
AP[®] Physics C: Mechanics

Scoring Guidelines

Question 1: Mathematical Routines (MR)

10 points

- A (i) For determining a correct expression for the magnitude of the horizontal acceleration of the cube **Point A1**

Example Response

$$a_{\text{cube}} = \frac{F_N}{M}$$

- (ii) For a multistep derivation that includes Newton's second law **Point A2**

For indicating that the magnitude of the acceleration is $\frac{bv}{2M}$ **Point A3**

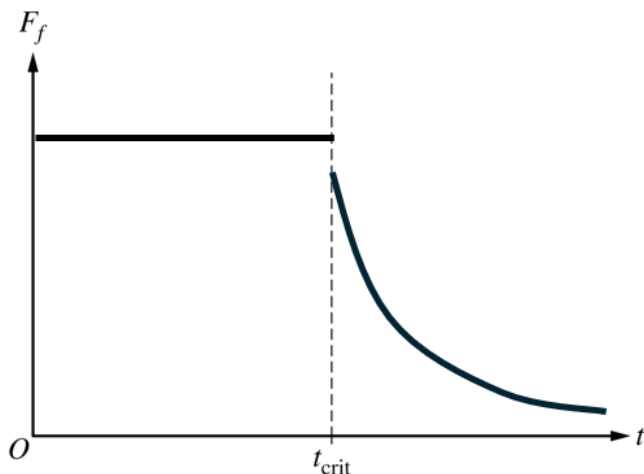
For separating dv and dt variables **Point A4**

For an exponential expression for F_N that evaluates to zero at infinite time **Point A5**

Example Response

$$\begin{aligned} a_{\text{cube}} &= a_{\text{box}} = a & a &= \frac{dv}{dt} = -\frac{bv}{2M} & \vec{F}_N &= Ma = M\left(\frac{-bv}{2M}\right) \\ a &= \frac{F_{\text{net}}}{m_{\text{total}}} = -\frac{bv}{2M} & \frac{dv}{v} &= -\frac{b}{2M} dt & \vec{F}_N &= -\frac{b}{2} v_0 e^{-\frac{bt}{2M}} \\ a &= -\frac{bv}{2M} & \int_{v_0}^v \frac{1}{v} dv &= -\frac{b}{2M} \int_0^t dt & F_N &= |\vec{F}_N| = \frac{b}{2} v_0 e^{-\frac{bt}{2M}} \\ & & \ln(v) - \ln(v_0) &= -\frac{bt}{2M} & & \\ & & \ln\left(\frac{v}{v_0}\right) &= -\frac{bt}{2M} & & \\ & & \frac{v}{v_0} &= e^{-\frac{bt}{2M}} & & \\ & & v &= v_0 e^{-\frac{bt}{2M}} & & \end{aligned}$$

(iii)	For sketching a horizontal line from $t = 0$ to $t = t_{\text{crit}}$	Point A6
	For sketching a concave up curve that never increases for $t > t_{\text{crit}}$	Point A7
Scoring Note: This point can be earned regardless of the discontinuity at $t = t_{\text{crit}}$.		

Example Response

B	For a multistep derivation that includes Newton's second law	Point B1
	For indicating that the sum of the frictional force and gravitational force exerted on the cube is zero at t_{crit}	Point B2
	For substituting the expression for F_N derived in part A into the expression for static friction	Point B3
Scoring Note: Responses that omit the subscript on μ_s may earn this point.		

