

*Advanced Placement Program*<sup>®</sup>

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1998 AP<sup>®</sup> CALCULUS AB  
EXAMINATION

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The College Board  
Educational Excellence for All Students

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**AREA 1 - COMPLETE THIS AREA AT EVERY EXAMINATION.**

**PLACE AP® NUMBER LABEL HERE.**

To maintain the security of the exam and the validity of my AP grade, I will allow no one other than myself to see the multiple-choice questions and will seal the appropriate section when asked to do so. In addition, I am aware of and agree to the Program's policies and procedures as outlined in the 1998 AP Bulletin for Students and Parents.

**A. SIGNATURE**

\_\_\_\_\_

Sign your name as it will appear on your college applications.

<b>B. YOUR AP NUMBER</b>															<b>C. NAME</b>															<b>M.I.</b>																																																																					
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**D. ADMIN. DAY IN MAY**

01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

**E. TIME OF DAY**

6 AM 7 AM 8 AM 9 AM 10 AM 11 AM 12 PM 1 PM 2 PM 3 PM 4 PM 5 PM

**F. AP EXAMINATION TO BE TAKEN USING THIS ANSWER SHEET**

Print examination name: \_\_\_\_\_

Fill in the appropriate oval below for examination name and number.

07	<input type="radio"/>	U.S. History	55	<input type="radio"/>	German Language
13	<input type="radio"/>	Art: History of	57	<input type="radio"/>	Gov. & Pol.: U.S.
14	<input type="radio"/>	Art: Studio Drawing	58	<input type="radio"/>	Gov. & Pol.: Comp.
15	<input type="radio"/>	Art: Studio General	59	<input type="radio"/>	International English Language
20	<input type="radio"/>	Biology	60	<input type="radio"/>	Latin: Vergil
25	<input type="radio"/>	Chemistry	60	<input type="radio"/>	Latin: Literature
31	<input type="radio"/>	Computer Science A	66	<input type="radio"/>	Calculus AB
33	<input type="radio"/>	Computer Science AB	68	<input type="radio"/>	Calculus BC
34	<input type="radio"/>	Economics: Micro	75	<input type="radio"/>	Music Theory
35	<input type="radio"/>	Economics: Macro	78	<input type="radio"/>	Physics B
36	<input type="radio"/>	Eng. Language & Comp.	80	<input type="radio"/>	Physics C: Mech.
37	<input type="radio"/>	Eng. Literature & Comp.	85	<input type="radio"/>	Psychology
40	<input type="radio"/>	Environmental Science	87	<input type="radio"/>	Spanish Language
43	<input type="radio"/>	European History	89	<input type="radio"/>	Spanish Literature
48	<input type="radio"/>	French Language	90	<input type="radio"/>	Statistics
51	<input type="radio"/>	French Literature			

**G. What language do you know best?**

English

English and another language

Another language

**H. DO NOT COMPLETE THIS SECTION UNLESS INSTRUCTED TO DO SO.**

**Essay Choices**

Fill in the ovals under the numbers of the essay questions you answered in this examination.

1	2	3	4	5	6	7	8	9	10	11	12
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**SCHOOL USE ONLY**

Fee Reduction Granted	Report to Teachers Section Designation
0 1 2 3 4 5 6 7 8 9	0 1 2 3 4 5 6 7 8 9

**AREA 2 - COMPLETE THIS AREA ONLY ONCE.**

**I. SEX**

1 Male 2 Female

**J. PRESENT GRADE LEVEL**

1 9th grade 2 10th grade 3 11th grade 4 12th grade 5 College 6 Other

**K. DATE OF BIRTH**

Month	Day	Year
01 Jan. 02 Feb. 03 Mar. 04 Apr. 05 May 06 June 07 July 08 Aug. 09 Sept. 10 Oct. 11 Nov. 12 Dec.	01 02 03 04 05 06 07 08 09 10 11 12	00 01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

**L. SOCIAL SECURITY NUMBER**

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**M. ETHNIC GROUP**

1 American Indian or Alaskan native 2 Black or African American 3 Mexican American or Chicano 4 Asian, Asian American, or Pacific Islander 5 Puerto Rican 6 South American, Latin American, Central American, or other Hispanic 7 White 8 Other

**N. EXPECTED DATE OF COLLEGE ENTRANCE**

F Fall 1998 W Winter/Spring 1999 S Summer 2000 U Undecided

**O. Will you be applying for Sophomore Standing at college?**

Yes  No

**ETS USE ONLY**

Exam	Exam
01 02 03 04 05 06 07 08 09	10 11 12 13 14 15 16 17 18 19

**P. STUDENT SEARCH SERVICE OF THE COLLEGE BOARD**

(Complete ONLY if you are a SOPHOMORE or a JUNIOR.)

Yes, I want the College Board to send information about me to colleges, universities, and governmental scholarship programs interested in students like me.

No, I do not want the College Board to send information about me to colleges, universities, and governmental scholarship programs through the Student Search Service.



**Q. THIS SECTION IS FOR THE SURVEY QUESTIONS IN THE CANDIDATE PACK. (DO NOT PUT RESPONSES TO EXAM QUESTIONS IN THIS SECTION.) BE SURE EACH MARK IS DARK AND COMPLETELY FILLS THE OVAL.**

- 1 (A) (B) (C) (D) (E)  
2 (A) (B) (C) (D) (E)  
3 (A) (B) (C) (D) (E)

- 4 (A) (B) (C) (D) (E)  
5 (A) (B) (C) (D) (E)

**DO NOT COMPLETE THIS SECTION UNLESS INSTRUCTED TO DO SO.**

R. If this answer sheet is for the French Language, French Literature, German Language, Spanish Language, or Spanish Literature Examination, please answer the following questions. (Your responses will not affect your grade.)

