2026

AP®

AP® Calculus AB

Scoring Guidelines

Part A (AB or BC): Graphing Calculator Required

Question 1 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Male birds of a certain species arrive at a nesting area over a thirty-day period. The rate at which the male birds arrive at the nesting area at time t days is modeled by a differentiable function M, where $M(t)$ is measured in number of birds per day. Selected values of $M(t)$ are shown in the table.

t (days)	0	5	10	15	20	25	30
$M(t)$ (birds per day)	2	7	16	6	5	2	0

(Note: Your calculator should be in radian mode.)

	Model Solution	Scoring	
A	Approximate $M'(7.5)$ using the average rate of change of M over the interval $5 \le t \le 10$. Show the work that leads to your answer, and indicate units of measure.		
	$M'(7.5) \approx \dfrac{M(10) - M(5)}{10 - 5} = \dfrac{16 - 7}{10 - 5} = \dfrac{9}{5}$ birds per day per day	Answer with setup	**Point 1 (P1)**
		Units	**Point 2 (P2)**

Scoring Notes for Part A

- To earn **P1**, a response must provide the correct numerical value for the average rate of change and include supporting work that shows both a difference and a quotient using values from the table.
  - $\dfrac{16-7}{10-5}$, $\dfrac{9}{10-5}$, $\dfrac{16-7}{5}$, $\dfrac{M(10)-M(5)}{10-5} = \dfrac{9}{5}$, or $\dfrac{M(10)-M(5)}{5} = \dfrac{9}{5}$ is sufficient to earn **P1**.
  - $\dfrac{M(10)-M(5)}{10-5}$ or $\dfrac{9}{5}$ by itself is not sufficient to earn **P1**.
- **P2** is earned for correct units, whether or not they are attached to a numerical value for the average rate of change.
- **P2** is also earned for the units "$\text{birds}/\text{day}^2$."

B	i. Use a midpoint Riemann sum with the three subintervals $[0, 10]$, $[10, 20]$, and $[20, 30]$ to approximate $\int_0^{30} M(t)\,dt$. Show the work that leads to your answer. ii. Interpret the meaning of $\int_0^{30} M(t)\,dt$ in the context of the problem.		
	i. $\int_0^{30} M(t)\,dt \approx M(5)\cdot(10-0) + M(15)\cdot(20-10) + M(25)\cdot(30-20)$	Form of midpoint Riemann sum	**Point 3 (P3)**
	$= 7\cdot 10 + 6\cdot 10 + 2\cdot 10 = 150$	Approximation with supporting work	**Point 4 (P4)**
	ii. $\int_0^{30} M(t)\,dt$ represents the number of male birds of this species that arrive at the nesting area from $t = 0$ days to $t = 30$ days.	Interpretation	**Point 5 (P5)**

Scoring Notes for Part B

- Read "$=$" as "$\approx$" for **P3**.
- The form of a midpoint Riemann sum includes three terms, each of which includes a product of two factors. To earn **P3**, at least five of the six factors must be correct. If any of the six factors is incorrect, the response does not earn **P4**.
- A response of $7\cdot 10 + 6\cdot 10 + 2\cdot 10$ or $10\cdot(7+6+2)$ earns **P3** and banks **P4** (i.e., subsequent errors in simplification will not be considered in scoring for **P4**).
- **P4** is earned for the value 150 (or equivalent) with supporting work. **P4** may be earned without **P3**. For example, $70 + 60 + 20$ or $10(15)$ earns **P4** but not **P3**.
- Consider the following examples:
  - $7\cdot(10-0) + 6\cdot(20-10) + 2\cdot(30-20)$ earns **P3** and is sufficient to earn **P4**.
  - $7\cdot 10 + 6\cdot 10 + 2\cdot 10$ earns **P3** and is sufficient to earn **P4**.
  - $M(5)\cdot 10 + M(15)\cdot 10 + M(25)\cdot 10$ earns **P3** and is eligible for **P4**.
  - $M(5)\cdot 10 + M(15)\cdot 10 + M(25)\cdot 10 = 150$ earns both **P3** and **P4**.
  - $5\cdot 10 + 6\cdot 10 + 2\cdot 10$ earns **P3** but is not eligible for **P4**.
    (Note that the factor of 5 in the first term of this expression is incorrect.)
  - An unsupported answer of 150 does not earn **P3** or **P4**.
- A completely correct left or right Riemann sum with supporting work does not earn **P3** and banks **P4** (i.e., subsequent errors in simplification will not be considered in scoring for **P4**). A response with any error or missing factors in a left or right Riemann sum does not earn **P3** or **P4**.
- **P5** can be earned whether or not **P3** and **P4** were earned.
- To earn **P5**, an interpretation must include both "number of [male] birds that arrive" and the time interval $t = 0$ to $t = 30$. The response need not include units of time or "nesting area."

C The rate at which female birds of the same species arrive at the same nesting area, in birds per day, is modeled by the function F defined as follows.

$$F(t) = \begin{cases} 0 & \text{for } 0 \le t < 15 \\ 18 + 16\sin\left(\frac{\pi}{20}(t+15)\right) & \text{for } 15 \le t \le 45 \end{cases}$$

How many female birds of this species arrive at the nesting area from $t = 15$ to $t = 45$? Show the setup for your calculations, and round your answer to the nearest integer.