Example Response

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = 0$$

$$F_f = F_g$$

$$\mu_s F_{N, \text{cube}} = F_g$$

$$\mu_s \left(\frac{b}{2} v_0 e^{-\frac{bt_{\text{crit}}}{2M}} \right) = Mg$$

$$e^{-\frac{bt_{\text{crit}}}{2M}} = \frac{2Mg}{\mu_s b v_0}$$

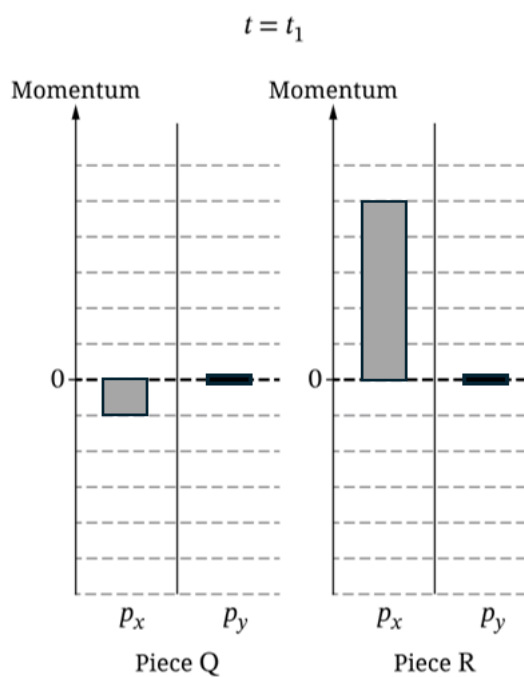
$$-\frac{bt_{\text{crit}}}{2M} = \ln \left(\frac{2Mg}{\mu_s b v_0} \right)$$

$$t_{\text{crit}} = -\frac{2M}{b} \ln \left(\frac{2Mg}{\mu_s b v_0} \right)$$

$$t_{\text{crit}} = \frac{2M}{b} \ln \left(\frac{\mu_s b v_0}{2Mg} \right)$$

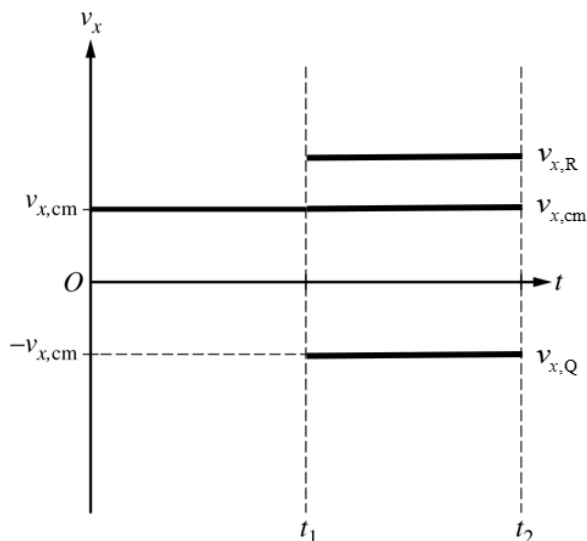
Question 2: Translation Between Representations (TBR)**12 points**

A	For drawing bars for p_x of Pieces Q and R that have heights which add to 4 units	Point A1
	For drawing lines on the zero-momentum line for p_y of Pieces Q and R	Point A2
	For drawing one of the following: • A bar for p_x of Piece Q with a height of -1 unit • A bar for p_x of Piece R with a height of 5 units	Point A3