1. Have you lived or studied for one month or more in a country where the language of the exam you are now taking is spoken?  Yes  No
2. Do you regularly speak or hear the language at home?  Yes  No

**INDICATE YOUR ANSWERS TO THE EXAM QUESTIONS IN THIS SECTION. IF A QUESTION HAS ONLY FOUR ANSWER OPTIONS, DO NOT MARK OPTION (E). YOUR ANSWER SHEET WILL BE SCORED BY MACHINE. USE ONLY NO. 2 PENCILS TO MARK YOUR ANSWERS ON PAGES 2 AND 3 (ONE RESPONSE PER QUESTION). AFTER YOU HAVE DETERMINED YOUR RESPONSE, BE SURE TO COMPLETELY FILL IN THE OVAL CORRESPONDING TO THE NUMBER OF THE QUESTION YOU ARE ANSWERING. STRAY MARKS AND SMUDGES COULD BE READ AS ANSWERS, SO ERASE CAREFULLY AND COMPLETELY. ANY IMPROPER GRIDDING MAY AFFECT YOUR GRADE.**

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FOR QUESTIONS 76-151, SEE PAGE 3.

**DO NOT WRITE IN THIS AREA.**



BE SURE EACH MARK IS DARK AND COMPLETELY FILLS THE OVAL. IF A QUESTION HAS ONLY FOUR ANSWER OPTIONS, DO NOT MARK OPTION E.

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ETS USE ONLY			
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PT1			
PT2			
PT3			
PT4			
TOT			
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TA1			
TA2			

DO NOT WRITE IN THIS AREA.



## CALCULUS AB

A CALCULATOR CANNOT BE USED ON PART A OF SECTION I. A GRAPHING CALCULATOR FROM THE APPROVED LIST IS REQUIRED ON PART B OF SECTION I AND FOR SECTION II OF THE EXAMINATION. CALCULATOR MEMORIES NEED NOT BE CLEARED. COMPUTERS, NONGRAPHING SCIENTIFIC CALCULATORS, CALCULATORS WITH QWERTY KEYBOARDS, AND ELECTRONIC WRITING PADS ARE NOT ALLOWED. CALCULATORS MAY NOT BE SHARED AND COMMUNICATION BETWEEN CALCULATORS IS PROHIBITED DURING THE EXAMINATION. ATTEMPTS TO REMOVE TEST MATERIALS FROM THE ROOM BY ANY METHOD WILL RESULT IN THE INVALIDATION OF TEST SCORES.

## SECTION I

Time—1 hour and 45 minutes

All questions are given equal weight.

Percent of total grade—50

Part A: 55 minutes, 28 multiple-choice questions

A calculator is NOT allowed.

Part B: 50 minutes, 17 multiple-choice questions

A graphing calculator is required.

Parts A and B of Section I are printed in this examination booklet; Section II, which consists of longer problems, is printed in a separate booklet.

General Instructions

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE INSTRUCTED TO DO SO.

INDICATE YOUR ANSWERS TO QUESTIONS IN PART A ON PAGE 2 OF THE SEPARATE ANSWER SHEET. THE ANSWERS TO QUESTIONS IN PART B SHOULD BE INDICATED ON PAGE 3 OF THE ANSWER SHEET. No credit will be given for anything written in this examination booklet, but you may use the booklet for notes or scratchwork. After you have decided which of the suggested answers is best, COMPLETELY fill in the corresponding oval on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely.

Example:

What is the arithmetic mean of the numbers 1, 3, and 6 ?

(A) 1

(B)  $\frac{7}{3}$

(C) 3

(D)  $\frac{10}{3}$

(E)  $\frac{7}{2}$

Sample Answer

(A) (B) (C) (D) (E)

Many candidates wonder whether or not to guess the answers to questions about which they are not certain. In this section of the examination, as a correction for haphazard guessing, one-fourth of the number of questions you answer incorrectly will be subtracted from the number of questions you answer correctly. It is improbable, therefore, that mere guessing will improve your score significantly; it may even lower your score, and it does take time. If, however, you are not sure of the best answer but have some knowledge of the question and are able to eliminate one or more of the answer choices as wrong, your chance of answering correctly is improved, and it may be to your advantage to answer such a question.

Use your time effectively, working as rapidly as you can without losing accuracy. Do not spend too much time on questions that are too difficult. Go on to other questions and come back to the difficult ones later if you have time. It is not expected that everyone will be able to answer all the multiple-choice questions.

CALCULUS AB  
SECTION I, Part A  
Time — 55 minutes  
Number of questions — 28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.

