2026

AP® Calculus BC

Scoring Guidelines

Part A (AB or BC): Graphing Calculator Required
Question 1 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Male birds of a certain species arrive at a nesting area over a thirty-day period. The rate at which the male birds arrive at the nesting area at time t days is modeled by a differentiable function M, where $M(t)$ is measured in number of birds per day. Selected values of $M(t)$ are shown in the table.

t (days)	0	5	10	15	20	25	30
$M(t)$ (birds per day)	2	7	16	6	5	2	0

(Note: Your calculator should be in radian mode.)

	Model Solution	Scoring	
A	Approximate $M'(7.5)$ using the average rate of change of M over the interval $5 \le t \le 10$. Show the work that leads to your answer, and indicate units of measure.		
	$M'(7.5) \approx \dfrac{M(10) - M(5)}{10 - 5} = \dfrac{16 - 7}{10 - 5} = \dfrac{9}{5}$ birds per day per day	Answer with setup	**Point 1 (P1)**
		Units	**Point 2 (P2)**

Scoring Notes for Part A

- To earn **P1**, a response must provide the correct numerical value for the average rate of change and include supporting work that shows both a difference and a quotient using values from the table.
  - $\dfrac{16-7}{10-5}$, $\dfrac{9}{10-5}$, $\dfrac{16-7}{5}$, $\dfrac{M(10)-M(5)}{10-5} = \dfrac{9}{5}$, or $\dfrac{M(10)-M(5)}{5} = \dfrac{9}{5}$ is sufficient to earn **P1**.
  - $\dfrac{M(10)-M(5)}{10-5}$ or $\dfrac{9}{5}$ by itself is not sufficient to earn **P1**.
- **P2** is earned for correct units, whether or not they are attached to a numerical value for the average rate of change.
- **P2** is also earned for the units "$\text{birds}/\text{day}^2$."

B

i. Use a midpoint Riemann sum with the three subintervals $[0, 10]$, $[10, 20]$, and $[20, 30]$ to approximate $\int_0^{30} M(t)\,dt$. Show the work that leads to your answer.

ii. Interpret the meaning of $\int_0^{30} M(t)\,dt$ in the context of the problem.

i. $\int_0^{30} M(t)\,dt \approx M(5)\cdot(10-0) + M(15)\cdot(20-10) + M(25)\cdot(30-20)$	Form of midpoint Riemann sum	**Point 3 (P3)**
$= 7\cdot 10 + 6\cdot 10 + 2\cdot 10 = 150$	Approximation with supporting work	**Point 4 (P4)**
ii. $\int_0^{30} M(t)\,dt$ represents the number of male birds of this species that arrive at the nesting area from $t = 0$ days to $t = 30$ days.	Interpretation	**Point 5 (P5)**

Scoring Notes for Part B

- Read "$=$" as "$\approx$" for **P3**.
- The form of a midpoint Riemann sum includes three terms, each of which includes a product of two factors. To earn **P3**, at least five of the six factors must be correct. If any of the six factors is incorrect, the response does not earn **P4**.
- A response of $7\cdot 10 + 6\cdot 10 + 2\cdot 10$ or $10\cdot(7+6+2)$ earns **P3** and banks **P4** (i.e., subsequent errors in simplification will not be considered in scoring for **P4**).
- **P4** is earned for the value 150 (or equivalent) with supporting work. **P4** may be earned without **P3**. For example, $70 + 60 + 20$ or $10(15)$ earns **P4** but not **P3**.
- Consider the following examples:
  - $7\cdot(10-0) + 6\cdot(20-10) + 2\cdot(30-20)$ earns **P3** and is sufficient to earn **P4**.
  - $7\cdot 10 + 6\cdot 10 + 2\cdot 10$ earns **P3** and is sufficient to earn **P4**.
  - $M(5)\cdot 10 + M(15)\cdot 10 + M(25)\cdot 10$ earns **P3** and is eligible for **P4**.
  - $M(5)\cdot 10 + M(15)\cdot 10 + M(25)\cdot 10 = 150$ earns both **P3** and **P4**.
  - $5\cdot 10 + 6\cdot 10 + 2\cdot 10$ earns **P3** but is not eligible for **P4**.
    (Note that the factor of 5 in the first term of this expression is incorrect.)
  - An unsupported answer of 150 does not earn **P3** or **P4**.
- A completely correct left or right Riemann sum with supporting work does not earn **P3** and banks **P4** (i.e., subsequent errors in simplification will not be considered in scoring for **P4**). A response with any error or missing factors in a left or right Riemann sum does not earn **P3** or **P4**.
- **P5** can be earned whether or not **P3** and **P4** were earned.
- To earn **P5**, an interpretation must include both "number of [male] birds that arrive" and the time interval $t = 0$ to $t = 30$. The response need not include units of time or "nesting area."