Example Response

B	For a multistep derivation that includes one of the following: - Any kinematics equation relevant to constant acceleration - Conservation of momentum	Point B1
	For indicating one of the following: - That the vertical component of the velocity of the projectile is zero at $t = t_1$ - That the vertical displacement of the projectile is zero at $t = t_2$	Point B2
	For one of the following: - Correctly using conservation of momentum to determine the horizontal component of the velocity of Piece R immediately after the projectile breaks apart - Correctly indicating the position of Piece R relative to the position of the center of mass of the projectile at $t = t_2$	Point B3
	For a correct expression for x_2 in terms of given quantities	Point B4
Example Responses		
<i>Solution 1</i> $\Delta y = v_{y0}t + \frac{1}{2}at^2$ $v_{y0} = v_0 \sin \theta$ $0 = v_0 \sin \theta t_2 + \frac{1}{2}(-g)t_2^2$ $t_2 = \frac{2v_0 \sin \theta}{g}$ $\sum \vec{p}_{0x} = \sum \vec{p}_{fx}$ $\vec{p}_{0x} = \vec{p}_{Qx} + \vec{p}_{Rx}$ $4Mv_0 \cos \theta = -Mv_0 \cos \theta + 3Mv_{Rf}$ $5Mv_0 \cos \theta = 3Mv_{Rf}$ $v_{Rf} = \frac{5}{3}v_0 \cos \theta$ $x_2 = x_{\text{proj}} + x_R$ $x_2 = v_{x0}t_{\text{up}} + v_{Rf}t_{\text{down}}$ $x_2 = (v_0 \cos \theta)\left(\frac{v_0 \sin \theta}{g}\right) + \left(\frac{5}{3}v_0 \cos \theta\right)\left(\frac{v_0 \sin \theta}{g}\right)$ $x_2 = \frac{v_0^2}{g} \cos \theta \sin \theta + \frac{5}{3} \frac{v_0^2}{g} \cos \theta \sin \theta$ $x_2 = \frac{8v_0^2}{3g} \cos \theta \sin \theta$ <i>Solution 2</i> $v_y = v_{y0} + a_y t$ $0 = v_0 \sin \theta + (-g)t_{\text{up}}$ $t_1 = \frac{v_0 \sin \theta}{g}$ $t_2 = 2t_1 = \frac{2v_0 \sin \theta}{g}$ $v_x = v_0 \cos \theta$ $x_{\text{CM}} = v_x t_2$ $x_{\text{CM}} = \frac{2v_0^2 \cos \theta \sin \theta}{g}$ $x_{\text{CM}} = \frac{M(0) + 3M(x_2)}{M + 3M}$ $x_{\text{CM}} = \frac{3}{4}x_2$ $\frac{3}{4}x_2 = \frac{2v_0^2 \cos \theta \sin \theta}{g}$ $x_2 = \frac{8v_0^2}{3g} \cos \theta \sin \theta$		

C	For sketching a labeled horizontal line for the center of mass that has a value equal to $v_{x,cm}$	Point C1
Scoring Note: Any clearly defined labels may earn this point.		
	For sketching a labeled horizontal line for Piece R that has a value greater than $v_{x,cm}$	Point C2
	For sketching a labeled horizontal line for Piece Q that has a value equal to $-v_{x,cm}$	Point C3

Example Response

D	For indicating $x_{\text{new}} < x_2$	Point D1
	For a correct justification, such as:	Point D2
	- Comparing the change in momentum of either piece in the two cases, the speed of the corresponding piece after the projectile breaks apart, and the distance traveled before reaching the ground - Relating the motion of the center of mass of the projectile to the mass of the pieces and the distance each piece lands from the center of mass	

Example Responses

In the case where Piece Q falls straight to the ground, it has zero momentum, as opposed to a negative momentum when it moved backwards. Because momentum is conserved when the pieces break apart, Piece R must have less positive momentum in the new case. Because Piece R has less momentum it has a lower speed. It takes the same time to fall to the ground in both cases, so Piece R travels a shorter distance before landing in the new case.

OR

In the case where Piece Q falls straight to the ground, Piece Q lands closer to the center of mass of the system than when it moved backward. Because the smaller mass piece lands closer to the CM, the larger mass piece also must land closer to the CM than it did before. This means Piece R travels a shorter distance before landing in the second case.

Question 3: Experimental Design and Analysis (LAB)**10 points****A (i)** For indicating **both** of the following quantities: **Point A1**

- The spring compression, x
- The distance the block slides to the right of $x = 0$, d

Scoring Note: The variables given are chosen only for illustrating algebraic relationships in parts A and B. Responses may earn this point by using any clearly defined quantities or variables.

(ii) For describing a reasonable method of reducing experimental uncertainty **Point A2**

Examples of acceptable responses include the following:

- Completing multiple trials using different values for the spring compression
- Repeat measurements for each trial
- Improved measurement technology to measure length more precisely

Example Response

Measure the distance the spring has been compressed from equilibrium. Release the block and measure the distance the block traveled from the point at which the block loses contact with the spring. Perform multiple trials using different initial spring compressions.