1. What is the  $x$ -coordinate of the point of inflection on the graph of  $y = \frac{1}{3}x^3 + 5x^2 + 24$  ?

(A) 5

(B) 0

(C)  $-\frac{10}{3}$

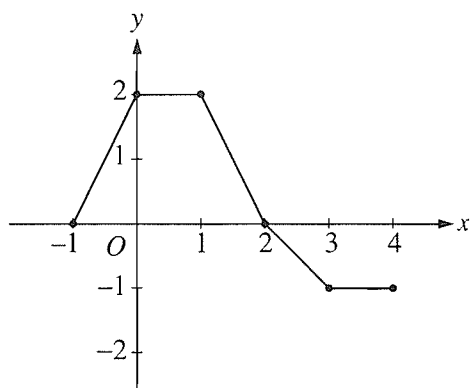
(D) -5

(E) -10

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**GO ON TO THE NEXT PAGE** 





2. The graph of a piecewise-linear function  $f$ , for  $-1 \leq x \leq 4$ , is shown above. What is the value of

$$\int_{-1}^4 f(x) dx ?$$

- (A) 1                      (B) 2.5                      (C) 4                      (D) 5.5                      (E) 8

3.  $\int_1^2 \frac{1}{x^2} dx =$

- (A)  $-\frac{1}{2}$                       (B)  $\frac{7}{24}$                       (C)  $\frac{1}{2}$                       (D) 1                      (E)  $2 \ln 2$

**GO ON TO THE NEXT PAGE**

4. If  $f$  is continuous for  $a \leq x \leq b$  and differentiable for  $a < x < b$ , which of the following could be false?

(A)  $f'(c) = \frac{f(b) - f(a)}{b - a}$  for some  $c$  such that  $a < c < b$ .

(B)  $f'(c) = 0$  for some  $c$  such that  $a < c < b$ .

(C)  $f$  has a minimum value on  $a \leq x \leq b$ .

(D)  $f$  has a maximum value on  $a \leq x \leq b$ .

(E)  $\int_a^b f(x) dx$  exists.

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5.  $\int_0^x \sin t dt =$

(A)  $\sin x$

(B)  $-\cos x$

(C)  $\cos x$

(D)  $\cos x - 1$

(E)  $1 - \cos x$

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**GO ON TO THE NEXT PAGE** 

6. If  $x^2 + xy = 10$ , then when  $x = 2$ ,  $\frac{dy}{dx} =$

(A)  $-\frac{7}{2}$

(B)  $-2$

(C)  $\frac{2}{7}$

(D)  $\frac{3}{2}$

(E)  $\frac{7}{2}$

7.  $\int_1^e \left(\frac{x^2 - 1}{x}\right) dx =$

(A)  $e - \frac{1}{e}$

(B)  $e^2 - e$

(C)  $\frac{e^2}{2} - e + \frac{1}{2}$

(D)  $e^2 - 2$

(E)  $\frac{e^2}{2} - \frac{3}{2}$

**GO ON TO THE NEXT PAGE** 

8. Let  $f$  and  $g$  be differentiable functions with the following properties:

(i)  $g(x) > 0$  for all  $x$

(ii)  $f(0) = 1$

If  $h(x) = f(x)g(x)$  and  $h'(x) = f(x)g'(x)$ , then  $f(x) =$

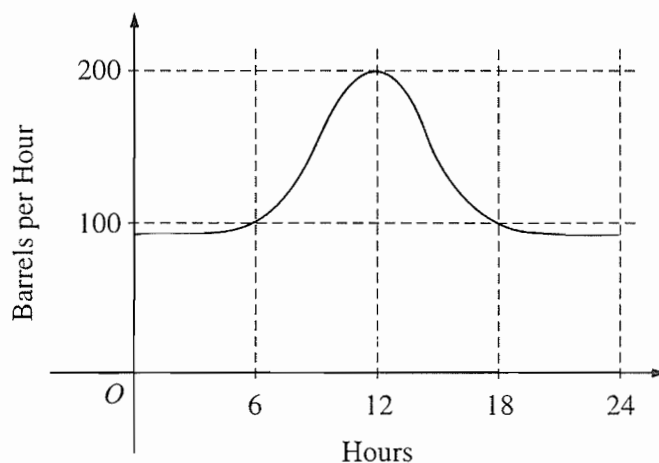
(A)  $f'(x)$

(B)  $g(x)$

(C)  $e^x$

(D) 0

(E) 1



9. The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500

(B) 600

(C) 2,400

(D) 3,000

(E) 4,800

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**GO ON TO THE NEXT PAGE** 

10. What is the instantaneous rate of change at  $x = 2$  of the function  $f$  given by  $f(x) = \frac{x^2 - 2}{x - 1}$  ?

- (A)  $-2$                       (B)  $\frac{1}{6}$                       (C)  $\frac{1}{2}$                       (D)  $2$                       (E)  $6$

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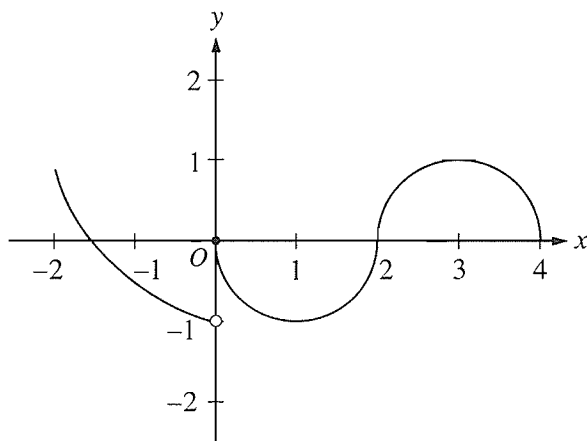
11. If  $f$  is a linear function and  $0 < a < b$ , then  $\int_a^b f''(x) dx =$

- (A)  $0$                       (B)  $1$                       (C)  $\frac{ab}{2}$                       (D)  $b - a$                       (E)  $\frac{b^2 - a^2}{2}$



12. If  $f(x) = \begin{cases} \ln x & \text{for } 0 < x \leq 2 \\ x^2 \ln 2 & \text{for } 2 < x \leq 4, \end{cases}$  then  $\lim_{x \rightarrow 2} f(x)$  is

- (A)  $\ln 2$       (B)  $\ln 8$       (C)  $\ln 16$       (D) 4      (E) nonexistent



13. The graph of the function  $f$  shown in the figure above has a vertical tangent at the point  $(2, 0)$  and horizontal tangents at the points  $(1, -1)$  and  $(3, 1)$ . For what values of  $x$ ,  $-2 < x < 4$ , is  $f$  not differentiable?

