

AP[®] Calculus BC

About the Advanced Placement Program[®] (AP[®])

The Advanced Placement Program[®] has enabled millions of students to take college-level courses and earn college credit, advanced placement, or both, while still in high school. AP Exams are given each year in May. Students who earn a qualifying score on an AP Exam are typically eligible, in college, to receive credit, placement into advanced courses, or both. Every aspect of AP course and exam development is the result of collaboration between AP teachers and college faculty. They work together to develop AP courses and exams, set scoring standards, and score the exams. College faculty review every AP teacher's course syllabus.

AP Calculus Program

AP Calculus AB and AP Calculus BC focus on students' understanding of calculus concepts and provide experience with methods and applications. Through the use of big ideas of calculus (e.g., modeling change, approximation and limits, and analysis of functions), each course becomes a cohesive whole, rather than a collection of unrelated topics. Both courses require students to use definitions and theorems to build arguments and justify conclusions.

The courses feature a multi-representational approach to calculus, with concepts, results, and problems expressed graphically, numerically, analytically, and verbally. Exploring connections among these representations builds understanding of how calculus applies limits to develop important ideas, definitions, formulas, and theorems. A sustained emphasis on clear communication of methods, reasoning, justifications, and conclusions is essential. Teachers and students should regularly use technology to reinforce relationships among functions, to confirm written work, to implement experimentation, and to assist in interpreting results.

AP Calculus BC Course Overview

AP Calculus BC is designed to be the equivalent to both first and second semester college calculus courses. AP Calculus BC applies the content and skills learned in AP Calculus AB to parametrically defined curves, polar curves, and vector-valued functions; develops additional integration techniques and applications; and introduces the topics of sequences and series.

PREREQUISITES

Before studying calculus, all students should complete the equivalent of four years of secondary mathematics designed for college-bound students: courses that should prepare them with a strong foundation in reasoning with algebraic symbols and working with algebraic structures. Prospective calculus students should take courses in which they study algebra, geometry, trigonometry, analytic geometry, and elementary functions. These functions include linear, polynomial, rational, exponential, logarithmic, trigonometric, inverse trigonometric, and piecewise-defined functions. In particular, before studying calculus, students must be familiar with the properties of functions, the composition of functions, the algebra of functions, and the graphs of functions.

Students must also understand the language of functions (domain and range, odd and even, periodic, symmetry, zeros, intercepts, and descriptors such as increasing and decreasing). Students should also know how the sine and cosine functions are defined from the unit circle and know the values of the trigonometric functions at the numbers 0, $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, and their multiples. Students who take AP Calculus BC should have basic familiarity with sequences and series, as well as some exposure to parametric and polar equations.

AP Calculus BC Course Content

The course content is organized into ten commonly taught units, which have been arranged in the following suggested, logical sequence:

- **Unit 1:** Limits and Continuity
- **Unit 2:** Differentiation: Definition and Fundamental Properties
- **Unit 3:** Differentiation: Composite, Implicit, and Inverse Functions
- **Unit 4:** Contextual Applications of Differentiation
- **Unit 5:** Analytical Applications of Differentiation
- **Unit 6:** Integration and Accumulation of Change
- **Unit 7:** Differential Equations
- **Unit 8:** Applications of Integration
- **Unit 9:** Parametric Equations, Polar Coordinates, and Vector-Valued Functions
- **Unit 10:** Infinite Sequences and Series

Each unit is broken down into teachable segments called topics.

In addition, the following big ideas serve as the foundation of the course, enabling students to create meaningful connections among concepts and develop deeper conceptual understanding:

- **Change:** Using derivatives to describe rates of change of one variable with respect to another or using definite integrals to describe the net change in one variable over an interval of another allows students to understand change in a variety of contexts.
- **Limits:** Beginning with a discrete model and then considering the consequences of a limiting case allows us to model real-world behavior and to discover and understand important ideas, definitions, formulas, and theorems in calculus.
- **Analysis of Functions:** Calculus allows us to analyze the behaviors of functions by relating limits to differentiation, integration, and infinite series and relating each of these concepts to the others.

AP Calculus BC Mathematical Practices

Students should develop the following mathematical practices while exploring course concepts:

- **Implementing Mathematical Processes:** Determine expressions and values using mathematical procedures and rules.
- **Connecting Representations:** Translate mathematical information from a single representation or across multiple representations.
- **Justification:** Justify reasoning and solutions.
- **Communication and Notation:** Use correct notation, language, and mathematical conventions to communicate results or solutions.

AP Calculus BC Exam Structure

AP CALCULUS BC EXAM: 3 HOURS, 15 MINUTES

Assessment Overview

The AP Calculus BC Exam assesses student understanding of the mathematical practices and learning objectives outlined in the course framework. The exam is 3 hours and 15 minutes long and includes 45 multiple-choice questions and 6 free-response questions.

Format of Assessment

Section I: Multiple-choice | 45 Questions | 105 Minutes | 50% of Exam Score

- **Part A:** 30 questions; 60 minutes (graphing calculator not permitted; **33.3% of Exam Score**).
 - **Part B:** 15 questions; 45 minutes (graphing calculator required; **16.7% of Exam Score**).
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Section II: Free-response | 6 Questions | 90 Minutes | 50% of Exam Score

- **Part A:** 2 questions; 30 minutes (graphing calculator required; **16.7% of Exam Score**).
- **Part B:** 4 questions; 60 minutes (graphing calculator not permitted; **33.3% of Exam Score**).

Exam Components

Sample Multiple-Choice Question

A curve is defined by the parametric equations $x(t) = t^2 + 3$ and $y(t) = \sin(t^2)$. Which of the following is an expression for d^2y/dx^2 in terms of t ?

- (A) $-\sin(t^2)$
- (B) $-2t\sin(t^2)$
- (C) $\cos(t^2) - 2t^2\sin(t^2)$
- (D) $2\cos(t^2) - 4t^2\sin(t^2)$

Sample Free-Response Question

A graphing calculator is required on this part of the exam.

For $0 \leq t \leq 5$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 1$, the particle is at position $(2, -7)$. It is known that $dx/dt = \sin(t/t + 3)$ and $dy/dt = e^{\cos t}$.

- (A) Write an equation for the line tangent to the curve at the point $(2, -7)$.
- (B) Find the y -coordinate of the position of the particle at time $t = 4$.
- (C) Find the total distance traveled by the particle from time $t = 1$ to time $t = 4$.
- (D) Find the time at which the speed of the particle is 2.5. Find the acceleration vector of the particle at this time.