C The rate at which female birds of the same species arrive at the same nesting area, in birds per day, is modeled by the function F defined as follows.

$$F(t) = \begin{cases} 0 & \text{for } 0 \le t < 15 \\ 18 + 16\sin\left(\frac{\pi}{20}(t+15)\right) & \text{for } 15 \le t \le 45 \end{cases}$$

How many female birds of this species arrive at the nesting area from $t = 15$ to $t = 45$? Show the setup for your calculations, and round your answer to the nearest integer.

$\int_{15}^{45} F(t)\,dt$	Integral	**Point 6 (P6)**
$= 641.859164$ To the nearest integer, the number of female birds that arrive from $t = 15$ to $t = 45$ is 642 birds.	Answer	**Point 7 (P7)**

Scoring Notes for Part C

- **P6** can be earned with or without the differential dt.
- **P6** can also be earned for the expression $\left[18t - \frac{320}{\pi}\cos\left(\frac{\pi}{20}(t+15)\right)\right]_{t=15}^{t=45}$.
- **P7** is earned for the correct answer, with or without supporting work.
- **P7** can also be earned for 641, 641.8, 641.9, 641.85, 641.86, 641.859, or the exact answer of $\frac{320}{\pi} + 540$.
- Units are not considered in scoring.

D	On the interval $15 < t < 30$, the difference in the rates at which male and female birds of this species arrive at the nesting area can be modeled by the differentiable function $D(t) = M(t) - F(t)$, where F is the function defined in part C. Is there a time t in the interval $15 < t < 20$ when $D(t) = 0$? Justify your answer.		
	Because D is differentiable, D is continuous.	Differentiable implies continuous	**Point 8 (P8)**
	$D(15) = M(15) - F(15) = 4 > 0$ $D(20) = M(20) - F(20) = -1.686292 < 0$ Therefore, by the Intermediate Value Theorem, there must be a time t, with $15 < t < 20$, such that $D(t) = 0$.	Answer with justification	**Point 9 (P9)**

Scoring Notes for Part D

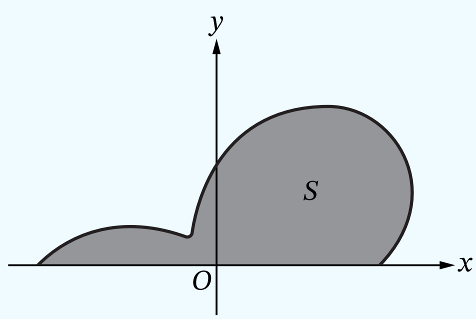
- To earn **P8**, a response must state that D is continuous because D is differentiable (or equivalent). A response that simply states "D is continuous" without justification does not earn **P8**.
- A response does not need to earn **P8** to be eligible for **P9**.
- To earn **P9**, a response must indicate that $D(15) > 0$ and $D(20) < 0$, state that "D is continuous," and answer "yes" in some way. A response of "yes, IVT" is not sufficient to earn **P9**.
  - A response that has earned **P8** does not need to restate "D is continuous" to earn **P9**.
- To earn **P9**, a response need not explicitly name the Intermediate Value Theorem, but if a theorem is named, it must be correct.

Part A (BC): Graphing Calculator Required
Question 2 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Let S be the shaded region bounded by the graph of the polar curve $r(\theta) = 3 + 2\sin(2\theta) + \cos(2\theta)$ for $0 \le \theta \le \pi$, as shown in the figure.



It can be shown that $r'(\theta) = 4\cos(2\theta) - 2\sin(2\theta)$.

(Note: Your calculator should be in radian mode.)

	Model Solution	Scoring	
A	Find the area of region S. Show the setup for your calculations.		
	$\frac{1}{2}\int_0^{\pi} (r(\theta))^2 \, d\theta$	Setup	**Point 1 (P1)**
	$= 18.064158$ The area of region S is 18.064.	Answer	**Point 2 (P2)**

Scoring Notes for Part A

- **P1** is earned for a definite integral with numerical limits and a correct integrand, such as $\int_0^{\pi} (r(\theta))^2\, d\theta$ or $\int_0^{\pi} (3 + 2\sin(2\theta) + \cos(2\theta))^2\, d\theta$, with or without the differential $d\theta$.
- **P2** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point.
- Incorrect or unclear communication between the correct integral and the correct answer is treated as scratch work and is not considered in scoring. For example, $\int_0^{\pi} (r(\theta))^2\, d\theta = \frac{36.128}{2}$ earns both **P1** and **P2**.
- The exact value of the area is $\frac{23\pi}{4}$.

B There is a point on the curve at which the slope of the line tangent to the curve is $-\frac{3}{7}$. At this point, $\frac{dy}{d\theta} = \frac{3\sqrt{2}}{2}$. Find $\frac{dx}{d\theta}$ at this point. Show the work that leads to your answer.