- (A) 0 only      (B) 0 and 2 only      (C) 1 and 3 only      (D) 0, 1, and 3 only      (E) 0, 1, 2, and 3

GO ON TO THE NEXT PAGE

14. A particle moves along the  $x$ -axis so that its position at time  $t$  is given by  $x(t) = t^2 - 6t + 5$ . For what value of  $t$  is the velocity of the particle zero?

- (A) 1                      (B) 2                      (C) 3                      (D) 4                      (E) 5

---

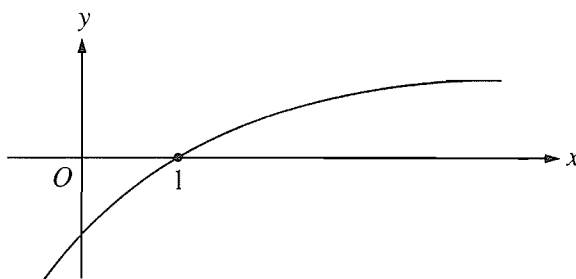
15. If  $F(x) = \int_0^x \sqrt{t^3 + 1} dt$ , then  $F'(2) =$

- (A) -3                      (B) -2                      (C) 2                      (D) 3                      (E) 18
- 



16. If  $f(x) = \sin(e^{-x})$ , then  $f'(x) =$

- (A)  $-\cos(e^{-x})$
- (B)  $\cos(e^{-x}) + e^{-x}$
- (C)  $\cos(e^{-x}) - e^{-x}$
- (D)  $e^{-x} \cos(e^{-x})$
- (E)  $-e^{-x} \cos(e^{-x})$



17. The graph of a twice-differentiable function  $f$  is shown in the figure above. Which of the following is true?

- (A)  $f(1) < f'(1) < f''(1)$
- (B)  $f(1) < f''(1) < f'(1)$
- (C)  $f'(1) < f(1) < f''(1)$
- (D)  $f''(1) < f(1) < f'(1)$
- (E)  $f''(1) < f'(1) < f(1)$

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18. An equation of the line tangent to the graph of  $y = x + \cos x$  at the point  $(0, 1)$  is

- (A)  $y = 2x + 1$       (B)  $y = x + 1$       (C)  $y = x$       (D)  $y = x - 1$       (E)  $y = 0$
- 

19. If  $f''(x) = x(x + 1)(x - 2)^2$ , then the graph of  $f$  has inflection points when  $x =$

- (A)  $-1$  only      (B)  $2$  only      (C)  $-1$  and  $0$  only      (D)  $-1$  and  $2$  only      (E)  $-1, 0,$  and  $2$  only
- 



20. What are all values of  $k$  for which  $\int_{-3}^k x^2 dx = 0$  ?

- (A)  $-3$                       (B)  $0$                       (C)  $3$                       (D)  $-3$  and  $3$                       (E)  $-3, 0,$  and  $3$
- 

21. If  $\frac{dy}{dt} = ky$  and  $k$  is a nonzero constant, then  $y$  could be

- (A)  $2e^{kty}$                       (B)  $2e^{kt}$                       (C)  $e^{kt} + 3$                       (D)  $kty + 5$                       (E)  $\frac{1}{2}ky^2 + \frac{1}{2}$
- 

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22. The function  $f$  is given by  $f(x) = x^4 + x^2 - 2$ . On which of the following intervals is  $f$  increasing?

(A)  $\left(-\frac{1}{\sqrt{2}}, \infty\right)$

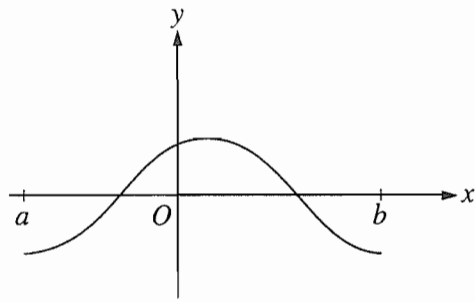
(B)  $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(C)  $(0, \infty)$