B (i) For indicating quantities that produce a graph that has a linear trend that can be used to determine μ_k **Point B1**

Examples of acceptable responses include the following:

- d as a function of x^2
- $\sqrt{d}$ as a function of x

Scoring Notes:

- Responses that include the reciprocals of the preceding examples, in addition to other equivalent graphs also earn this point.
- This point may be earned independently of the response in part A.

(ii) For correctly describing the relationship of the slope of the best-fit line to μ_k **Point B2**

Examples of acceptable responses include the following:

Graph	Analysis
d as a function of x^2	slope = $\frac{k}{2\mu_k mg}$
$\sqrt{d}$ as a function of x	slope = $\sqrt{\frac{k}{2\mu_k mg}}$

Example Response

Plotting sliding distance on the vertical axis and the square of the compression distance on the horizontal axis will yield a linear graph. The slope of the graph will be equal to

$$\frac{k}{2\mu_k mg}, \text{ so } \mu_k = \frac{k}{2mg(\text{slope})}.$$

- C (i) For labeling the axes with appropriate quantities (including units) that could be plotted to produce a linear graph that can be used to determine k_{new} , such as: **Point C1**

- h as a function of s^2
- $\sqrt{h}$ as a function of s
- mgh as a function of $\frac{1}{2}s^2$

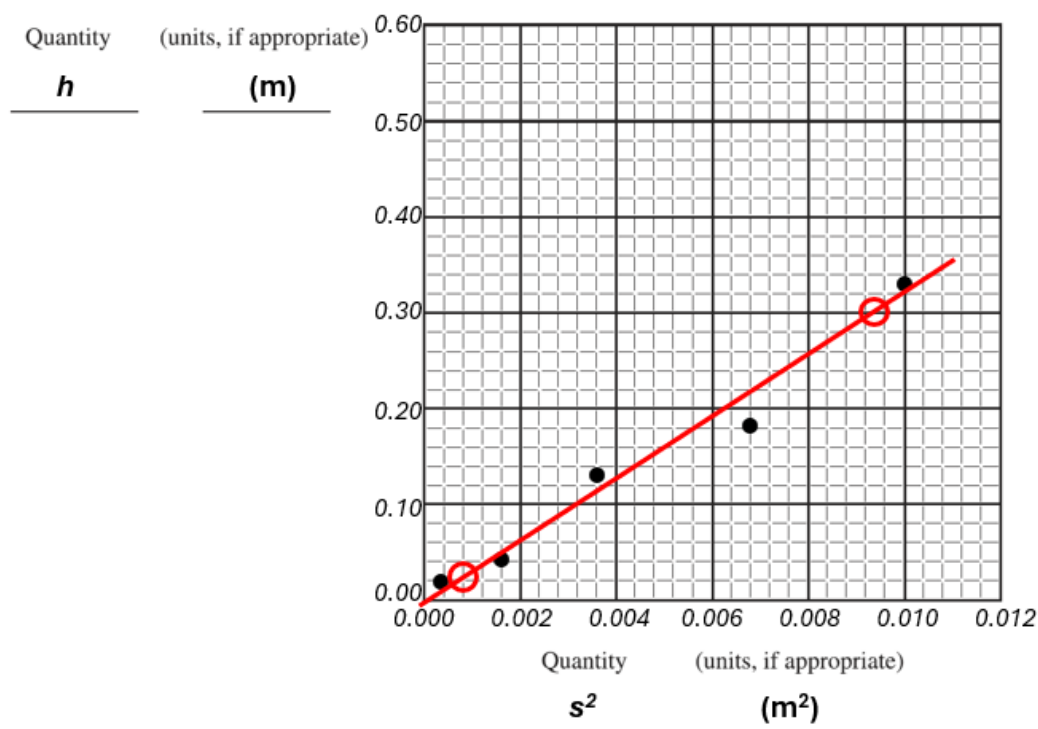
Scoring Note: Responses that include the reciprocals of the preceding examples, in addition to other equivalent graphs, also earn this point.