$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} \Rightarrow \frac{dx}{d\theta} = \frac{dy/d\theta}{dy/dx}$	$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$	**Point 3 (P3)**
At the point, $\frac{dy}{dx} = -\frac{3}{7}$ and therefore, $\frac{dx}{d\theta} = \frac{\frac{3\sqrt{2}}{2}}{-\frac{3}{7}} = -\frac{7\sqrt{2}}{2} = -4.950$ (or -4.949).	$\frac{dy}{dx} = -\frac{3}{7}$	**Point 4 (P4)**
	Answer	**Point 5 (P5)**

Scoring Notes for Part B

- **P3** can be earned for $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$, $\frac{dx}{d\theta} = \frac{dy/d\theta}{dy/dx}$, $\frac{dx}{d\theta} = \frac{dy}{d\theta} \cdot \frac{dx}{dy}$, or equivalent.
- **P4** can be earned by stating that $\frac{dy}{dx} = -\frac{3}{7}$ or by substituting $-\frac{3}{7}$ for $\frac{dy}{dx}$ in an equation involving $\frac{dx}{d\theta}$.
- A response of $\frac{dx}{d\theta} = \frac{3\sqrt{2}/2}{-3/7}$ earns **P3** and **P4** and earns **P5** if there are no subsequent errors in simplification.
- A response of $-\frac{3}{7} = \frac{dy/d\theta}{dx/d\theta}$ earns **P3** and **P4** and is eligible to earn **P5**.
- **P5** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.

C

i. Find the value of θ in the interval $0 < \theta < \frac{\pi}{2}$ at which r has a critical point.

ii. Use a derivative test to determine whether the critical point is the location of a relative minimum, a relative maximum, or neither for r.

i.	$r'(\theta) = 0 \;\Rightarrow\; \theta = 0.553574$	Value of θ	**Point 6 (P6)**
ii.	By the First Derivative Test, r has a relative maximum at $\theta = 0.554$ (or 0.553) because $r'(\theta)$ changes from positive to negative there.	Answer using derivative test	**Point 7 (P7)**

Scoring Notes for Part C

- **P6** is earned for presenting $\theta = 0.554$ (or 0.553). A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.
- Any other identified critical values (with or without subsequent analysis for these values) can be ignored.
- A response must earn **P6** to be eligible for **P7**.
- To earn **P7**, a response does not need to cite a derivative test by name, nor does it need to restate the value of $\theta = 0.554$ found earlier in part C. It does, however, need to correctly classify the critical point with analysis using a derivative test.
- Alternate solution using Second Derivative Test:

  i. $r'(\theta) = 0 \;\Rightarrow\; \theta = 0.553574$

  ii. $r''(0.553574) < 0$

  Because $r'(0.553574) = 0$ and $r''(0.553574) < 0$, by the Second Derivative Test, r has a relative maximum at $\theta = 0.554$ (or 0.553).

  - The value of $r''(0.553574)$ is -8.944. This value isn't necessary for the analysis to be correct; however, if a value is reported, it must be accurate to the number of digits reported (up to the third decimal place).
- A Candidates Test is not sufficient justification to earn **P7**.
- The exact value of θ at which r has a critical point is $\frac{1}{2}\arctan 2$.

D Find the average distance from the origin to a point on the polar curve $r(\theta) = 3 + 2\sin(2\theta) + \cos(2\theta)$ for $\frac{\pi}{2} \le \theta \le \pi$. Show the setup for your calculations.

$\dfrac{1}{\pi - \frac{\pi}{2}} \displaystyle\int_{\pi/2}^{\pi} r(\theta)\, d\theta$	Average value formula	**Point 8 (P8)**
$= 1.726760$ The average distance from the origin to a point on the polar curve r for $\frac{\pi}{2} \le \theta \le \pi$ is 1.727 (or 1.726).	Answer	**Point 9 (P9)**

Scoring Notes for Part D

- **P8** is earned for the correct integral, with or without the differential $d\theta$, along with evidence of division by $\pm\frac{\pi}{2}$. In the presence of the correct integral, the correct answer will suffice as evidence of division by $\frac{\pi}{2}$. These may appear all in one step, as in the model solution, or in multiple steps.
- **P9** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.
- Incorrect or unclear communication between the correct integral and the correct answer is treated as scratch work and is not considered in scoring. The following responses earn **P8** for the correct integral with the correct answer as evidence of division by $\frac{\pi}{2}$ and earn **P9** for the correct answer:
  - $\int_{\pi/2}^{\pi} r(\theta)\, d\theta = 2.712$, so the average value is 1.727.
  - $\int_{\pi/2}^{\pi} r(\theta)\, d\theta = \frac{2.712}{\pi/2} = 1.727$.

    (In this instance, incorrect linkage is not considered in scoring.)
  - $\int_{\pi/2}^{\pi} r(\theta)\, d\theta = 1.727$.

    (In this instance, incorrect linkage is not considered in scoring.)
- The exact value of the average distance is $\frac{3\pi - 4}{\pi}$.