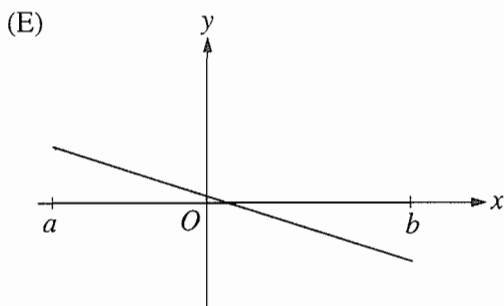
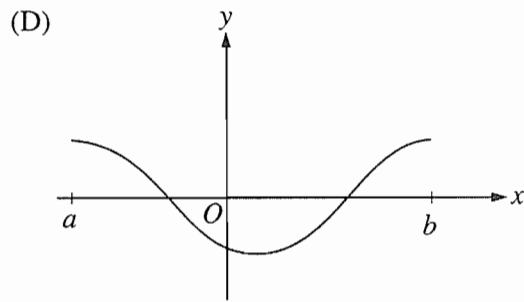
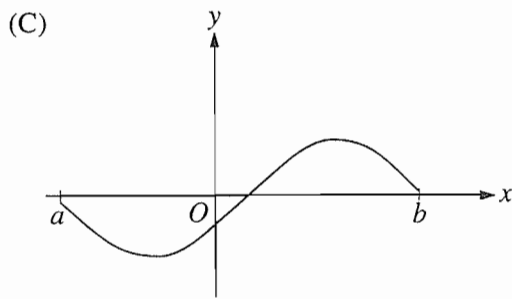
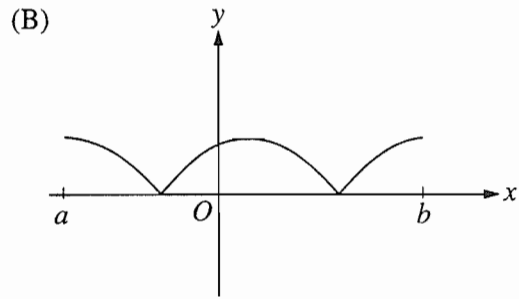
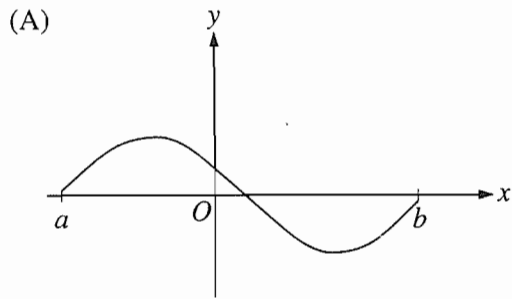
(D)  $(-\infty, 0)$

(E)  $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

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23. The graph of  $f$  is shown in the figure above. Which of the following could be the graph of the derivative of  $f$ ?



GO ON TO THE NEXT PAGE

24. The maximum acceleration attained on the interval  $0 \leq t \leq 3$  by the particle whose velocity is given by  $v(t) = t^3 - 3t^2 + 12t + 4$  is
- (A) 9                      (B) 12                      (C) 14                      (D) 21                      (E) 40

---

25. What is the area of the region between the graphs of  $y = x^2$  and  $y = -x$  from  $x = 0$  to  $x = 2$ ?

- (A)  $\frac{2}{3}$                       (B)  $\frac{8}{3}$                       (C) 4                      (D)  $\frac{14}{3}$                       (E)  $\frac{16}{3}$
- 



$x$	0	1	2
$f(x)$	1	$k$	2

26. The function  $f$  is continuous on the closed interval  $[0, 2]$  and has values that are given in the table above. The equation  $f(x) = \frac{1}{2}$  must have at least two solutions in the interval  $[0, 2]$  if  $k =$

- (A) 0                      (B)  $\frac{1}{2}$                       (C) 1                      (D) 2                      (E) 3
- 

27. What is the average value of  $y = x^2\sqrt{x^3 + 1}$  on the interval  $[0, 2]$  ?

- (A)  $\frac{26}{9}$                       (B)  $\frac{52}{9}$                       (C)  $\frac{26}{3}$                       (D)  $\frac{52}{3}$                       (E) 24
- 

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28. If  $f(x) = \tan(2x)$ , then  $f'\left(\frac{\pi}{6}\right) =$

(A)  $\sqrt{3}$

(B)  $2\sqrt{3}$

(C) 4

(D)  $4\sqrt{3}$

(E) 8

---

END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS PART ONLY.  
DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB  
SECTION I, Part B  
Time — 50 minutes  
Number of questions — 17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON  
THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

**BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76-92.**

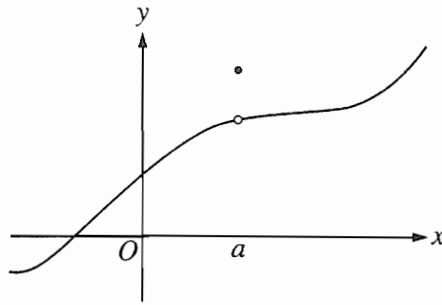
**YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.**

In this test:

- (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- (2) Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.

**GO ON TO THE NEXT PAGE** 





76. The graph of a function  $f$  is shown above. Which of the following statements about  $f$  is false?

- (A)  $f$  is continuous at  $x = a$ .
- (B)  $f$  has a relative maximum at  $x = a$ .
- (C)  $x = a$  is in the domain of  $f$ .
- (D)  $\lim_{x \rightarrow a^+} f(x)$  is equal to  $\lim_{x \rightarrow a^-} f(x)$ .
- (E)  $\lim_{x \rightarrow a} f(x)$  exists.

