

## About the Advanced Placement Program<sup>®</sup> (AP<sup>®</sup>)

The Advanced Placement Program<sup>®</sup> has enabled millions of students to take college-level courses and earn college credit, advanced placement, or both, while still in high school. AP Exams are given each year in May. Students who earn a qualifying score on an AP Exam are typically eligible to receive college credit, placement into advanced courses, or both. Every aspect of AP course and exam development is the result of collaboration between AP teachers and college faculty. They work together to develop AP courses and exams, set scoring standards, and score the exams. College faculty review every AP teacher's course syllabus.

## AP Calculus Program

AP Calculus AB and AP Calculus BC focus on students' understanding of calculus concepts and provide experience with methods and applications. Although computational competence is an important outcome, the main emphasis is on a multirepresentational approach to calculus, with concepts, results, and problems being expressed graphically, numerically, analytically, and verbally. The connections among these representations are important.

Teachers and students should regularly use technology to reinforce relationships among functions, to confirm written work, to implement experimentation, and to assist in interpreting results. Through the use of the unifying themes of calculus (e.g., derivatives, integrals, limits, approximation, and applications and modeling) the courses become cohesive rather than a collection of unrelated topics.

## AP Calculus BC Course Overview

AP Calculus BC is roughly equivalent to both first and second semester college calculus courses. It extends the content learned in AB to different types of equations (polar, parametric, vector-valued) and new topics (such as Euler's method, integration by parts, partial fraction decomposition, and improper integrals), and introduces the topic of sequences and series. The AP course covers topics in differential and integral calculus, including concepts and skills of limits, derivatives, definite integrals, the Fundamental Theorem of Calculus, and series. The course teaches students to approach calculus concepts and problems when they are represented graphically, numerically, analytically, and verbally, and to make connections amongst these representations.

Students learn how to use technology to help solve problems, experiment, interpret results, and support conclusions.

### RECOMMENDED PREREQUISITES

Before studying calculus, all students should complete the equivalent of four years of secondary mathematics designed for college-bound students: courses which should prepare them with a strong foundation in reasoning with algebraic symbols and working with algebraic structures. Prospective calculus students should take courses in which they study algebra, geometry, trigonometry, analytic geometry, and elementary functions. These functions include linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise-defined functions. In particular, before studying calculus, students must be familiar with the properties of functions, the composition of functions, the algebra of functions, and the graphs of functions. Students must also understand the language of functions (domain and range, odd and even, periodic, symmetry, zeros, intercepts, and descriptors such as increasing and decreasing). Students should also know how the sine and cosine functions are defined from the unit circle and know the values of the trigonometric functions at the numbers  $0$ ,  $\pi/6$ ,  $\pi/4$ ,  $\pi/3$ ,  $\pi/2$ , and their multiples. Students who take AP Calculus BC should have basic familiarity with sequences and series, as well as some exposure to polar equations.

## Use of Graphing Calculators

Professional mathematics organizations have strongly endorsed the use of calculators in mathematics instruction and testing. The use of a graphing calculator in AP Calculus BC is considered an integral part of the course.

## AP Calculus BC Course Content

The course is organized around the foundational concepts of calculus:

### I. Limits:

Students must have a solid, intuitive understanding of limits and be able to compute one-sided limits, limits at infinity, the limit of a sequence, and infinite limits. They should be able to apply limits to understand the behavior of a function near a point and understand how limits are used to determine continuity.

### II. Derivatives:

Students should be able to use different definitions of the derivative, estimate derivatives from tables and graphs, and apply various derivative rules and properties. Students should also be able to solve separable differential equations, understand and be able to apply the Mean Value Theorem, and be familiar with a variety of real-world applications, including related rates, optimization, and growth and decay models.

### III. Integrals and the Fundamental Theorem of Calculus:

Students should be familiar with basic techniques of integration, including basic antiderivatives and substitution, and properties of integrals. Students should also understand area, volume, and motion applications of integrals, as well as the use of the definite integral as an accumulation function. It is critical that students understand the relationship between integration and differentiation as expressed in the Fundamental Theorem of Calculus.

### IV. Series:

Students should be familiar with various methods for determining convergence and divergence of a series, Maclaurin series for common functions, general Taylor series representations, radius and interval of convergence, and operations on power series. The technique of using power series to approximate an arbitrary function near a specific value allows for an important connection back to the tangent-line problem.

## Mathematical Practices for AP Calculus

The Mathematical Practices for AP Calculus (MPACs) capture important aspects of the work that mathematicians engage in, at the level of competence expected of AP Calculus students. These MPACs are highly interrelated tools that should be used frequently and in diverse contexts to support conceptual understanding of calculus.

1. Reasoning with definitions and theorems
2. Connecting concepts
3. Implementing algebraic/computational processes
4. Connecting multiple representations
5. Building notational fluency
6. Communicating

## AP Calculus BC Exam Structure

### AP CALCULUS BC EXAM: 3 HOURS 15 MINUTES

#### Assessment Overview

The AP Calculus BC Exam questions measure students' understanding of the concepts of calculus, their ability to apply these concepts, and their ability to make connections among graphical, numerical, analytical, and verbal representations of mathematics. Adequate preparation for the exam also includes a strong foundation in algebra, geometry, trigonometry, and elementary functions, though the course necessarily focuses on differential and integral calculus. Students may not take both the Calculus AB and Calculus BC Exams within the same year. A Calculus AB sub-score is reported based on performance on the portion of the Calculus BC Exam devoted to Calculus AB topics.

The free-response section tests students' ability to solve problems using an extended chain of reasoning. During the second timed portion of the free-response section (Part B), students are permitted to continue work on problems in Part A, but they are not permitted to use a calculator during this time.

#### Format of Assessment

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**Section I:** Multiple Choice | 45 Questions | 1 Hour, 45 Minutes | 50% of Exam Score

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- **Part A:** 30 questions; 60 minutes (calculator not permitted)
  - **Part B:** 15 questions; 45 minutes (graphing calculator required)
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**Section II:** Free Response | 6 Questions | 1 Hour, 30 Minutes | 50% of Exam Score

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- **Part A:** 2 questions; 30 minutes (graphing calculator required)
- **Part B:** 4 questions; 60 minutes (calculator not permitted)

## AP CALCULUS BC SAMPLE EXAM QUESTIONS

### Sample Multiple-Choice Question

Which of the following statements about the series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{1+\sqrt{n}}$  is true?

- (A) The series converges absolutely.
- (B) The series converges conditionally.
- (C) The series converges, but neither conditionally nor absolutely.
- (D) The series diverges.

## AP Calculus BC Exam Structure

### Sample Free-Response Question

#### Free Response: Section II, Part B

No calculator is allowed for problems on this part of the exam.

The function  $f$  has derivatives of all orders at  $x = 0$ , and the Maclaurin series for  $f$  is

$$\sum_{n=2}^{\infty} \frac{\ln n}{3^n n^3} x^n.$$

- (a) Find  $f(0)$  and  $f^{(4)}(0)$ .
- (b) Does  $f$  have a relative minimum, a relative maximum, or neither at  $x = 0$ ? Justify your answer.
- (c) Using the ratio test, determine the interval of convergence of the Maclaurin series for  $f$ . Justify your answer.