$\int_{15}^{45} F(t)\, dt$	Integral	**Point 6 (P6)**
$= 641.859164$ To the nearest integer, the number of female birds that arrive from $t = 15$ to $t = 45$ is 642 birds.	Answer	**Point 7 (P7)**

Scoring Notes for Part C

- **P6** can be earned with or without the differential dt.
- **P6** can also be earned for the expression $\left[18t - \frac{320}{\pi}\cos\left(\frac{\pi}{20}(t+15)\right)\right]_{t=15}^{t=45}$.
- **P7** is earned for the correct answer, with or without supporting work.
- **P7** can also be earned for 641, 641.8, 641.9, 641.85, 641.86, 641.859, or the exact answer of $\frac{320}{\pi} + 540$.
- Units are not considered in scoring.

D	On the interval $15 < t < 30$, the difference in the rates at which male and female birds of this species arrive at the nesting area can be modeled by the differentiable function $D(t) = M(t) - F(t)$, where F is the function defined in part C. Is there a time t in the interval $15 < t < 20$ when $D(t) = 0$? Justify your answer.		
	Because D is differentiable, D is continuous.	Differentiable implies continuous	**Point 8 (P8)**
	$D(15) = M(15) - F(15) = 4 > 0$ $D(20) = M(20) - F(20) = -1.686292 < 0$ Therefore, by the Intermediate Value Theorem, there must be a time t, with $15 < t < 20$, such that $D(t) = 0$.	Answer with justification	**Point 9 (P9)**

Scoring Notes for Part D

- To earn **P8**, a response must state that D is continuous because D is differentiable (or equivalent). A response that simply states "D is continuous" without justification does not earn **P8**.
- A response does not need to earn **P8** to be eligible for **P9**.
- To earn **P9**, a response must indicate that $D(15) > 0$ and $D(20) < 0$, state that "D is continuous," and answer "yes" in some way. A response of "yes, IVT" is not sufficient to earn **P9**.
  - A response that has earned **P8** does not need to restate "D is continuous" to earn **P9**.
- To earn **P9**, a response need not explicitly name the Intermediate Value Theorem, but if a theorem is named, it must be correct.

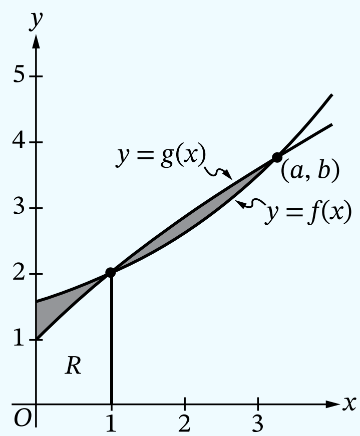
Part A (AB): Graphing Calculator Required
Question 2 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

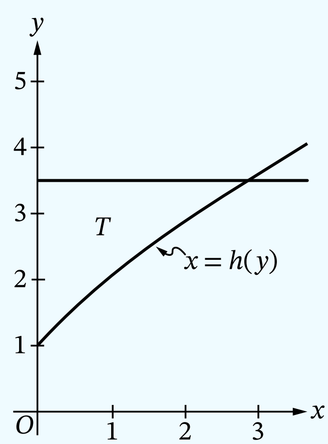
The function *f* is defined by $f(x) = 1.43^x + 0.57$, and the function *g* is defined by $g(x) = \frac{14x + 12}{x + 12}$. The graphs of *f* and *g* intersect at the points $(1, 2)$ and (a, b), as shown in Figure 1.

Figure 1



For $x \ge 0$, the equation $y = g(x)$ can be rewritten as $x = h(y) = \frac{12y - 12}{14 - y}$, where *h* is a function of *y*. The graph of $x = h(y)$ and the horizontal line $y = 3.5$ are shown in Figure 2.

Figure 2



	Model Solution	Scoring	
A	Let R be the region bounded by the graph of g, the x-axis, the y-axis, and the vertical line $x = 1$, as shown in Figure 1. Find the area of region R. Show the setup for your calculations.		
	$\int_0^1 g(x)\,dx$	Integral	**Point 1 (P1)**
	$= 1.513338$ The area of region R is 1.513.	Answer	**Point 2 (P2)**

Scoring Notes for Part A

- **P1** is earned for expressing the area of R as $\int_0^1 g(x)\,dx$ or $\int_0^1 \frac{14x+12}{x+12}\,dx$, with or without the differential dx.
- **P2** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point.
- Incorrect communication between the correct integral and the correct answer is treated as scratch work and is not considered in scoring.
- The exact answer is $14 + 156\ln\left(\frac{12}{13}\right)$.
- Note: **P1** could also be earned with $1 + \int_1^2 (1 - h(y))\,dy$ or $2 - \int_1^2 h(y)\,dy$.