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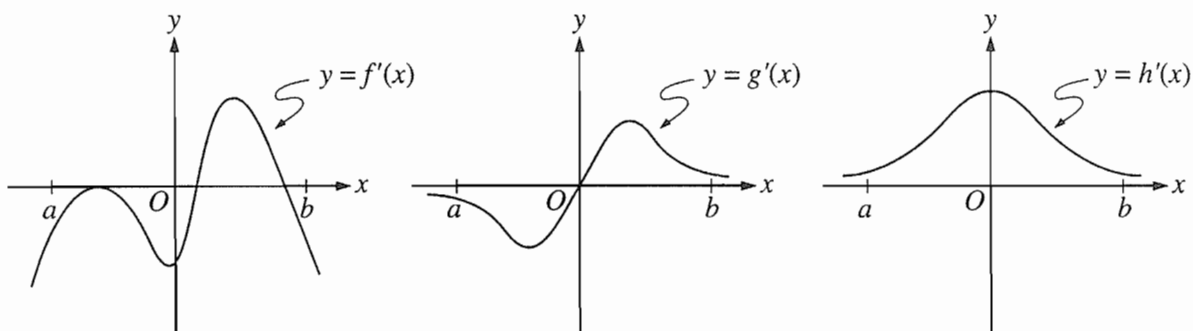
77. Let  $f$  be the function given by  $f(x) = 3e^{2x}$  and let  $g$  be the function given by  $g(x) = 6x^3$ . At what value of  $x$  do the graphs of  $f$  and  $g$  have parallel tangent lines?

- (A)  $-0.701$
  - (B)  $-0.567$
  - (C)  $-0.391$
  - (D)  $-0.302$
  - (E)  $-0.258$
- 

78. The radius of a circle is decreasing at a constant rate of 0.1 centimeter per second. In terms of the circumference  $C$ , what is the rate of change of the area of the circle, in square centimeters per second?

- (A)  $-(0.2)\pi C$
  - (B)  $-(0.1)C$
  - (C)  $-\frac{(0.1)C}{2\pi}$
  - (D)  $(0.1)^2 C$
  - (E)  $(0.1)^2 \pi C$
- 

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79. The graphs of the derivatives of the functions  $f$ ,  $g$ , and  $h$  are shown above. Which of the functions  $f$ ,  $g$ , or  $h$  have a relative maximum on the open interval  $a < x < b$ ?

- (A)  $f$  only
- (B)  $g$  only
- (C)  $h$  only
- (D)  $f$  and  $g$  only
- (E)  $f$ ,  $g$ , and  $h$

80. The first derivative of the function  $f$  is given by  $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$ . How many critical values does  $f$  have on the open interval  $(0, 10)$ ?

- (A) One
- (B) Three
- (C) Four
- (D) Five
- (E) Seven

**GO ON TO THE NEXT PAGE**

81. Let  $f$  be the function given by  $f(x) = |x|$ . Which of the following statements about  $f$  are true?

- I.  $f$  is continuous at  $x = 0$ .
- II.  $f$  is differentiable at  $x = 0$ .
- III.  $f$  has an absolute minimum at  $x = 0$ .

(A) I only      (B) II only      (C) III only      (D) I and III only      (E) II and III only

---

82. If  $f$  is a continuous function and if  $F'(x) = f(x)$  for all real numbers  $x$ , then  $\int_1^3 f(2x)dx =$

- (A)  $2F(3) - 2F(1)$
  - (B)  $\frac{1}{2}F(3) - \frac{1}{2}F(1)$
  - (C)  $2F(6) - 2F(2)$
  - (D)  $F(6) - F(2)$
  - (E)  $\frac{1}{2}F(6) - \frac{1}{2}F(2)$
- 

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83. If  $a \neq 0$ , then  $\lim_{x \rightarrow a} \frac{x^2 - a^2}{x^4 - a^4}$  is

- (A)  $\frac{1}{a^2}$       (B)  $\frac{1}{2a^2}$       (C)  $\frac{1}{6a^2}$       (D) 0      (E) nonexistent

84. Population  $y$  grows according to the equation  $\frac{dy}{dt} = ky$ , where  $k$  is a constant and  $t$  is measured in years. If the population doubles every 10 years, then the value of  $k$  is

- (A) 0.069      (B) 0.200      (C) 0.301      (D) 3.322      (E) 5.000

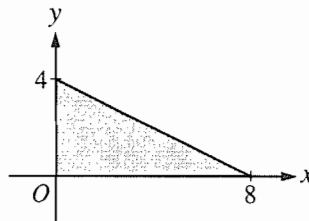
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$x$	2	5	7	8
$f(x)$	10	30	40	20

85. The function  $f$  is continuous on the closed interval  $[2, 8]$  and has values that are given in the table above. Using the subintervals  $[2, 5]$ ,  $[5, 7]$ , and  $[7, 8]$ , what is the trapezoidal approximation of

$$\int_2^8 f(x) dx ?$$

- (A) 110                      (B) 130                      (C) 160                      (D) 190                      (E) 210
- 



86. The base of a solid is a region in the first quadrant bounded by the  $x$ -axis, the  $y$ -axis, and the line  $x + 2y = 8$ , as shown in the figure above. If cross sections of the solid perpendicular to the  $x$ -axis are semicircles, what is the volume of the solid?

- (A) 12.566                      (B) 14.661                      (C) 16.755                      (D) 67.021                      (E) 134.041
- 

**GO ON TO THE NEXT PAGE** 

87. Which of the following is an equation of the line tangent to the graph of  $f(x) = x^4 + 2x^2$  at the point where  $f'(x) = 1$  ?

- (A)  $y = 8x - 5$
- (B)  $y = x + 7$
- (C)  $y = x + 0.763$
- (D)  $y = x - 0.122$
- (E)  $y = x - 2.146$

88. Let  $F(x)$  be an antiderivative of  $\frac{(\ln x)^3}{x}$ . If  $F(1) = 0$ , then  $F(9) =$

- (A) 0.048
- (B) 0.144
- (C) 5.827
- (D) 23.308
- (E) 1,640.250



89. If  $g$  is a differentiable function such that  $g(x) < 0$  for all real numbers  $x$  and if  $f'(x) = (x^2 - 4)g(x)$ , which of the following is true?

