

# AP Physics C: Electricity and Magnetism

# **Question 1: Mathematical Routines (MR)**

10 points

A (i) For a multistep derivation that includes the equation  $\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$ 

Point A1

Scoring Note: Vector notation is not required for this point to be earned.

For a correct substitution of the area of an appropriate Gaussian surface with nonzero flux for the region  $R_1 < r < R_2$  (e.g.,  $2\pi r\ell$ )

Point A2

For a correct expression for the enclosed charge (e.g.,  $\sigma_1(2\pi R_1 \ell)$ )

Point A3

### **Example Response**

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

$$E(2\pi r\ell) = \frac{\sigma_1(2\pi R_1 \ell)}{\varepsilon_0}$$

$$E = \frac{\sigma_1 R_1}{\varepsilon_0 r}$$

(ii) For substituting the expression for E from part A (i) into  $\Delta V = -\int_a^b \vec{E} \cdot d\vec{r}$ 

Point A4

#### **Scoring Notes:**

- Vector notation is not required for this point to be earned.
- The sign of  $\Delta V$  is not considered for this point to be earned.

For an attempt to solve the integral for  $|\Delta V|$  that includes correct limits

Point A5

(e.g., 
$$|\Delta V| = \frac{\sigma_1 R_1}{\varepsilon_0} \int_{R_1}^{R_2} \frac{1}{r} dr$$
)

**Scoring Note:** This point may be earned regardless of the order of the limits of integration; for example,  $\frac{\sigma_1 R_1}{\varepsilon_0} \int_{R_2}^{R_1} \frac{1}{r} dr$ .

# **Example Response**

$$\begin{aligned} \left| \Delta V \right| &= \left| -\int_{a}^{b} \vec{E} \cdot d\vec{r} \right| \\ \left| \Delta V \right| &= \int_{R_{1}}^{R_{2}} \frac{\sigma_{1} R_{1}}{r \varepsilon_{0}} dr = \frac{\sigma_{1} R_{1}}{\varepsilon_{0}} \int_{R_{1}}^{R_{2}} \frac{1}{r} dr \\ \left| \Delta V \right| &= \frac{\sigma_{1} R_{1}}{\varepsilon_{0}} \ln \left( r \right) \bigg|_{R_{1}}^{R_{2}} &= \frac{\sigma_{1} R_{1}}{\varepsilon_{0}} \ln \left( \frac{R_{2}}{R_{1}} \right) \end{aligned}$$

(iii) For sketching a graph that is zero for both  $0 < r < R_1$  and  $r > R_2$ 

Point A6

For sketching a graph that is decreasing and concave up for  $R_1 < r < R_2$ 

Point A7

**Scoring Note:** The curve does not have to intersect the vertical dashed lines for this point to be earned.

#### **Example Response**

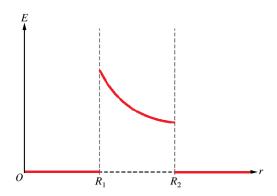


Figure 2

B For a multistep derivation that includes the equation  $C = \frac{Q}{\Delta V}$  Point B1

For indicating that C with the dielectric material inserted is C without the dielectric material inserted multiplied by  $\kappa$  (e.g.,  $C = \kappa \frac{Q}{\Delta V}$ )

For substitutions of both the total charge Q and the potential difference  $\Delta V$  that are consistent with part A

## **Example Response**

Without the dielectric material

$$C = \frac{Q}{\Delta V}$$

$$C = \frac{\sigma_1(2\pi L R_1)}{\left(\frac{\sigma_1 R_1}{\varepsilon_0} \ln\left(\frac{R_2}{R_1}\right)\right)}$$

$$C = \frac{2\pi L \varepsilon_0}{\ln\left(\frac{R_2}{R_1}\right)}$$

With the dielectric material

$$C = \kappa \frac{Q}{\Delta V}$$

$$C = \frac{2\pi L \kappa \varepsilon_0}{\ln\left(\frac{R_2}{R_1}\right)}$$

# **Question 2: Translation Between Representations (TBR)**

12 points

A For drawing a bar at  $\frac{1}{4}T$ 

Point A1

For drawing a bar at  $\frac{1}{4}T$  that has a height of 3 units

Point A2

For indicating that  $|\mathcal{E}|$  is zero at times t = 0 and  $\frac{1}{2}T$ 

Point A3

#### **Example Response**

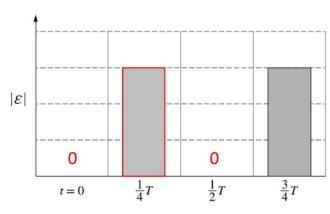


Figure 2

B For a multistep derivation that includes the equation  $\mathcal{E} = -\frac{d\Phi_{\rm B}}{dt}$ 