B Region R, described in part A, is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is $\frac{1}{3}$ times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

Volume $= \frac{1}{3}\int_0^1 (g(x))^2\,dx = \frac{1}{3}\int_0^1 \left(\frac{14x+12}{x+12}\right)^2 dx$	Form of integrand	**Point 3 (P3)**
	Answer	**Point 4 (P4)**

Scoring Notes for Part B

- **P3** is earned for a response with an integrand of the form $k(g(x))^2$ in a definite or indefinite integral, where k is any nonzero constant.
- **P4** is earned for the correct definite integral, with or without the differential dx.
- Any of $\frac{1}{3}\int_0^1 (g(x))^2\,dx$, $\frac{1}{3}\int_0^1 \left(\frac{14x+12}{x+12}\right)^2 dx$, or a mathematically equivalent expression is sufficient to earn both **P3** and **P4**.
- **Special case:** If the response did not earn **P1** in part A with $\int_0^1 f(x)\,dx$ and then in part B presents $\int_0^1 (f(x))^2\,dx$, the response earns **P3** and is eligible for **P4.**
- **Special case:** If the response did not earn **P1** in part A with either $\int_0^1 (f(x)-g(x))\,dx$ or $\int_0^1 (g(x)-f(x))\,dx$ and then in part B presents $\int_0^1 (f(x)-g(x))^2\,dx$ or $\int_0^1 (g(x)-f(x))^2\,dx$, the response earns **P3** and is eligible for **P4.**

C The shaded region in Figure 1 is bounded by the graphs of f and g on the interval from $x = 0$ to $x = a$. Find the area of the shaded region. Show the setup for your calculations.

$f(x) = g(x) \Rightarrow x = 1,\ x = 3.255817$	Form of integrand	**Point 5 (P5)**
$\int_0^{3.255817} \lvert f(x) - g(x)\rvert\, dx$	Integral	**Point 6 (P6)**
$= 0.631784$ The area of the shaded region is 0.632 (or 0.631).	Answer	**Point 7 (P7)**

Scoring Notes for Part C

- **P5** is earned for a response that presents an integrand of $f(x) - g(x)$, $\lvert f(x) - g(x)\rvert$, $g(x) - f(x)$, or $\lvert g(x) - f(x)\rvert$ in a definite integral, with or without the differential dx.
- **P6** is earned for the definite integral $\int_0^a \lvert f(x) - g(x)\rvert\, dx$, $\int_0^a \lvert g(x) - f(x)\rvert\, dx$, or a mathematically equivalent expression.
- Note: **P6** could also be earned for a sum of definite integrals, such as $\int_0^1 (f(x) - g(x))\, dx + \int_1^a (g(x) - f(x))\, dx$, $\int_0^1 (g(x) - f(x))\, dx + \int_1^a (f(x) - g(x))\, dx$, or a mathematically equivalent expression.
- Note: If the response presents a value for a, it must be either 3, 3.2, 3.3, 3.25, 3.26, 3.255, or 3.256 to be eligible for **P6**. Any additional decimal places after the third decimal place are ignored.
- **P7** is earned for the correct answer, with or without supporting work. A reported answer should be accurate to three places after the decimal point, rounded or truncated. An inappropriately rounded answer does not earn the point, unless an earlier point was not earned due to inappropriate rounding.
- Incorrect communication between the correct integral and the correct answer is treated as scratch work and is not considered in scoring.
  - $\int_0^1 (g(x) - f(x))\, dx + \int_1^{3.255817} (f(x) - g(x))\, dx = -0.632$, so the area is 0.632.

    Note: This response earns **P5** and **P6** for the integral. It also earns **P7** for the correct answer.
  - $\int_0^1 (g(x) - f(x))\, dx + \int_1^{3.255817} (f(x) - g(x))\, dx = 0.632$

    Note: This response earns **P5** and **P6** for the integral. It also earns **P7** for the correct answer. (In this instance, incorrect linkage is not considered in scoring.)

D	Let T be the region bounded by the graph of $x = h(y)$, the y-axis, and the horizontal line $y = 3.5$, as shown in Figure 2. Write, but do not evaluate, an integral expression that gives the volume of the solid generated when region T is revolved about the y-axis.		
	Volume $= \pi \int_1^{3.5} (h(y))^2 \, dy$	Form of integrand	**Point 8 (P8)**
		Answer	**Point 9 (P9)**

Scoring Notes for Part D

- **P8** is earned for a response with an integrand of the form $k(h(y))^2$ in a definite or indefinite integral, where k is any nonzero constant.
- **P9** is earned for the correct definite integral, including the constant π, limits $y = 1$ to $y = 3.5$, and differential dy.
  - Note: **P8** and **P9** can be earned by using the shell method, $2\pi \int_0^{20/7} x(3.5 - g(x)) \, dx$.

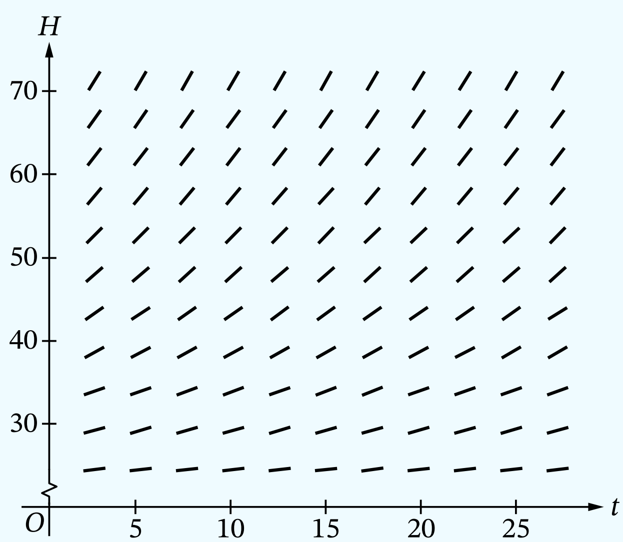
Part B (AB or BC): Graphing Calculator Not Allowed
Question 3 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

A pie is taken from a hot oven and put on a table. The internal temperature of the pie at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{15}(H - 20)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 75$. For $t > 0$, it is known that $20 < H(t) < 75$.