- (A)  $f$  has a relative maximum at  $x = -2$  and a relative minimum at  $x = 2$ .
  - (B)  $f$  has a relative minimum at  $x = -2$  and a relative maximum at  $x = 2$ .
  - (C)  $f$  has relative minima at  $x = -2$  and at  $x = 2$ .
  - (D)  $f$  has relative maxima at  $x = -2$  and at  $x = 2$ .
  - (E) It cannot be determined if  $f$  has any relative extrema.
- 

90. If the base  $b$  of a triangle is increasing at a rate of 3 inches per minute while its height  $h$  is decreasing at a rate of 3 inches per minute, which of the following must be true about the area  $A$  of the triangle?

- (A)  $A$  is always increasing.
  - (B)  $A$  is always decreasing.
  - (C)  $A$  is decreasing only when  $b < h$ .
  - (D)  $A$  is decreasing only when  $b > h$ .
  - (E)  $A$  remains constant.
- 

**GO ON TO THE NEXT PAGE** 



91. Let  $f$  be a function that is differentiable on the open interval  $(1, 10)$ . If  $f(2) = -5$ ,  $f(5) = 5$ , and  $f(9) = -5$ , which of the following must be true?
- I.  $f$  has at least 2 zeros.
  - II. The graph of  $f$  has at least one horizontal tangent.
  - III. For some  $c$ ,  $2 < c < 5$ ,  $f(c) = 3$ .
- (A) None  
 (B) I only  
 (C) I and II only  
 (D) I and III only  
 (E) I, II and III

92. If  $0 \leq k < \frac{\pi}{2}$  and the area under the curve  $y = \cos x$  from  $x = k$  to  $x = \frac{\pi}{2}$  is 0.1, then  $k =$
- (A) 1.471                      (B) 1.414                      (C) 1.277                      (D) 1.120                      (E) 0.436

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY.  
 CHECK YOUR WORK ON PART B ONLY.  
 DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

MAKE SURE YOU HAVE PLACED YOUR AP NUMBER LABEL ON YOUR  
 ANSWER SHEET AND HAVE WRITTEN AND GRIDDED YOUR AP NUMBER  
 IN THE APPROPRIATE SECTION OF YOUR ANSWER SHEET.

AFTER TIME HAS BEEN CALLED, ANSWER QUESTIONS 93-96.

93. Which graphing calculator did you use during the examination?
- (A) Casio 6300, Casio 7000, Casio 7300, Casio 7400, or Casio 7700
  - (B) Texas Instruments TI-80 or TI-81
  - (C) Casio 9700, Casio 9800, Casio 9850, Sharp 9200, Sharp 9300, Texas Instruments TI-82, Texas Instruments TI-83, Texas Instruments TI-85, or Texas Instruments TI-86
  - (D) Hewlett-Packard HP-48 series or HP-38G
  - (E) Some other calculator
94. During your Calculus AB course, which of the following best describes your calculator use?
- (A) I used my own graphing calculator.
  - (B) I used a graphing calculator furnished by my school, both in class and at home.
  - (C) I used a graphing calculator furnished by my school only in class.
  - (D) I used a graphing calculator furnished by my school mostly in class, but occasionally at home.
  - (E) I did not use a graphing calculator.
95. During your Calculus AB course, which of the following describes approximately how often a graphing calculator was used by you or your teacher in classroom learning activities?
- (A) Almost every class
  - (B) About three-quarters of the classes
  - (C) About one-half of the classes
  - (D) About one-quarter of the classes
  - (E) Seldom or never
96. During your Calculus AB course, which of the following describes approximately how often you were allowed to use a graphing calculator on tests?
- (A) Almost all of the time
  - (B) About three-quarters of the time
  - (C) About one-half of the time
  - (D) About one-quarter of the time
  - (E) Seldom or never

## CALCULUS AB

## SECTION II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

## GENERAL INSTRUCTIONS

You may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight. The problems are printed in the booklet and in the green insert; it may be easier for you to first look over all problems in the insert. When you are told to begin, open your booklet, carefully tear out the green insert, and start to work.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- You should write all work for each part of each problem in the space provided for that part in the booklet. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. You will be graded on the correctness and completeness of your methods as well as the accuracy of your final answers. Correct answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.
- You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example,  $\int_1^5 x^2 dx$  may not be written as `fnInt(X^2, X, 1, 5)`.
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.
- Unless otherwise specified, the domain of a function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number.

CALCULUS AB

SECTION II

Time — 1 hour and 30 minutes

Number of problems — 6

Percent of total grade — 50

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

**REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.**

General instructions for this section are printed on the back cover of this booklet.

1. Let  $R$  be the region bounded by the  $x$ -axis, the graph of  $y = \sqrt{x}$ , and the line  $x = 4$ .

(a) Find the area of the region  $R$ .

---

(b) Find the value of  $h$  such that the vertical line  $x = h$  divides the region  $R$  into two regions of equal area.

**GO ON TO THE NEXT PAGE** 

(c) Find the volume of the solid generated when  $R$  is revolved about the  $x$ -axis.

---

(d) The vertical line  $x = k$  divides the region  $R$  into two regions such that when these two regions are revolved about the  $x$ -axis, they generate solids with equal volumes. Find the value of  $k$ .