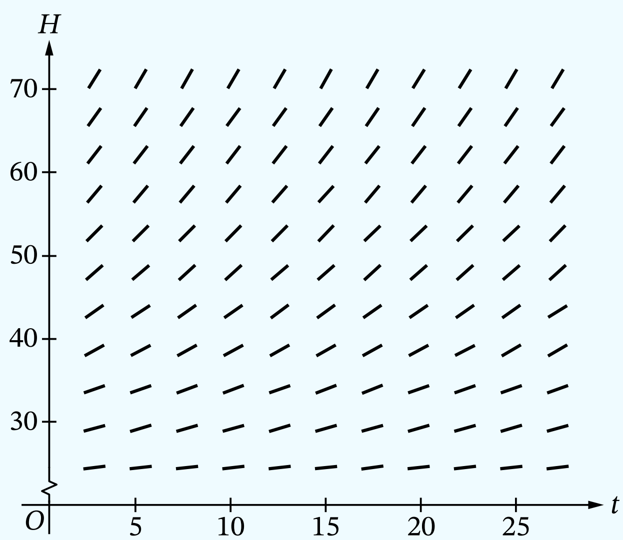
Part B (AB or BC): Graphing Calculator Not Allowed
Question 3 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

A pie is taken from a hot oven and put on a table. The internal temperature of the pie at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{15}(H - 20)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 75$. For $t > 0$, it is known that $20 < H(t) < 75$.

	Model Solution	Scoring	
A	Explain why the following could not be a slope field for the differential equation $\frac{dH}{dt} = -\frac{1}{15}(H - 20)$.		



Model Solution	Scoring	
The slopes of the line segments in a slope field for the given differential equation should all be negative because $\frac{dH}{dt} < 0$ on $20 < H < 75$. However, the slopes of the line segments in the slope field given are all positive.	Explanation	**Point 1 (P1)**

Scoring Notes for Part A

- To earn **P1**, the response must reference the sign of the slopes of the line segments or the slope field's implication for the graph of H. Referencing the sign of $\frac{dH}{dt}$ without connection to the slope field does not earn **P1**.
- Common communication errors include references to the sign of the slope field, the slope of the slope field, the slope of the differential equation, and the increasing behavior of the slope field. Such responses do not earn **P1**.
- **P1** can be earned for a response of "because there are no negative slopes shown" or "because the slopes shown are positive."
- **P1** can be earned by using a local argument at any point (t, H) with $0 < t < 30$ and $20 < H < 75$, rather than a global argument.

B Find the slope of the line tangent to the graph of H at time $t = 0$. Show the work that leads to your answer.

$\left.\frac{dH}{dt}\right\rvert_{(0,\,75)} = -\frac{1}{15}(75 - 20) = -\frac{11}{3}$	Answer with supporting work	**Point 2 (P2)**

Scoring Notes for Part B

- A response can earn **P2** with $-\frac{1}{15}(75 - 20)$, $-\frac{1}{15}(55)$, or $\left.\frac{dH}{dt}\right\rvert_{(0,\,75)} = -\frac{11}{3}$.
- A response of $-\frac{1}{15}(75 - 20)$ or $-\frac{1}{15}(55)$ banks **P2** (i.e., subsequent errors in simplification will not be considered in scoring for **P2**).
- Presentation of a tangent line equation will not be considered in scoring for **P2**.

C It can be shown that $\frac{d^2H}{dt^2} = \frac{1}{225}(H - 20)$. The line tangent to the graph of H at time $t = 0$ is used to approximate $H(5)$, the internal temperature of the pie at time $t = 5$. Is this approximation an overestimate or an underestimate for the actual value of $H(5)$? Give a reason for your answer.

For $t > 0$, $20 < H < 75$, so $\frac{d^2H}{dt^2} > 0$.	Considers sign of $\frac{d^2H}{dt^2}$	**Point 3 (P3)**
Because $\frac{d^2H}{dt^2} > 0$, the graph of H is concave up. Therefore, the tangent line lies below the graph, so the approximation is an underestimate for $H(5)$.	Answer with reason	**Point 4 (P4)**

Scoring Notes for Part C

- **P3** can be earned with $\frac{d^2H}{dt^2} = 0$, $\frac{d^2H}{dt^2} > 0$, or $\frac{d^2H}{dt^2} < 0$. **P3** can be earned symbolically or in words.
- A response is eligible for **P4** only if it has earned **P3**.
- A response that states "concave up, so underestimate" earns **P4** if **P3** was earned. Referencing the concavity of $\frac{dH}{dt}$ or $\frac{d^2H}{dt^2}$ does not earn **P4**.
- Although a response that presents an argument based on $\frac{d^2H}{dt^2}$ or concavity at a single point can earn **P3** with an explicit consideration of the sign of $\frac{d^2H}{dt^2}$, such an argument does not earn **P4**.

D Use separation of variables to find an expression for $H(t)$, the particular solution to the given differential equation with initial condition $H(0) = 75$.