	Model Solution	Scoring	
A	Explain why the following could not be a slope field for the differential equation $\frac{dH}{dt} = -\frac{1}{15}(H - 20)$.		



Model Solution	Scoring	
The slopes of the line segments in a slope field for the given differential equation should all be negative because $\frac{dH}{dt} < 0$ on $20 < H < 75$. However, the slopes of the line segments in the slope field given are all positive.	Explanation	**Point 1 (P1)**

Scoring Notes for Part A

- To earn **P1**, the response must reference the sign of the slopes of the line segments or the slope field's implication for the graph of H. Referencing the sign of $\frac{dH}{dt}$ without connection to the slope field does not earn **P1**.
- Common communication errors include references to the sign of the slope field, the slope of the slope field, the slope of the differential equation, and the increasing behavior of the slope field. Such responses do not earn **P1**.
- **P1** can be earned for a response of "because there are no negative slopes shown" or "because the slopes shown are positive."
- **P1** can be earned by using a local argument at any point (t, H) with $0 < t < 30$ and $20 < H < 75$, rather than a global argument.

B Find the slope of the line tangent to the graph of H at time $t = 0$. Show the work that leads to your answer.

$\left.\frac{dH}{dt}\right\|_{(0,\,75)} = -\frac{1}{15}(75-20) = -\frac{11}{3}$	Answer with supporting work	**Point 2 (P2)**

Scoring Notes for Part B

- A response can earn **P2** with $-\frac{1}{15}(75-20)$, $-\frac{1}{15}(55)$, or $\left.\frac{dH}{dt}\right|_{(0,\,75)} = -\frac{11}{3}$.
- A response of $-\frac{1}{15}(75-20)$ or $-\frac{1}{15}(55)$ banks **P2** (i.e., subsequent errors in simplification will not be considered in scoring for **P2**).
- Presentation of a tangent line equation will not be considered in scoring for **P2**.

C It can be shown that $\frac{d^2H}{dt^2} = \frac{1}{225}(H-20)$. The line tangent to the graph of H at time $t = 0$ is used to approximate $H(5)$, the internal temperature of the pie at time $t = 5$. Is this approximation an overestimate or an underestimate for the actual value of $H(5)$? Give a reason for your answer.

For $t > 0$, $20 < H < 75$, so $\frac{d^2H}{dt^2} > 0$.	Considers sign of $\frac{d^2H}{dt^2}$	**Point 3 (P3)**
Because $\frac{d^2H}{dt^2} > 0$, the graph of H is concave up. Therefore, the tangent line lies below the graph, so the approximation is an underestimate for $H(5)$.	Answer with reason	**Point 4 (P4)**

Scoring Notes for Part C

- **P3** can be earned with $\frac{d^2H}{dt^2} = 0$, $\frac{d^2H}{dt^2} > 0$, or $\frac{d^2H}{dt^2} < 0$. **P3** can be earned symbolically or in words.
- A response is eligible for **P4** only if it has earned **P3**.
- A response that states "concave up, so underestimate" earns **P4** if **P3** was earned. Referencing the concavity of $\frac{dH}{dt}$ or $\frac{d^2H}{dt^2}$ does not earn **P4**.
- Although a response that presents an argument based on $\frac{d^2H}{dt^2}$ or concavity at a single point can earn **P3** with an explicit consideration of the sign of $\frac{d^2H}{dt^2}$, such an argument does not earn **P4**.

D Use separation of variables to find an expression for $H(t)$, the particular solution to the given differential equation with initial condition $H(0) = 75$.

$\frac{dH}{H-20} = -\frac{1}{15}\,dt$ $\int \frac{dH}{H-20} = \int -\frac{1}{15}\,dt$	Separates variables	**Point 5 (P5)**
$\ln\lvert H-20\rvert = -\frac{t}{15} + C$	Finds one antiderivative	**Point 6 (P6)**
	Finds other antiderivative	**Point 7 (P7)**
$\ln\lvert 75-20\rvert = -\frac{1}{15}(0) + C \;\Rightarrow\; \ln 55 = C$	Constant of integration and uses initial condition	**Point 8 (P8)**
$\ln\lvert H-20\rvert = -\frac{t}{15} + \ln 55$ $\lvert H-20\rvert = 55e^{-t/15}$ $H-20 = \pm 55e^{-t/15}$ Because $H(0) = 75$, $H(t) = 20 + 55e^{-t/15}$.	Solves for $H(t)$	**Point 9 (P9)**