- (ii) For labeling **both** axes with a linear scale **Point C2**

For plotting data points consistent with the quantities labeled in part C (i) **Point C3**

- (iii) For drawing a line or curve that approximates the trend of the plotted data **Point C4**

Example Response



DFor correctly relating the slope of the best-fit line to the value of k_{new} **Point D1**

Examples of acceptable responses include the following:

Graph	Analysis
h as a function of s^2	$\text{slope} = \frac{k_{\text{new}}}{2mg}$
$\sqrt{h}$ as a function of s	$\text{slope} = \sqrt{\frac{k_{\text{new}}}{2mg}}$
mgh as a function of $\frac{1}{2}s^2$	$\text{slope} = k_{\text{new}}$

For calculating an experimental value of k_{new} within a range of 1150 N/m to 1400 N/m**Point D2****Example Response**

$$mgh = \frac{1}{2}k_{\text{new}}s^2$$

$$\text{slope} = \frac{k_{\text{new}}}{2mg}$$

$$k_{\text{new}} = 2mg(\text{slope})$$

$$\text{slope} = \frac{0.30 \text{ m} - 0.024 \text{ m}}{0.0094 \text{ m}^2 - 0.0008 \text{ m}^2} = 32 \text{ m}^{-1}$$

$$k_{\text{new}} = 2mg(\text{slope})$$

$$k_{\text{new}} = 2(2.0 \text{ kg})(9.8 \text{ m/s}^2)(32 \text{ m}^{-1})$$

$$k_{\text{new}} = 1254 \text{ N/m}$$

Question 4: Qualitative Quantitative Translation (QQT)

8 points

A	For indicating $\omega_2 > \omega_1$	Point A1
	For a justification that correctly relates force and crank arm length to the torque exerted on the crank arm	Point A2
	For a justification that correctly relates torque exerted over a single rotation to one of the following: - The rotational kinetic energy of the wheel - The angular acceleration	Point A3

Example Response

$\omega_2 > \omega_1$. The torque exerted on the crank arm will be greater at the end of the crank arm with a greater length, and a greater torque exerted over one rotation means a greater change in rotational kinetic energy of the wheel. A greater rotational kinetic energy for the same wheel with the same rotational inertia means the wheel has a greater angular speed.

B	For a multistep derivation that includes one of the following: - Conservation of energy that indicates work is done on the wheel - The work-energy theorem - Newton's second law in rotational form	Point B1
	For indicating one of the following: - The work done by F on the crank arm is $2\pi F\ell_1$. - The magnitude of the angular acceleration of the wheel is $\alpha = \frac{F\ell_1}{I_W}$.	Point B2
	For a correct expression for ω_1 in terms of given quantities	Point B3

Example Responses*Solution 1*

$$W = \Delta K$$

$$\tau \Delta \theta = \Delta K_{\text{rot}}$$

$$(F\ell_1)(2\pi) = \frac{1}{2}I\omega_1^2$$

$$\frac{4\pi F\ell_1}{I_W} = \omega_1^2$$

$$\omega_1 = \sqrt{\frac{4\pi F\ell_1}{I_W}}$$

Solution 2

$$\alpha_{\text{sys}} = \frac{\sum \tau}{I_{\text{sys}}}$$

$$\alpha = \frac{Fr}{I} = \frac{F\ell_1}{I_W}$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\omega_1^2 = 2\alpha(2\pi)$$

$$\omega_1^2 = 4\pi\alpha$$

$$\omega_1^2 = 4\pi \frac{F\ell_1}{I_W}$$

$$\omega_1 = \sqrt{\frac{4\pi F\ell_1}{I_W}}$$

C	For indicating $\omega_3 < \omega_1$	Point C1
	For a justification that indicates one of the following: • The work done to turn the crank arm is converted into translational and rotational kinetic energies. • The torque exerted on the wheel by friction changes the net torque on the wheel.	Point C2
	Example Response <i>$\omega_3 < \omega_1$. When the unicycle is stationary, the work done by the force of magnitude F to turn the crank arm one rotation is converted into only rotational kinetic energy. When the unicycle can move forward, the same amount of work done is converted into translational and rotational kinetic energies. This means the wheel has to turn more slowly when the unicycle can move forward.</i>	