Point B1

**Scoring Note:** The negative sign does not have to be present in the expression for this point to be earned.

For a correct expression for the induced emf (e.g.,  $\mathcal{E} = BA\omega \sin(\omega t)$ )

Point B2

For using Ohm's law to relate current and emf (e.g.,  $I = \frac{\mathcal{E}}{R}$ )

Point B3

For a correct expression for the absolute value of the maximum induced current

Point B4

(e.g., 
$$I = \frac{BA\omega}{R}$$
)

#### **Example Response**

$$\mathcal{E} = -\frac{d\Phi_{\rm B}}{dt}$$

$$\mathcal{E} = -\frac{d}{dt} \Big[ BA \cos(\omega t) \Big] = BA\omega \sin(\omega t)$$

$$\mathcal{E}_{\rm max} = BA\omega$$

$$I = \frac{\Delta V}{R} = \frac{\mathcal{E}}{R}$$
 
$$I_{\text{max}} = \frac{\mathcal{E}_{\text{max}}}{R} = \frac{BA\omega}{R}$$

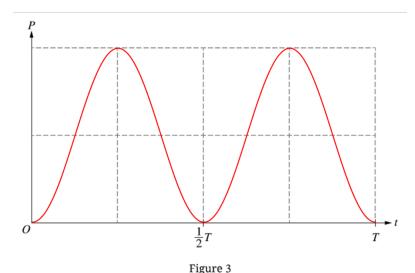
C For a curve that is approximately sinusoidal Point C1

For a curve that shows exactly two cycles Point C2

For a curve that starts at the origin and has equal maximum values and equal minimum values

**Scoring Note:** A curve that starts and ends at P = 0 and has a single maximum can earn this point.

#### **Example Response**



For correctly indicating whether the representations are consistent in parts A and C

For a correct justification that indicates that the maxima and/or minima of the graph in part C align with the bars drawn in part A because P is proportional to  $\mathcal{E}^2$ 

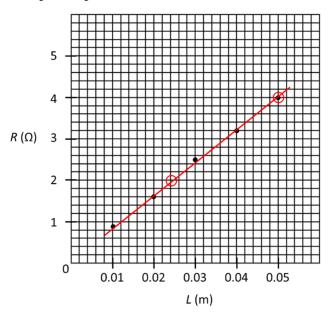
#### **Example Response**

D

Yes, part C is consistent with part A. P is proportional to  $\mathcal{E}^2$ . When  $|\mathcal{E}|$  is maximum, P is maximum.

Questi	on 3: Experimental Design and Analysis (LAB)	10 points
A	For a procedure in which the length of and the current in the circuit element are measured, and the cross-sectional area of the circuit element is determined	Point A1
	For a procedure that indicates a reasonable method of reducing experimental uncertainty	Point A2
	Examples of acceptable responses may include the following:	
	<ul> <li>Making different measurements of the current in the circuit element</li> <li>Varying the potential difference of the power supply</li> </ul>	
	Example Response	
	Measure the length and diameter of the circuit element. Use the diameter to calculate the area of the circuit element. Measure the current in the circuit element. Repeat the procedure multiple times for different potential difference settings on the variable power supply.	
В	For indicating quantities that can be graphed appropriately to determine $\rho_1$ , consistent with the procedure from part A, such as potential difference as a function of current	Point B1
	<ul> <li>Scoring Notes:</li> <li>Responses that include the reciprocals of the preceding example, in addition to other equivalent graphs, also earn this point.</li> <li>This point may be earned independently of the response in part A.</li> </ul>	
	For a correct analysis of the graph, consistent with the first point of part B	Point B2
	Example Response	
	Graph the potential difference across the circuit element as a function of the current in	
	the circuit element. Use the slope of the graph, which is $\frac{\rho_1 \ell}{\pi r^2}$ , where $\ell$ is the length of	
	the circuit element and $r$ is equal to the radius of the circuit element, to determine $\rho_1$ .	
C (i)	For indicating appropriate quantities that could be plotted on the graph to determine $\rho_2$ , for example $R$ as a function of $L$	Point C1
	<ul> <li>Scoring Note: This point may be earned:</li> <li>if the vertical and horizontal variables are reversed.</li> <li>for other equivalent graphs.</li> </ul>	
(ii)	For labeling the axes (including units) with a linear scale	Point C2
	For correctly plotting data points consistent with <b>one</b> of the following:	Point C3
	• The quantities indicated in part C (i)	
	<ul> <li>The quantities provided in the table</li> <li>The axes indicated in the first point of part C (ii)</li> </ul>	
(iii)	For drawing a line or curve that approximates the trend of the plotted data	Point C4