Scoring Notes for Part D

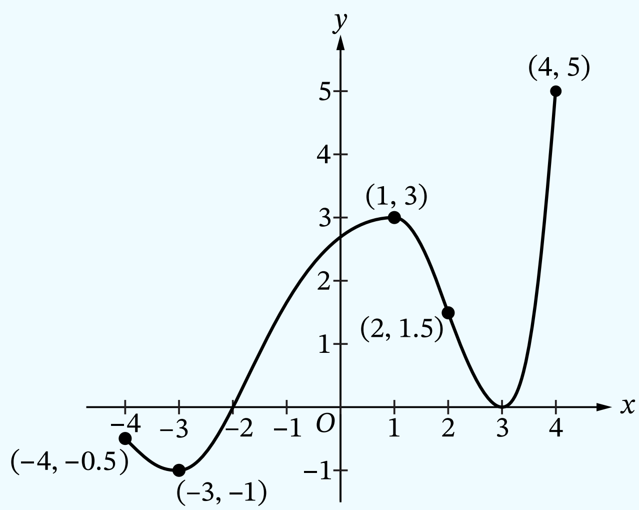
- A response with no separation of variables does not earn **P5**, **P6**, **P7**, **P8**, or **P9**.
- A response earns **P5** if the separation of variables results in an equation of the form $A\frac{dH}{H-20} = B\,dt$, where A and B are nonzero constants.
- A response that presents a consistent antiderivative of the form $A\ln(H-20)$ (with either parentheses or absolute value) or Bt earns **P6**. If the response presents two consistent antiderivatives, it also earns **P7**.
- **Special case:** If the response begins with a separation of the form $A\frac{dH}{H+20} = B\,dt$, where A and B are correct coefficients, it earns **P5**, can earn **P6** with one consistent antiderivative, can earn **P7** with two consistent antiderivatives, and is eligible to earn **P8** but not **P9**.
- A response with no constant of integration is eligible to earn **P5**, **P6**, and **P7**, but does not earn **P8** or **P9**.
- A response is eligible for **P8** if it has earned **P5** and **P6**.
- An eligible response earns **P8** by correctly including the constant of integration in an equation and substituting 0 for t and 75 for H.
- A response is eligible for **P9** only if it has earned **P5**, **P6**, and **P7**. Responses that do not explicitly show the constant of integration and use of initial condition can still earn **P9** with a correct $H(t)$.
- A response earns **P9** for an answer of $H(t) = 20 + 55e^{-t/15}$, $H(t) = 20 + e^{-\frac{t}{15}+\ln 55}$, or equivalent. The response does not need to provide justification for their choice of the positive coefficient on the exponential.

Part B (AB or BC): Graphing Calculator Not Allowed
Question 4 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Let f be a twice-differentiable function on the closed interval $[-4, 4]$ with $f(2) = 3$. The graph of f', the derivative of f, is shown.



Graph of f'

	Model Solution	Scoring	
A	For $x > 0$, the function g is defined by $g(x) = f(x) - \ln x$. Find $g'(2)$. Show the work that leads to your answer.		
	$g'(x) = f'(x) - \frac{1}{x}$	Setup	**Point 1 (P1)**
	$g'(2) = f'(2) - \frac{1}{2} = 1.5 - \frac{1}{2} = 1$	Answer	**Point 2 (P2)**

Scoring Notes for Part A

- **P1** can be earned with $g'(x) = f'(x) - \frac{1}{x}$ or $g'(2) = f'(2) - \frac{1}{2}$.
- An answer of $1.5 - \frac{1}{2}$ earns **P1** and **P2**.
- A response does not need to simplify, but an error in simplification does not earn **P2**.
- An unsupported answer of 1 earns **P2** but not **P1**.

B Find all values of x on the open interval $0 < x < 3$ at which the graph of f has a point of inflection. Give a reason for your answer.

The graph of f has a point of inflection where f' changes from decreasing to increasing or from increasing to decreasing. The graph of f has a point of inflection at $x = 1$ because f' changes from increasing to decreasing there.	Answer	**Point 3 (P3)**
	Reason	**Point 4 (P4)**

Scoring Notes for Part B

- **P3** is earned for an answer of $x = 1$. If any other/additional values of x in $0 < x < 3$ are declared to be locations for points of inflection, the response does not earn either **P3** or **P4**. A response that also declares $x = -3$ and/or $x = 3$ as locations for points of inflection is eligible for both **P3** and **P4**.
- A response that declares a point $(1, a)$, where a is any real number, as a point of inflection earns **P3** but is not eligible for **P4**.
- A response must earn **P3** to be eligible for **P4**.
- To earn **P4**, a response must tie the reason to the given graph of f'.
  - A response of "f has a point of inflection at $x = 1$ because f' changes from increasing to decreasing there" earns both **P3** and **P4**.
  - A response of "f has a point of inflection at $x = 1$ because the slope of f' changes sign there" earns both **P3** and **P4**.
  - A response of "f has a point of inflection at $x = 1$ because f' attains a relative extremum there" earns both **P3** and **P4**.
  - A response of "f has a point of inflection at $x = 1$ because f changes concavity there" earns **P3** but not **P4**.
  - A response of "f has a point of inflection at $x = 1$ because f'' changes sign there" earns **P3** but not **P4**.
  - A response that relies upon an ambiguous term such as "the function" or "the graph" does not earn **P4**.

C	For $-4 \le x \le 4$, on what open intervals, if any, is the graph of f both increasing and concave down? Give a reason for your answer.		
	The graph of function f is increasing on an interval when $f'(x) > 0$ and concave down on an interval when f' is decreasing. Therefore, f is increasing and concave down on the interval $1 < x < 3$.	Answer	**Point 5 (P5)**
		Reason	**Point 6 (P6)**

Scoring Notes for Part C

- **P5** is earned by an answer of $1 < x < 3$, or this interval including one or both endpoints.
- A response must earn **P5** to be eligible for **P6**.
- To earn **P6**, a response must refer to f' and correctly cite both $f'(x) > 0$ and f' is decreasing.
- A response of "$f' > 0$ and f' is decreasing on $1 < x < 3$" earns both **P5** and **P6**.
- **Special case:** A response of "f is increasing on $-2 < x < 3$ and $3 < x < 4$ because $f'(x) > 0$ there" earns **P5** but not **P6**.
- **Special case:** A response of "f is concave down on $-4 < x < -3$ and $1 < x < 3$ because f' is decreasing there" earns **P5** but not **P6**.