$\frac{dH}{H-20} = -\frac{1}{15}\,dt$ $\int \frac{dH}{H-20} = \int -\frac{1}{15}\,dt$	Separates variables	**Point 5 (P5)**
$\ln\lvert H-20\rvert = -\frac{t}{15} + C$	Finds one antiderivative	**Point 6 (P6)**
	Finds other antiderivative	**Point 7 (P7)**
$\ln\lvert 75-20\rvert = -\frac{1}{15}(0) + C \;\Rightarrow\; \ln 55 = C$	Constant of integration and uses initial condition	**Point 8 (P8)**
$\ln\lvert H-20\rvert = -\frac{t}{15} + \ln 55$ $\lvert H-20\rvert = 55e^{-t/15}$ $H - 20 = \pm 55e^{-t/15}$ Because $H(0) = 75$, $H(t) = 20 + 55e^{-t/15}$.	Solves for $H(t)$	**Point 9 (P9)**

Scoring Notes for Part D

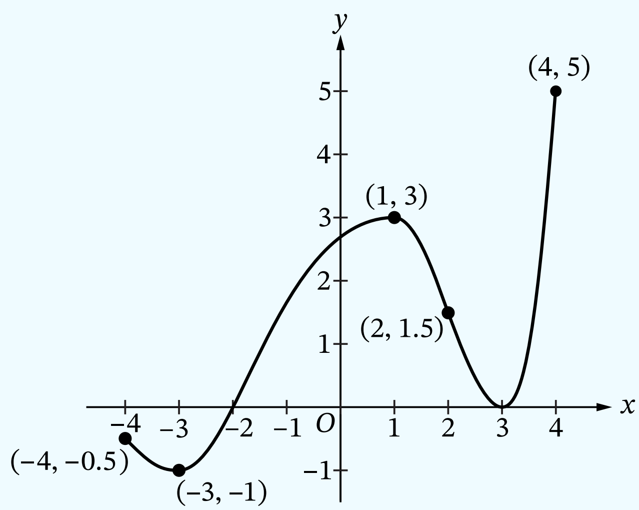
- A response with no separation of variables does not earn **P5**, **P6**, **P7**, **P8**, or **P9**.
- A response earns **P5** if the separation of variables results in an equation of the form $A\dfrac{dH}{H-20} = B\,dt$, where A and B are nonzero constants.
- A response that presents a consistent antiderivative of the form $A\ln(H-20)$ (with either parentheses or absolute value) or Bt earns **P6**. If the response presents two consistent antiderivatives, it also earns **P7**.
- **Special case:** If the response begins with a separation of the form $A\dfrac{dH}{H+20} = B\,dt$, where A and B are correct coefficients, it earns **P5**, can earn **P6** with one consistent antiderivative, can earn **P7** with two consistent antiderivatives, and is eligible to earn **P8** but not **P9**.
- A response with no constant of integration is eligible to earn **P5**, **P6**, and **P7**, but does not earn **P8** or **P9**.
- A response is eligible for **P8** if it has earned **P5** and **P6**.
- An eligible response earns **P8** by correctly including the constant of integration in an equation and substituting 0 for t and 75 for H.
- A response is eligible for **P9** only if it has earned **P5**, **P6**, and **P7**. Responses that do not explicitly show the constant of integration and use of initial condition can still earn **P9** with a correct $H(t)$.
- A response earns **P9** for an answer of $H(t) = 20 + 55e^{-t/15}$, $H(t) = 20 + e^{-\frac{t}{15}+\ln 55}$, or equivalent. The response does not need to provide justification for their choice of the positive coefficient on the exponential.

Part B (AB or BC): Graphing Calculator Not Allowed
Question 4 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Let *f* be a twice-differentiable function on the closed interval $[-4, 4]$ with $f(2) = 3$. The graph of f', the derivative of *f*, is shown.



Graph of f'

	Model Solution	Scoring	
A	For $x > 0$, the function g is defined by $g(x) = f(x) - \ln x$. Find $g'(2)$. Show the work that leads to your answer.		
	$g'(x) = f'(x) - \frac{1}{x}$	Setup	**Point 1 (P1)**
	$g'(2) = f'(2) - \frac{1}{2} = 1.5 - \frac{1}{2} = 1$	Answer	**Point 2 (P2)**

Scoring Notes for Part A

- **P1** can be earned with $g'(x) = f'(x) - \frac{1}{x}$ or $g'(2) = f'(2) - \frac{1}{2}$.
- An answer of $1.5 - \frac{1}{2}$ earns **P1** and **P2**.
- A response does not need to simplify, but an error in simplification does not earn **P2**.
- An unsupported answer of 1 earns **P2** but not **P1**.

B Find all values of x on the open interval $0 < x < 3$ at which the graph of f has a point of inflection. Give a reason for your answer.