### **Example Response**



**D** For correctly relating the slope of the best-fit line to  $\rho_2$ 

Point D1

(e.g., 
$$\frac{\Delta R}{\Delta L}$$
 = slope =  $\frac{\rho_2}{A}$ )

For a value for  $\rho_2$  that is between  $3.5 \times 10^{-4} \ \Omega \cdot m$  and  $4.5 \times 10^{-4} \ \Omega \cdot m$ 

Point D2

## **Example Response**

slope = 
$$\frac{\Delta R}{\Delta L}$$
  
 $\frac{\Delta R}{\Delta L} = \frac{4.0 \Omega - 2.0 \Omega}{0.050 \text{ m} - 0.024 \text{ m}}$   
 $\frac{\Delta R}{\Delta L} = 77 \frac{\Omega}{\text{m}}$ 

$$R = \frac{\rho L}{A}$$

$$R = \left(\frac{\rho_2}{A}\right) L$$
slope =  $\frac{\rho_2}{A}$ 

$$\rho_2 = A(\text{slope})$$

$$\rho_2 = (5.0 \times 10^{-6} \text{ m}^2) \left(77 \frac{\Omega}{\text{m}}\right)$$

$$\rho_2 \approx 3.9 \times 10^{-4} \ \Omega \cdot \text{m}$$

 $B_{\text{tot}} = \frac{\mu_0 I}{\pi d}$ 

Ques	stion 4: Qualitative Quantitative Translation (QQT)	8 points
A	For indicating that $F_2 > F_1$	Point A
	For correctly relating the magnitude of the magnetic force to the magnitude of the magnetic field	Point A2
	For indicating that the magnitude of the magnetic field at the location of Sphere 2 is greater than the magnitude of the magnetic field at the location of Sphere 1 by referring to either:	Point A3
	<ul> <li>The relative distance from the spheres to Wires S and T</li> <li>The direction of the magnetic fields from Wires S and T at the locations of the spheres</li> </ul>	
	Example Response	
	The magnitude of the magnetic force exerted on a sphere is directly proportional to $Q$ , $v$ , and $B$ . Because $Q$ and $v$ are the same for both spheres, the difference in the magnitudes of the magnetic forces is due to the difference in the magnitudes of the magnetic fields at the locations of the spheres. The magnitude of the magnetic field is greater at the location of Sphere 2 than at the location of Sphere 1 because the fields from the wires are in the same direction. Therefore, $F_2 > F_1$ .	
В	For a multistep derivation that includes the equation $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\rm enc}$ or	Point B1
	$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I(d\vec{\ell} \times \hat{r})}{r^2}$	
	Scoring Note: Vector notation is not required for this point to be earned. A multistep	
	derivation that includes $B = \frac{\mu_0 I}{2\pi d}$ can earn this point.	
	For a correct expression for the magnitude of the magnetic field due to the current in one wire at the location of Sphere 2	Point B2
	For indicating that the magnitude of the total field is twice that due to one wire	Point B3
	Example Response	
	Determine the magnitude B of the magnetic field due to one wire. $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\rm enc}$	
	$B(2\pi d) = \mu_0 I$	
	$B = \frac{\mu_0 I}{2\pi d}$ Determine B	
	Determine $B_{\text{tot}}$ . $B_{\text{tot}} = \frac{\mu_0 I}{2\pi d} + \frac{\mu_0 I}{2\pi d}$	

C For indicating  $F_{\text{new}} = F_2$  Point C1

For indicating  $B_{\text{tot}} = \frac{3\mu_0 I}{2\pi d} - \frac{\mu_0 I}{2\pi d}$ 

Point C2

OR

For indicating that the net magnitude of the magnetic field at the location of Sphere 2 will remain the same even though the field from Wire T will change

#### **Example Response**

The magnetic field at the location of Sphere 2 due to the current in Wire T is in the opposite direction to the magnetic field due to the current in Wire S. The current in

Wire T is increased by a factor of 3. Therefore, 
$$B_{\text{tot}} = \frac{3\mu_0 I}{2\pi d} - \frac{\mu_0 I}{2\pi d} = \frac{\mu_0 I}{\pi d}$$
 in this

scenario. Thus,  $F_{\text{new}} = F_2$ .