D	For $-4 \le x \le 4$, find the value of x at which f has an absolute minimum and the value of x at which f has an absolute maximum. Give reasons for your answers.		
	For $-4 \le x \le 4$, f attains an absolute extremum when $f'(x) = 0$ or at an endpoint. $f'(x) = 0 \;\Rightarrow\; x = -2,\ x = 3$ Because $f'(x) < 0$ for $-4 < x < -2$, f is decreasing there. Because $f'(x) > 0$ for $-2 < x < 3$, f is increasing there. Because $f'(x) > 0$ for $3 < x < 4$, f is increasing there. Therefore, f has an absolute minimum at $x = -2$. $f(4) - f(-4) = \int_{-4}^{4} f'(x)\,dx > 0$, because on the interval $-4 \le x \le 4$, the area bounded by the graph of f' above the x-axis is greater than the area bounded by the graph of f' below the x-axis. Therefore, $f(4) > f(-4)$ and f has an absolute maximum at $x = 4$.	Considers $f'(x) = 0$ Absolute minimum with reason Absolute maximum with reason	**Point 7 (P7)** **Point 8 (P8)** **Point 9 (P9)**

Scoring Notes for Part D

- **P7** is earned for considering $f'(x) = 0$. **P7** is not earned by just presenting $x = -2$ and $x = 3$.
  - A response that discusses the sign of $f'(x)$ changing or uses the phrase “critical points of f” also earns **P7**.
- A response that correctly declares “minimum at $x = -2$” with correct justification earns **P8**.
- A response that correctly declares “maximum at $x = 4$” with correct justification earns **P9**.
- **Special case:** A response that correctly declares a “minimum at $x = -2$” and “maximum at $x = 4$” with an incorrect or insufficient attempt at justification earns **P8** but not **P9**.

Part B (AB): Graphing Calculator Not Allowed
Question 5 — 9 points

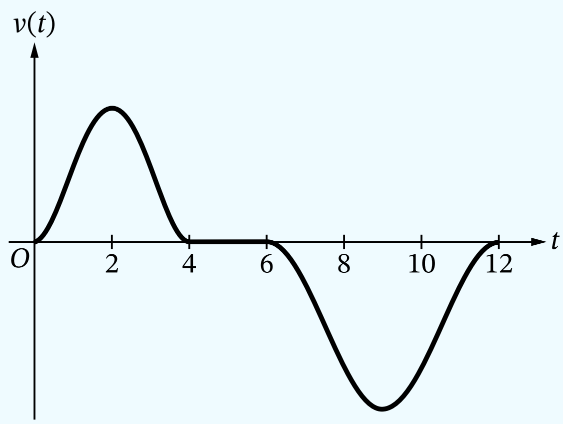
General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

A remote-controlled toy car moves back and forth along a straight path so that its velocity at time t is given by the function v, where $v(t)$ is measured in feet per second and t is measured in seconds.

$$v(t) = \begin{cases} t^4 - 8t^3 + 16t^2 & \text{for } 0 \le t \le 4 \\ 0 & \text{for } 4 < t < 6 \\ 10\cos\left(\dfrac{\pi}{3}t\right) - 10 & \text{for } 6 \le t \le 12 \end{cases}$$

The graph of $v(t)$ is shown.



	Model Solution	Scoring	
A	Find the acceleration of the car at time $t = 1$ second. Show the work that leads to your answer.		
	For $0 < t < 4$, $v'(t) = a(t) = 4t^3 - 24t^2 + 32t$.	Considers $v'(t)$	**Point 1 (P1)**
	$v'(1) = a(1) = 4(1)^3 - 24(1)^2 + 32(1) = 12$	Answer	**Point 2 (P2)**

Scoring Notes for Part A

- **P1** can be earned by presenting $v'(t)$, $v'(1)$, $4t^3 - 24t^2 + 32t$, or $4(1)^3 - 24(1)^2 + 32(1)$.
- A response of $4(1)^3 - 24(1)^2 + 32(1)$ need not be simplified, but if a value is presented, it must be correct to earn **P2**.
- An unsupported answer of 12 earns **P2** but not **P1**.
- Note: In the event of a copy error where $v(t)$ is a fourth-degree polynomial with at most one error, the response is eligible for **P1** with a presented correct derivative but cannot earn **P2**.
  - If this copy error is continued in part B and/or part C, the response is eligible for **P3** through **P6**.

	Model Solution	Scoring	
B	Is the car speeding up or slowing down at time $t = 1$ second? Give a reason for your answer.		
	$v(1) > 0$ and $v'(1) > 0$. Because $v(1)$ and $v'(1)$ have the same sign, the car is speeding up at time $t = 1$ second.	Answer with reason	**Point 3 (P3)**

Scoring Notes for Part B

- To earn **P3**, a response must consider both velocity and acceleration either numerically or graphically.
- An evaluation of $v(1)$ is not necessary, but if a value is presented, it must be correct. The correct value is $v(1) = 9$.
- A response can either import the implied sign of $v'(1)$ from part A or restart.
- A response that imports a negative value for $v'(1)$ from part A can earn **P3** if it also considers a positive velocity and states that the car is slowing down.
- Alternate justification: A response can earn **P3** if it states that the car is speeding up because the graph of velocity is positive and increasing at $t = 1$ or because the graph of velocity is moving away from the horizontal axis at $t = 1$.