**GO ON TO THE NEXT PAGE** 

2. Let  $f$  be the function given by  $f(x) = 2xe^{2x}$ .

(a) Find  $\lim_{x \rightarrow -\infty} f(x)$  and  $\lim_{x \rightarrow \infty} f(x)$ .

---

(b) Find the absolute minimum value of  $f$ . Justify that your answer is an absolute minimum.

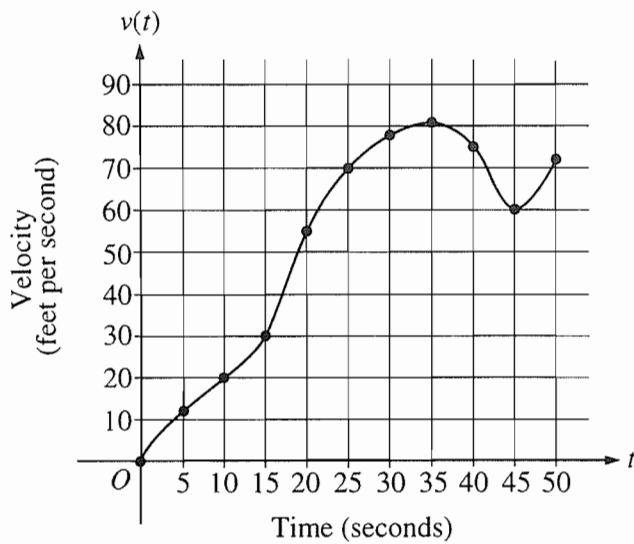
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(c) What is the range of  $f$ ?

---

(d) Consider the family of functions defined by  $y = bxe^{bx}$ , where  $b$  is a nonzero constant. Show that the absolute minimum value of  $bxe^{bx}$  is the same for all nonzero values of  $b$ .

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$t$ (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

3. The graph of the velocity  $v(t)$ , in ft/sec, of a car traveling on a straight road, for  $0 \leq t \leq 50$ , is shown above. A table of values for  $v(t)$ , at 5 second intervals of time  $t$ , is shown to the right of the graph.
- (a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

- 
- (b) Find the average acceleration of the car, in ft/sec<sup>2</sup>, over the interval  $0 \leq t \leq 50$ .

GO ON TO THE NEXT PAGE



- (c) Find one approximation for the acceleration of the car, in  $\text{ft}/\text{sec}^2$ , at  $t = 40$ . Show the computations you used to arrive at your answer.

- 
- (d) Approximate  $\int_0^{50} v(t) dt$  with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

**GO ON TO THE NEXT PAGE** 

4. Let  $f$  be a function with  $f(1) = 4$  such that for all points  $(x, y)$  on the graph of  $f$  the slope is given by  $\frac{3x^2 + 1}{2y}$ .

(a) Find the slope of the graph of  $f$  at the point where  $x = 1$ .

---

(b) Write an equation for the line tangent to the graph of  $f$  at  $x = 1$  and use it to approximate  $f(1.2)$ .

**GO ON TO THE NEXT PAGE** 

- (c) Find  $f(x)$  by solving the separable differential equation  $\frac{dy}{dx} = \frac{3x^2 + 1}{2y}$  with the initial condition  $f(1) = 4$ .

- 
- (d) Use your solution from part (c) to find  $f(1.2)$ .

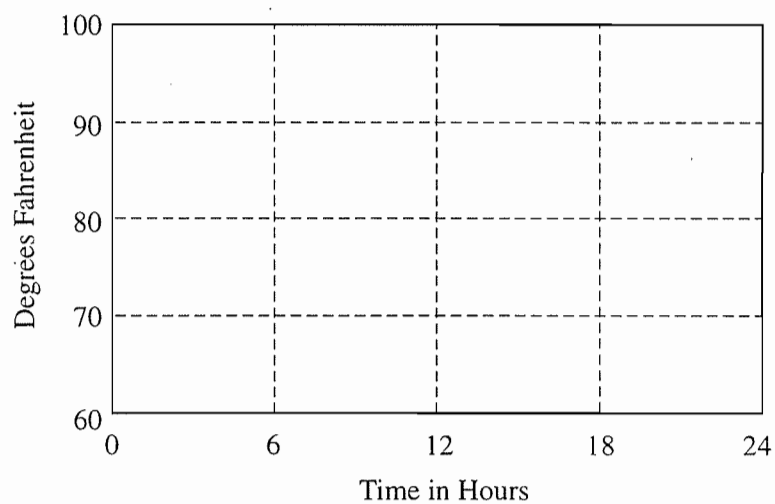
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5. The temperature outside a house during a 24-hour period is given by

$$F(t) = 80 - 10 \cos\left(\frac{\pi t}{12}\right), \quad 0 \leq t \leq 24,$$

where  $F(t)$  is measured in degrees Fahrenheit and  $t$  is measured in hours.

(a) Sketch the graph of  $F$  on the grid below.



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(b) Find the average temperature, to the nearest degree Fahrenheit, between  $t = 6$  and  $t = 14$ .

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- (c) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of  $t$  was the air conditioner cooling the house?

- 
- (d) The cost of cooling the house accumulates at the rate of \$0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24-hour period?

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6. Consider the curve defined by  $2y^3 + 6x^2y - 12x^2 + 6y = 1$ .

(a) Show that  $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$ .

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(b) Write an equation of each horizontal tangent line to the curve.

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- (c) The line through the origin with slope  $-1$  is tangent to the curve at point  $P$ . Find the  $x$ - and  $y$ -coordinates of point  $P$ .

END OF EXAMINATION

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