> <li>1. Correct conclusion (<i>Yes</i>) in context (<i>lead concentration</i>)</li> <li>2. Justification based on the <math>p</math>-value, <b>but without</b> comparison of the <math>p</math>-value to the significance level, <math>\alpha</math></li> </ol> <p><b>Incorrect (I)</b> if the response does not satisfy the criteria for E or P.</p>

<b>Scoring for Question 1</b>	<b>Score</b>
<b>Complete Response</b> Both parts essentially correct	<b>4</b>
<b>Substantial Response</b> One part essentially correct and one part partially correct	<b>3</b>
<b>Developing response</b> Both parts partially correct OR one part essentially correct and one part incorrect	<b>2</b>
<b>Minimal Response</b> One part partially correct and one part incorrect	<b>1</b>

## Question 2: Focus on Probability and Sampling Distributions

2. Past records from a certain movie theater indicate that 80 percent of moviegoers make a purchase at the movie theater's snack bar. A random sample of 3 moviegoers from the theater will be selected at random. Let the random variable  $S$  represent the number of moviegoers who make a purchase at the snack bar from the 3 selected.

(A) Complete the following table to create the probability distribution of  $S$ .

$s$	0	1	2	3
$P(s)$	0.008			0.512

(B) Calculate and interpret the expected value of  $S$ .

## Scoring Guidelines for Question 2: Focus on Probability and Sampling Distributions

4 points

### General Scoring Notes

Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct, partially correct, or incorrect. The response is then categorized based on the scores assigned to each letter part and awarded a score between 0 and 4 (see the table at the end of each question).

**Learning Objectives:** **UNC-3.B** **VAR-5.A** **VAR-5.C** **UNC-3.C** **VAR-5.D**

	Model Solution	Scoring										
<p><b>Part (A)</b></p> <p><b>2.B</b></p> <p><b>3.A</b></p> <p><b>UNC-3.B</b></p> <p><b>VAR-5.A</b></p>	<p>Random variable <math>S</math> has a binomial distribution with <math>n = 3</math> and <math>p = 0.8</math>.</p> $P(s = 1) = \binom{3}{1}(0.8)^1(0.2)^2 = 0.096$ $P(s = 2) = \binom{3}{2}(0.8)^2(0.2)^1 = 0.384$ <table border="1"> <thead> <tr> <th><math>s</math></th> <th>0</th> <th>1</th> <th>2</th> <th>3</th> </tr> </thead> <tbody> <tr> <td><math>P(s)</math></td> <td>0.008</td> <td>0.096</td> <td>0.384</td> <td>0.512</td> </tr> </tbody> </table>	$s$	0	1	2	3	$P(s)$	0.008	0.096	0.384	0.512	<p><b>Essentially correct (E)</b> if the response includes the following three components.</p> <ol style="list-style-type: none"> <li>Shows evidence that the calculation of at least one probability necessary to complete the table is based on a binomial with <math>n = 3</math> and <math>p = 0.8</math>, either by using the binomial formula with correct values substituted in, or by using the notation <math>B(3, 0.8)</math>, or by using a calculator command, such as <code>binompdf</code> (<math>n = 3, p = 0.8, s = 2</math>). However, an unlabeled calculator command such as <code>binompdf(3, 0.8, 2)</code> does not satisfy component 1.</li> </ol> <p><b>Note:</b></p> <ul style="list-style-type: none"> <li>A normal approximation is not appropriate because the sample size is too small.</li> </ul> <ol style="list-style-type: none"> <li>Correct calculation of <math>P(s = 1)</math> or <math>P(s = 2)</math>.</li> <li>The probabilities of the probability distribution sum to 1.</li> </ol> <p><b>Partially correct (P)</b> if the response includes only two of the three components.</p> <p><b>Incorrect (I)</b> if the response does not satisfy the criteria for E or P.</p>
$s$	0	1	2	3								
$P(s)$	0.008	0.096	0.384	0.512								
<p><b>Part (B)</b></p> <p><b>3.B</b></p> <p><b>4.B</b></p> <p><b>VAR-5.C</b></p> <p><b>UNC-3.C</b></p> <p><b>VAR-5.D</b></p>	<p><b>Calculation, Option 1:</b></p> <p>The expected value of <math>S</math> is <math>E(S) = \sum sP(s)</math></p> $= 0(0.008) + 1(0.096) + 2(0.384) + 3(0.512)$ $= 2.4$ <p><b>Calculation, Option 2:</b></p> <p>Because <math>S</math> is a binomial random variable, <math>E(S) = np = 3(0.8) = 2.4</math>.</p> <p><b>Interpretation:</b></p> <p>If the process of randomly selecting 3 moviegoers is repeated many times, the average number of moviegoers who make a purchase at the snack bar will be very close to 2.4.</p>	<p><b>Essentially correct (E)</b> if the response satisfies the following four components.</p> <ol style="list-style-type: none"> <li>Calculates the expected value correctly.</li> </ol> <p>A correct expected value calculation that is consistent with incorrect probabilities reported in part (a) satisfies component 1. If the expected value calculation is rounded to an integer, the response does not satisfy component 1.</p> <ol style="list-style-type: none"> <li>Interpretation—includes reference to long run, many samples or repeated sampling.</li> <li>Interpretation—includes reference of average.</li> <li>Context.</li> </ol> <p><b>Partially correct (P)</b> if the response includes only three of the four components.</p> <p><b>Incorrect (I)</b> if the response does not satisfy the criteria for E or P.</p>										

<b>Scoring for question 2</b>	<b>Score</b>
<b>Complete Response</b> Both parts essentially correct	<b>4</b>
<b>Substantial Response</b> One part essentially correct and one part partially correct	<b>3</b>
<b>Developing response</b> Both parts partially correct OR one part essentially correct and one part incorrect	<b>2</b>
<b>Minimal Response</b> One part partially correct and one part incorrect	<b>1</b>