C	Find the distance, in feet, that the car traveled over the time interval $0 \le t \le 4$ seconds. Show the work that leads to your answer.		
	$\int_0^4 v(t)\,dt = \int_0^4 \left(t^4 - 8t^3 + 16t^2\right) dt$	Integral	**Point 4 (P4)**
	$= \left.\frac{t^5}{5} - 2t^4 + \frac{16}{3}t^3\right\|_0^4$	Antiderivative	**Point 5 (P5)**
	$= \frac{4^5}{5} - 2\cdot 4^4 + \frac{16}{3}\cdot 4^3 - 0 = 4^3\left(\frac{16}{5} - 8 + \frac{16}{3}\right) = \frac{512}{15}$ Over the time interval $0 \le t \le 4$ seconds, the car traveled a distance of $\frac{512}{15}$ feet.	Answer	**Point 6 (P6)**

Scoring Notes for Part C

- To earn **P4**, a response must present a definite integral with an integrand of $v(t)$ or $t^4 - 8t^3 + 16t^2$, with or without the differential dt. The use of absolute value in the integrand (e.g., $|v(t)|$ or $\left|t^4 - 8t^3 + 16t^2\right|$) is acceptable for **P4** (although there is no change in sign on $0 \le t \le 4$).
- **P5** is earned for the correct antiderivative.
- A response of $\frac{4^5}{5} - 2\cdot 4^4 + \frac{16}{3}\cdot 4^3$ earns **P5** and banks **P6** (i.e., subsequent errors in simplification will not be considered in scoring for **P6**).

D Find the average velocity of the car over the time interval $6 \le t \le 12$ seconds. Show the work that leads to your answer.

$\frac{1}{12-6}\int_6^{12} v(t)\,dt = \frac{1}{6}\int_6^{12}\left(10\cos\left(\frac{\pi}{3}t\right)-10\right)dt$	Average value formula	**Point 7 (P7)**
$= \frac{1}{6}\left(10\cdot\frac{3}{\pi}\sin\left(\frac{\pi}{3}t\right)-10t\right)\Big\vert_6^{12}$	Form of antiderivative	**Point 8 (P8)**
$= \frac{1}{6}(-120-(-60)) = -10$ Over the time interval $6 \le t \le 12$ seconds, the average velocity of the car is -10 feet per second.	Answer	**Point 9 (P9)**

Scoring Notes for Part D

- **P7** is earned for the correct definite integral with an integrand of either $v(t)$ or $\left(10\cos\left(\frac{\pi}{3}t\right)-10\right)$, with or without the differential dt, along with evidence of division by 6. In the presence of the correct integral, the correct answer will suffice as evidence of division by 6. These may appear all in one step, as in the model solution, or in multiple steps.
  - **Special case:** A copy error using an integrand of $10\cos\left(\frac{\pi}{3}t\right)$ is eligible for **P7** and can earn **P8** with the correct antiderivative of $\frac{30}{\pi}\sin\left(\frac{\pi}{3}t\right)$ but cannot earn **P9**.
- **P8** is earned for an antiderivative of the form $k\sin\left(\frac{\pi}{3}t\right)-10t$, $\frac{1}{6}\left(k\sin\left(\frac{\pi}{3}t\right)-10t\right)$, or equivalent, for $k > 0$ or $k = -\frac{30}{\pi}$. If $k \ne \frac{30}{\pi}$, then the response is not eligible to earn **P9**.
- A response of $\frac{1}{6}\left(\left(10\cdot\frac{3}{\pi}\sin\left(\frac{\pi}{3}\cdot 12\right)-10\cdot 12\right)-\left(10\cdot\frac{3}{\pi}\sin\left(\frac{\pi}{3}\cdot 6\right)-10\cdot 6\right)\right)$, $\frac{1}{6}(-120-(-60))$, or equivalent banks **P9** (i.e., subsequent errors in simplification will not be considered in scoring for **P9**).
  - Note: An ambiguous response, such as unmatched parentheses in $\frac{1}{6}\left(\left(10\cdot\frac{3}{\pi}\sin\left(\frac{\pi}{3}\cdot 12\right)-10\cdot 12\right)-\left(10\cdot\frac{3}{\pi}\sin\left(\frac{\pi}{3}\cdot 6\right)-10\cdot 6\right)\right.$ or $\frac{1}{6}(-120-(-60)$, does not bank **P9**. Therefore, to earn **P9,** the response must resolve the ambiguity with a correct final answer, such as $\frac{1}{6}(-120-(-60))$ or $\frac{1}{6}(-60)$.

Part B (AB): Graphing Calculator Not Allowed
Question 6 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

The function f is twice differentiable. The table gives values of f and its derivative f' at selected values of x.

x	0	2	3	6
$f(x)$	-1	3	8	5
$f'(x)$	-5	4	9	-2

	Model Solution	Scoring	
A	Find $\lim_{x\to 2} \frac{f(x)}{x}$, or state that the limit does not exist.		
	$\lim_{x\to 2} \frac{f(x)}{x} = \frac{f(2)}{2} = \frac{3}{2}$	Answer	**Point 1 (P1)**

Scoring Notes for Part A

- An answer of $\frac{3}{2}$ earns **P1**, with or without supporting work.
- To earn **P1**, a response must resolve the limit; for example, writing only $\lim_{x\to 2} \frac{3}{2}$ is not sufficient.

