1. A power supply is set to \( \varepsilon = 24 \text{ V} \) and is connected to resistors \( R_1 = 9.0 \Omega \) and \( R_2 = 3.0 \Omega \), capacitor \( C = 12 \mu \text{F} \), and switch \( S \), as shown in the figure. Initially, the capacitor is uncharged, and switch \( S \) is open.

(a) At time \( t = 0 \), the switch is then closed.
   i. Calculate the current through \( R_1 \) immediately after the switch is closed.
   ii. Determine the current through \( R_2 \) immediately after the switch is closed.

A long time after the switch is closed, the circuit reaches steady-state conditions.

(b) Calculate the potential difference across \( R_2 \).

(c) Calculate the magnitude of the charge \( Q \) on the positive plate of the capacitor.

(d) On the axes shown, sketch a graph of the potential difference \( V_C \) across the capacitor as a function of time \( t \).
   Explicitly label any intercepts, asymptotes, maxima, or minima with values or expressions, as appropriate.

After steady-state conditions are reached, the switch is now opened, and time is reset to \( t = 0 \).

(e) Using integral calculus, derive an expression for the charge \( q(t) \) on the capacitor as a function of time \( t \) after the switch is opened. Express your answer in terms of \( Q \).
The capacitor is discharged, and a third resistor is added to the circuit, as shown above. The switch is then closed

(f) Does the time it takes for the charge on the capacitor to reach $\frac{2}{3}$ of its maximum value increase, decrease, or stay the same as compared to the circuit in part (a)?

__________ Increase  __________ Decrease  __________ Stay the same

Justify your answer.
Scoring Guidelines for Question 1  
**15 points**

**Learning Objectives:**  
CNV-6.C  
CNV-7.B  
CNV-7.C  
CNV-7.D.a  
CNV-7.E.a

(a)  
i. Calculate the current through $R_1$ immediately after the switch is closed.  
One point for correctly applying Ohm's law to the circuit treating the capacitor as a short circuit.  
$$V = \varepsilon = IR_1$$  
One point for using the piston to change the volume of the gas.  
$$I = \frac{\varepsilon}{R_1} = \frac{(24 \text{ V})}{(9.0 \text{ }\Omega)} = 2.67 \text{ A}$$  
ii. Determine the current through $R_2$ immediately after the switch is closed.  
One point for indicating the current in $R_2$ is zero.  
$$I_2 = 0$$  
**Total for Part (a) 3 points**

(b)  
Calculate the potential difference across $R_2$.  
One point for correctly applying Ohm's law to the circuit treating the capacitor as an open circuit.  
$$V = \varepsilon = I(R_1 + R_2)$$  
One point for a correct substitution into the above equation  
$$I = \frac{\varepsilon}{R_1} = \frac{(24 \text{ V})}{(9.0 \text{ }\Omega + 3.0 \text{ }\Omega)} = 2.0 \text{ A}$$  
One point for correctly applying Ohm's law to calculate the potential difference across $R_2$  
$$V = IR_2 = (2.0 \text{ A})(3.0 \text{ }\Omega) = 6.0 \text{ V}$$  
**Total for Part (b) 3 points**

(c)  
Calculate the magnitude of the charge $Q$ on the positive plate of the capacitor.  
One point for a correct substitution into an equation to solve for the charge stored on the capacitor.  
$$Q = CV = (12 \mu F)(6.0\text{ V}) = 72 \mu \text{C}$$  
**1 point**

(d)  
Sketch a graph of the potential difference $V_C$ across the capacitor as a function of time $t$.  
One point for a concave up curve.  
Explicitly label any intercepts, asymptotes, maxima, or minima with values or expressions, as appropriate.  
One point for a horizontal asymptote at the maximum charge and correctly labeling the maximum charge.  
**Total for Part (d) 2 points**
(e) Derive an expression for the charge $q(t)$ on the capacitor as a function of time $t$ after the switch is opened. One point for an expression of Kirchhoff’s loop equation.

$$V_R + V_C = 0$$

$$iR_2 = -\frac{q}{C}$$

One point for expressing the equation as a differential equation.