The graph of f has a point of inflection where f' changes from decreasing to increasing or from increasing to decreasing. The graph of f has a point of inflection at $x = 1$ because f' changes from increasing to decreasing there.	Answer	**Point 3 (P3)**
	Reason	**Point 4 (P4)**

Scoring Notes for Part B

- **P3** is earned for an answer of $x = 1$. If any other/additional values of x in $0 < x < 3$ are declared to be locations for points of inflection, the response does not earn either **P3** or **P4**. A response that also declares $x = -3$ and/or $x = 3$ as locations for points of inflection is eligible for both **P3** and **P4**.
- A response that declares a point $(1, a)$, where a is any real number, as a point of inflection earns **P3** but is not eligible for **P4**.
- A response must earn **P3** to be eligible for **P4**.
- To earn **P4**, a response must tie the reason to the given graph of f'.
    - A response of "f has a point of inflection at $x = 1$ because f' changes from increasing to decreasing there" earns both **P3** and **P4**.
    - A response of "f has a point of inflection at $x = 1$ because the slope of f' changes sign there" earns both **P3** and **P4**.
    - A response of "f has a point of inflection at $x = 1$ because f' attains a relative extremum there" earns both **P3** and **P4**.
    - A response of "f has a point of inflection at $x = 1$ because f changes concavity there" earns **P3** but not **P4**.
    - A response of "f has a point of inflection at $x = 1$ because f'' changes sign there" earns **P3** but not **P4**.
    - A response that relies upon an ambiguous term such as "the function" or "the graph" does not earn **P4**.

C	For $-4 \le x \le 4$, on what open intervals, if any, is the graph of f both increasing and concave down? Give a reason for your answer.		
	The graph of function f is increasing on an interval when $f'(x) > 0$ and concave down on an interval when f' is decreasing. Therefore, f is increasing and concave down on the interval $1 < x < 3$.	Answer Reason	**Point 5 (P5)** **Point 6 (P6)**

Scoring Notes for Part C

- **P5** is earned by an answer of $1 < x < 3$, or this interval including one or both endpoints.
- A response must earn **P5** to be eligible for **P6**.
- To earn **P6**, a response must refer to f' and correctly cite both $f'(x) > 0$ and f' is decreasing.
- A response of "$f' > 0$ and f' is decreasing on $1 < x < 3$" earns both **P5** and **P6**.
- **Special case:** A response of "f is increasing on $-2 < x < 3$ and $3 < x < 4$ because $f'(x) > 0$ there" earns **P5** but not **P6**.
- **Special case:** A response of "f is concave down on $-4 < x < -3$ and $1 < x < 3$ because f' is decreasing there" earns **P5** but not **P6**.

D	For $-4 \le x \le 4$, find the value of x at which f has an absolute minimum and the value of x at which f has an absolute maximum. Give reasons for your answers.	
	For $-4 \le x \le 4$, f attains an absolute extremum when $f'(x) = 0$ or at an endpoint. $f'(x) = 0 \Rightarrow x = -2,\ x = 3$ Because $f'(x) < 0$ for $-4 < x < -2$, f is decreasing there. Because $f'(x) > 0$ for $-2 < x < 3$, f is increasing there. Because $f'(x) > 0$ for $3 < x < 4$, f is increasing there. Therefore, f has an absolute minimum at $x = -2$. $f(4) - f(-4) = \int_{-4}^{4} f'(x)\,dx > 0$, because on the interval $-4 \le x \le 4$, the area bounded by the graph of f' above the x-axis is greater than the area bounded by the graph of f' below the x-axis. Therefore, $f(4) > f(-4)$ and f has an absolute maximum at $x = 4$.	Considers $f'(x) = 0$ — **Point 7 (P7)** Absolute minimum with reason — **Point 8 (P8)** Absolute maximum with reason — **Point 9 (P9)**

Scoring Notes for Part D

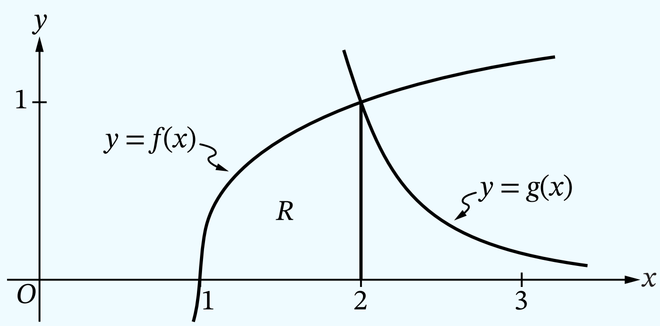
- **P7** is earned for considering $f'(x) = 0$. **P7** is not earned by just presenting $x = -2$ and $x = 3$.
  - A response that discusses the sign of $f'(x)$ changing or uses the phrase "critical points of f" also earns **P7**.
- A response that correctly declares "minimum at $x = -2$" with correct justification earns **P8**.
- A response that correctly declares "maximum at $x = 4$" with correct justification earns **P9**.
- **Special case:** A response that correctly declares a "minimum at $x = -2$" and "maximum at $x = 4$" with an incorrect or insufficient attempt at justification earns **P8** but not **P9**.

Part B (BC): Graphing Calculator Not Allowed
Question 5 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Let f be the function defined by $f(x) = \sqrt[3]{x-1}$, and let g be the function defined by $g(x) = e^{(-2x+4)}$. The graphs of f and g intersect at the point $(2, 1)$. Let R be the region bounded by the graph of f, the x-axis, and the vertical line $x = 2$, as shown in the figure.