B	Let $g(x) = f(f(x))$. Find $g'(2)$. Show the work that leads to your answer.		
	$g'(x) = f'(f(x)) \cdot f'(x)$	Chain rule	**Point 2 (P2)**
	$g'(2) = f'(f(2)) \cdot f'(2) = f'(3) \cdot f'(2) = 9 \cdot 4 = 36$	Answer	**Point 3 (P3)**

Scoring Notes for Part B

- **P2** is earned with either $f'(f(x)) \cdot f'(x)$, $f'(f(2)) \cdot f'(2)$, or $f'(3) \cdot f'(2)$.
- To be eligible for **P3**, a response must have earned **P2**. **P3** is earned for an answer of 36 or equivalent.

C	Let h be a differentiable function such that $h(0) = 10$ and $h'(x) = f'(3x)$. Find $h(2)$. Show the work that leads to your answer.		
	$h(2) = h(0) + \int_0^2 h'(x)\,dx = 10 + \int_0^2 f'(3x)\,dx$	Integral	**Point 4 (P4)**
	$= 10 + \left(\frac{1}{3}f(3x)\Big\vert_0^2\right) = 10 + \frac{1}{3}(f(6) - f(0))$	Antiderivative	**Point 5 (P5)**
	$= 10 + \frac{1}{3}(5 - (-1)) = 10 + \frac{1}{3}(6) = 12$	Answer	**Point 6 (P6)**

Scoring Notes for Part C

- To earn **P4**, a response must present an indefinite or definite integral with an integrand of $h'(x)$ or $f'(3x)$, with or without the initial condition of $h(0) = 10$. (See below for notes on how to handle a missing differential dx.)
- **P5** is earned for an antiderivative of the form $k\,f(3x)$ or equivalent, for $k > 0$. If $k \neq \frac{1}{3}$, then the response is not eligible to earn **P6**.
- A response of $10 + \frac{1}{3}(5 - (-1))$ or equivalent banks **P6** (i.e., subsequent errors in simplification will not be considered in scoring for **P6**).
  - Note: An ambiguous response, such as $10 + \frac{1}{3}(5 - (-1)$, does not bank **P6** and therefore must go on to resolve the ambiguity with a correct final answer (e.g., $10 + \frac{1}{3}(6)$ or 12) to earn **P6**.
- If the differential dx is missing:
  - Writing $\int_0^2 h'(x)$ or $\int_0^2 f'(3x)$ earns **P4** and is eligible to earn **P5** and **P6**.
  - Writing $10 + \int_0^2 h'(x)$ or $10 + \int_0^2 f'(3x)$ earns **P4** and is eligible to earn **P5** and **P6**.
  - Writing $\int_0^2 h'(x) + 10$ or $\int_0^2 f'(3x) + 10$ introduces an ambiguity for the intended integrand.
    - $\int_0^2 h'(x) + 10 = \left[\frac{1}{3}f(3x)\right]_0^2 + 10$ or $\int_0^2 f'(3x) + 10 = \left[\frac{1}{3}f(3x)\right]_0^2 + 10$ resolves the ambiguity. Therefore, this earns **P4** and **P5** and is eligible for **P6**.
    - $\int_0^2 h'(x) + 10 = \left[\frac{1}{3}f(3x) + 10x\right]_0^2$ or $\int_0^2 f'(3x) + 10 = \left[\frac{1}{3}f(3x) + 10x\right]_0^2$ confirms that an incorrect integrand was used. Therefore, this does not earn **P4**, earns **P5**, and is not eligible for **P6**.
    - If the ambiguity is not resolved, this does not earn **P4**, **P5**, or **P6**.
- Alternate solution using indefinite integral:

$$\int f'(3x)\,dx = \frac{1}{3}f(3x) + C$$

$$h(0) = 10 = \frac{1}{3}f(0) + C = \frac{1}{3}(-1) + C \;\Rightarrow\; C = \frac{31}{3}$$

$$h(x) = \frac{1}{3}f(3x) + \frac{31}{3}$$

$$h(2) = \frac{1}{3}f(6) + \frac{31}{3} = \frac{5}{3} + \frac{31}{3} = 12$$

D Let k be the function defined by $k(x) = \int_0^x t^2 f(t)\,dt$.

i. Find $k'(x)$.

ii. Find $k''(3)$. Show the work that leads to your answer.

i. $k'(x) = \frac{d}{dx}\left(\int_0^x t^2 \cdot f(t)\,dt\right) = x^2 \cdot f(x)$	$k'(x)$	**Point 7 (P7)**
ii. $k''(x) = x^2 \cdot f'(x) + 2x \cdot f(x)$	Product rule	**Point 8 (P8)**
$k''(3) = 3^2 \cdot f'(3) + 2(3) \cdot f(3) = 9 \cdot 9 + 6 \cdot 8 = 129$	Answer	**Point 9 (P9)**

Scoring Notes for Part D

- **P7** is earned for $k'(x) = x^2 \cdot f(x)$.
- **P8** is earned with either $x^2 \cdot f'(x) + 2x \cdot f(x)$ or $3^2 \cdot f'(3) + 2(3) \cdot f(3)$.
- A response must have earned **P7** and **P8** to be eligible for **P9**. **P9** is earned for an answer of 129 or equivalent.