$$\frac{dq}{dt} = -\frac{q}{RC}$$

One point for integrating with correct limits or constant of integration.

$$\int_{q(0)}^{q(t)} \frac{1}{q} dq = -\int_{t=0}^{t} \frac{1}{R_2C} dt$$

$$\left[ \ln q \right]_{q(0)}^{q(t)} = -\frac{1}{R_2C} (t-0) = -\frac{t}{R_2C}$$

$$q(t) = Q e^{-\frac{t}{R_2C}}$$

One point for correctly substituting into the above equation.

$$q(t) = Q e^{-\frac{t}{R_2C}} = (72 \mu C)e^{-\frac{t}{\frac{1}{30 \Omega} \cdot \frac{1}{12 \mu F}}} = (72 \mu C)e^{-\frac{t}{1(10^{-12})}}$$

Total for Part (e) 4 points

(f) Does the time it takes for the charge on the capacitor to reach $\frac{2}{3}$ of its maximum value increase, decrease, or stay the same as compared to the circuit in part (a)? One point for selecting that the time for the charge “Increases”.

One point for a correct justification.

Example of acceptable justification:

- Adding a resistor in series increases the resistance of the circuit. Since the time constant for a capacitor is equal to $RC$, increasing the resistance increases the time constant, and it takes more time to charge the capacitor.

Total for part (f) 2 points

Total for question 1 15 points
Question 2

2. A coil of wire is used to create a solenoid as depicted in the figure shown. The right end of the coil goes up in front of the \( y \)-axis, and the left end of the coil goes down behind the \( y \)-axis. The solenoid has radius \( a \), length \( L \), and \( N \) turns of wire in its coil. A power supply of variable emf is set to provide a potential difference of \( \epsilon \) and is connected to the solenoid. The figure shows an \( xyz \)-coordinate axis in which the \( y \)-axis is along the central axis of the solenoid and point \( P \) is at the origin of the coordinate system. The resistance of the solenoid is \( R \).

(a) Indicate below the direction of the magnetic field at point \( P \).

\[ +x \quad +y \quad +z \]
\[ -x \quad -y \quad -z \]

Justify your answer.

(b) An axial view of the solenoid is shown. The \(+y\) direction is out of the page. Point \( P \) is shown as are two other points, \( Q \) and \( R \), which are located with \( P \) in the \( xz \) plane. Point \( Q \) is a distance \( \frac{a}{2} \) from point \( P \), and point \( R \) is a distance \( 4a \) from \( P \).

i. Indicate on the figure the directions of the magnetic field at points \( Q \) and \( R \). If the magnitude of the magnetic field is zero, indicate this by writing \( B = 0 \) next to that point.

ii. Is the magnitude of the magnetic field at point \( Q \) greater than, less than, or equal to the magnitude of the magnetic field at point \( P \)?

\[ \text{Greater than} \quad \text{Less than} \quad \text{Equal to} \]

Justify your answer.
i. On the figure above, draw an Amperian loop that can be used to determine the magnetic field along the central axis of the solenoid.

ii. Use Ampere's law to derive an expression for the magnetic field strength at point P. Express your answer in terms of ε, R, L, N, a, and physical constants, as appropriate.

Students conduct an experiment with this apparatus in which they vary the emf of the power supply and measure the resulting magnetic field strength at the center of the solenoid. The data are shown in the table. The students also note that the solenoid has 160 turns, the radius $a = 0.015$ m, and the length $L = 0.140$ m.

\[
\begin{array}{|c|c|c|c|c|}
\hline
\varepsilon (V) & 1.48 & 2.94 & 4.36 & 5.81 & 7.31 \\
B (10^{-5} \text{T}) & 4.7 & 10.7 & 13.7 & 20.4 & 24.9 \\
\hline
\end{array}
\]

(d) Plot these data on the axes provided and draw a best-fit line for the data.

(e) Use the best-fit line to calculate the resistance of the circuit used in the experiment.

A resistor is added in parallel with the solenoid.