	Model Solution	Scoring	
A	Evaluate $\int_1^2 f(x)\,dx$. Show the work that leads to your answer.		
	$\int_1^2 f(x)\,dx = \int_1^2 \sqrt[3]{x-1}\,dx = \frac{3}{4}(x-1)^{4/3}\Big\vert_1^2$	Antiderivative	**Point 1 (P1)**
	$= \frac{3}{4}(2-1)^{4/3} - \frac{3}{4}(1-1)^{4/3} = \frac{3}{4}$	Answer	**Point 2 (P2)**

Scoring Notes for Part A

- An answer of $\frac{3}{4}(2-1)^{4/3} - \frac{3}{4}(1-1)^{4/3}$ earns both **P1** and **P2**.
- An unsupported answer of $\frac{3}{4}$ earns **P2** but not **P1**.

B	Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid generated when region R is rotated about the x-axis.		
	Volume $= \pi \int_1^2 (f(x))^2\, dx$	Form of integrand	**Point 3 (P3)**
		Answer	**Point 4 (P4)**

Scoring Notes for Part B

- **P3** is earned for a response with an integrand of the form $k(f(x))^2$ in a definite or indefinite integral, where k is any nonzero constant.
- **P4** is earned for the correct definite integral, with or without the differential dx.
- A response of $\pi \int_1^2 (f(x))^2\, dx$, $\pi \int_1^2 (x-1)^{2/3}\, dx$, or a mathematically equivalent expression earns both **P3** and **P4**.
- A response that imports an incorrect $(x-1)^n$ from part A, where $n \neq 0$, is eligible for **P3** and **P4**.
- **Special case: P3** is earned for a response that correctly rotates region R about the y-axis and obtains an answer of $\pi \int_0^1 \left(4 - \left(y^3+1\right)^2\right) dy$ or equivalent. Such a response is not eligible for **P4**.

C	Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of region R.		
	Perimeter $= 1 + 1 + \int_1^2 \sqrt{1 + (f'(x))^2}\, dx$	Form of arc length integral	**Point 5 (P5)**
	$= 2 + \int_1^2 \sqrt{1 + \left(\frac{1}{3}(x-1)^{-2/3}\right)^2}\, dx$	Answer	**Point 6 (P6)**

Scoring Notes for Part C

- **P5** is earned for an integral of the form $\int_1^2 \sqrt{1 + (f'(x))^2}\, dx$, with or without the differential dx. The expression for $f'(x)$ and the linear boundaries, 2, are assessed in **P6**.
- Note: The limits do not have to be correct to earn **P5**, but they must be numerical.
- A response that imports an incorrect $(x-1)^n$ from part A, where $n \neq 0$, is eligible for **P5** and **P6**.
- To be eligible for **P6**, the integrand must be of the form $\sqrt{1 \pm (f'(x))^n}$, where $n = 1$ or 2.

D	Evaluate $\int_2^{\infty} g(x)\,dx$. Show the work that leads to your answer.		
	$\int_2^{\infty} e^{(-2x+4)}\,dx = \lim_{b\to\infty}\int_2^{b} e^{(-2x+4)}\,dx$	Limit notation	**Point 7 (P7)**
	$= \lim_{b\to\infty} -\frac{1}{2}e^{(-2x+4)}\Big\vert_2^b$	Antiderivative	**Point 8 (P8)**
	$= \lim_{b\to\infty}\left(-\frac{1}{2}e^{(-2b+4)} + \frac{1}{2}e^{(-4+4)}\right) = \frac{1}{2}$	Answer	**Point 9 (P9)**

Scoring Notes for Part D

- To earn **P7**, a response must correctly use limit notation applied to an integral with an integrand of the form ke^{-2x} or e^{-2x+4}, or to an antiderivative of the form ke^{-2x} or ke^{-2x+4}, where k is a nonzero constant. For example, **P7** may be earned for any of the following: $\lim_{b\to\infty}\int_2^{b} e^{(-2x+4)}\,dx$, $\lim_{b\to\infty} -\frac{e^4}{2}e^{-2x}\Big\vert_2^b$, or $\lim_{b\to\infty}\left(-\frac{1}{2}e^{(-2b+4)} + \frac{1}{2}e^{(-4+4)}\right)$.
- Once earned, **P7** is banked. Subsequently presenting arithmetic with infinity (e.g., $-\frac{1}{2}e^{(-2\cdot\infty+4)} + \frac{1}{2}$) does not affect scoring. Because **P7** is for correctly using limit notation, a response that only presents an expression including arithmetic with infinity, such as $-\frac{1}{2}e^{(-2\cdot\infty+4)} + \frac{1}{2}$, does not earn **P7**.
- **P8** can be earned by finding an antiderivative of the form $ke^{(-2x+4)}$ or ke^{-2x}, where k is a nonzero constant, with or without correct limit notation.
- A response must earn **P8** to be eligible for **P9**.
- **P9** is earned for an answer of $\frac{1}{2}$ or equivalent.

Part B (BC): Graphing Calculator Not Allowed
Question 6 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

The Maclaurin series for a function g is given by
$\sum_{n=0}^{\infty} 2\left(\frac{-x}{5}\right)^n = 2 - \frac{2}{5}x + \frac{2}{25}x^2 - \frac{2}{125}x^3 + \frac{2}{625}x^4 - \cdots + 2\left(\frac{-x}{5}\right)^n + \cdots$ and converges to $g(x)$ for $|x| < 5$.

	Model Solution	Scoring	
A	For $x > 0$, the Maclaurin series for g is an alternating geometric series. Find $g(3)$. Show the work that leads to your answer.		
	For this geometric series, the first term is $a = 2$ and the common ratio is $r = -\frac{3}{5}$. Therefore, $g(3) = \frac{2}{1-\left(-\frac{3}{5}\right)} = \frac{2}{\frac{8}{5}} = \frac{5}{4}$.	Uses $\frac{a}{1-r}$	**Point 1 (P1)**
		Answer	**Point 2 (P2)**

Scoring Notes for Part A

- **P1** is earned for using an expression of the form $\frac{a}{1-r}$, where $a \neq 0$ and $r = \pm\frac{3}{5}$.
- **P2** is earned for the correct answer, with or without supporting work.
- A response of $\frac{2}{1-\left(-\frac{3}{5}\right)}$ earns both **P1** and **P2**.

B	The function f is defined by $f(x) = g'(x)$. Write the first four nonzero terms of the Maclaurin series for f.		
	$f(x) = -\frac{2}{5} + \frac{4}{25}x - \frac{6}{125}x^2 + \frac{8}{625}x^3 - \cdots$	Two terms	**Point 3 (P3)**
		Remaining two terms	**Point 4 (P4)**

Scoring Notes for Part B

- To earn **P3** and **P4**, the terms of the series can be presented in a list or as part of a polynomial or series. Extraneous terms are treated as scratch work and do not impact scoring of **P3** and **P4**.
- A response must earn **P3** to be eligible for **P4**.
- Read "$=$" as "$\approx$" as needed.

C	The second-degree Taylor polynomial for f about $x = 0$ is used to approximate $f\left(\frac{5}{2}\right)$ as $-\frac{3}{10}$, where f is the function defined in part B. Justify that $\left\lvert f\left(\frac{5}{2}\right) - \left(-\frac{3}{10}\right)\right\rvert \le \frac{1}{5}$.		
	The series for $f\left(\frac{5}{2}\right)$ is an alternating series whose terms decrease in absolute value to 0 for $n > 1$, so $\left\lvert f\left(\frac{5}{2}\right) - \left(-\frac{3}{10}\right)\right\rvert \le \left\lvert \frac{8}{625}\left(\frac{5}{2}\right)^3 \right\rvert = \frac{8}{625} \cdot \frac{125}{8} = \frac{1}{5}$.	Uses the first omitted term	**Point 5 (P5)**
		Justification	**Point 6 (P6)**

Scoring Notes for Part C

- A response earns **P5** by correctly evaluating the third-degree term found in part B at $x = \frac{5}{2}$, regardless of whether that term is correct. If the third-degree term is incorrect, the response is not eligible to earn **P6**.
- To earn **P6**, a response must have earned **P5** and indicate that the series for $f\left(\frac{5}{2}\right)$ is a convergent alternating series.
- A response that declares the error is equal to $\frac{1}{5}$ (or any equivalent form of this value) does not earn **P6**.

D The function h is defined by $h(x) = 25g(x) - 2e^x$.

i. Write the first three nonzero terms of the Maclaurin series for e^x.

ii. Write the first three nonzero terms of the Maclaurin series for h.

i.	The first three nonzero terms of the Maclaurin series for e^x are $1 + x + \frac{x^2}{2!}$.	Terms of series for e^x	**Point 7 (P7)**
ii.	The first three nonzero terms of the Maclaurin series for h are: $25\left(2 - \frac{2}{5}x + \frac{2}{25}x^2\right) - 2\left(1 + x + \frac{x^2}{2}\right)$ $= 50 - 10x + 2x^2 - 2 - 2x - x^2$ $= 48 - 12x + x^2$	Two terms of series for h	**Point 8 (P8)**
		Remaining term of series for h	**Point 9 (P9)**

Scoring Notes for Part D

- To earn **P7**, **P8**, and **P9**, the terms of the series can be presented in a list or as part of a polynomial or series.
- **P7** and **P8** are independent of each other, and a response can earn one without earning the other.
- A response must earn **P8** to be eligible for **P9**.
  - **Special case:** A response that presents a polynomial that could be simplified to the correct three terms will earn **P8** but not **P9**.
- Extraneous terms are treated as scratch work and do not impact scoring of **P7**, **P8**, or **P9**.
- Read "$=$" as "$\approx$" as needed.