(f) Will the magnetic field on the central axis of the solenoid increase, decrease, or stay the same?

_________ Increase _________ Decrease _________ Stay the same

Justify your answer.
Scoring Guidelines for Question 2  
15 points

Learning Objectives:  
CNV-8.C.d

(a) One point for selecting “-y”  

Indicate below the direction of the magnetic field at point P.

\[ +x \quad +y \quad +z \quad -x \quad -y \quad -z \]

Justify your answer.
One point for a justification describing the correct use of the right-hand rule.
Example of acceptable justification:
- The current leaves the battery to the right and comes up in front of the coil and goes down behind the coil. Using the right-hand rule, if the current comes up in from of the coil, the magnetic field is directed to the left. This is the -y direction.

Total for Part (a)  
2 points

(b) i. Indicate on the figure the directions of the magnetic field at points Q and R.
One point for indicating that the magnetic field at point Q is in the same direction as indicated in part (a).

One point for indicating that the magnetic field at point R is zero.

ii. Is the magnitude of the magnetic field at point Q greater than, less than, or equal to the magnitude of the magnetic field at point P?
One point for selecting “Less than” and for a correct justification.
Example of an acceptable justification:
- For a coil of wire, the magnetic field is at a maximum at its center; thus, the magnetic field is less at point Q than at point P.
Note: If the coil is considered a long solenoid, then the magnetic field inside the solenoid is constant and selecting “Equal to” with this justification receives full credit.

Total for Part (b)  
3 points
(c) i. Draw an Amperian loop.
   One point for drawing an appropriate Amperian loop.

ii. Derive an expression for the magnetic field strength at point P.
   One point for a correct expression of Ampere’s law consistent with the loop drawn in part (c)(i).
   \[ \int B \, ds = \mu_0 I_{enc} \]
   \[ \int B_1 \, dl_1 + \int B_2 \, dl_2 + \int B_3 \, dl_3 + \int B_4 \, dl_4 = \mu_0 I_{enc} \]
   One point for correctly evaluating the integration of the magnetic field around the loop.
   \[ BL = \mu_0 NI \]
   \[ B = \frac{\mu_0 N I}{L} \]

Total for Part (c) 3 points

(d) Plot these data on the axes provided and draw a best-fit line for the data.
   One point for correctly plotting the data points.
   One point for drawing reasonable best fit line. That is, the straight line drawn should have roughly the same amount of points above and below.

Total for Part (d) 2 points
(e) Use the best-fit line to calculate the resistance of the circuit used in the experiment. 

One point for calculating the slope from the best fit line and not the data points.

\[ \text{slope} = m = \frac{\Delta y}{\Delta x} = \frac{(20 - 5) \times 10^{-5} \, \text{T}}{(5.8 - 1.7) \, \text{V}} = 3.65 \times 10^{-5} \, \Omega \]

One point for a correct substitution into an equation relating the slope to the resistance.

One point for a correct use of Ohm’s law.

\[ B = \mu_0 \frac{N}{L} I = \mu_0 \frac{N \, R}{L} \]

\[ m = \frac{\mu_0 N}{LR} \quad R = \frac{\mu_0 N}{L \, m} = \frac{(4 \pi \times 10^{-7} \, \text{T} \cdot \text{m})/(160)}{(0.140 \, \text{m})(3.65 \times 10^{-5} \, \Omega)} \]

\[ R = \frac{\mu_0 N}{L \, m} = \frac{(4 \pi \times 10^{-7} \, \text{T} \cdot \text{m})/(160)}{(0.140 \, \text{m})(3.65 \times 10^{-5} \, \Omega)} = 39.3 \, \Omega \]

Total for Part (e) 3 points

(f) Will the magnetic field on the central axis of the solenoid increase, decrease, or stay the same? 

One point for selecting “Stays the same”.

One point for a correct justification.

Example of acceptable justification:

- Adding the resistor in parallel does not change the current in the solenoid, so the magnetic field on the central axis will stay the same.

Total for part (f) 2 points

Total for question 2 15 points