Chief Reader Report on Student Responses:

2025 AP® Precalculus Free-Response Questions

• Number of Students Scored	254,469		
• Number of Readers	714		
• Score Distribution	Exam Score	N	%At
	5	71,426	28.1%
	4	65,578	25.8%
	3	68,335	26.9%
	2	28,696	11.3%
	1	20,434	8.0%
• Global Mean	3.55		

The following comments on the 2025 free-response questions for AP® Precalculus were written by the Chief Reader, Michael Boardman of Pacific University. They give an overview of each free-response question and of how students performed on the question, including typical student errors. General comments regarding the skills and content that students frequently have the most problems with are included. Some suggestions for improving student preparation in these areas are also provided. Teachers are encouraged to attend a College Board workshop to learn strategies for improving student performance in specific areas.

Question 1

Topic: Function Concepts

	Max Points:	Mean Score:
Point A1	1	0.64
Point A2	1	0.72
Point B1	1	0.56
Point B2	1	0.56
Point C1	1	0.73
Point C2	1	0.30

Overall Mean Score: 3.51

What were the responses to this question expected to demonstrate?

This question assesses knowledge of and skill with specific function concepts from the course framework. It involves functions that are presented in different representations—one given in a table, the other given analytically.

- To complete part A (i), one must identify from the table the output of the function *f* when 1 is the input (Skill 2.A) and use it to compute the value of the composition $g(f(1))$ (Skill 2.A, LO 2.7.A, and EK 2.7.A.2).
- In part A (ii) the response must demonstrate an understanding of inverse function notation and an ability to identify from the table of values inputs of *f* that produce the specific output 3.5 (Skill 2.A, LO 2.8.A, and EK 2.8.A.2). In this case, there was one such input: $x = 0$.
- In part B (i) one must do similar work to part A (ii) but with the analytically presented function *g*. A graphing calculator is used to solve an equation to find all domain values that produce an output of 0 (Skill 1.A, LO 1.5.A, and EK 1.5.A.3). There were three such domain values: $x = -1.233$, $x = 1.578$, and $x = 5.643$.
- Part B (ii) asks for the right end behavior of *g*, a cubic function. The response requires the use of proper limit notation in stating $\lim_{x \to \infty} g(x) = -\infty$ (Skill 3.A, LO 1.6.A, and EK 1.6.A.1).
- Part C requires (i) determining which function type, among linear, quadratic, exponential or logarithmic, best models the function *f* (Skill 1.C, LO 2.5.A, and EK 2.5.A.1) and (ii) giving reasoning for this answer that is based on the connection between the change in outputs and the change in inputs of *f* (Skill 3.C, LO 2.5.A, and EK 2.5.A.1).

How well did the responses address the course content related to this question? How well did the responses integrate the skill(s) required on this question?

- Part A (i): Most responses indicated facility with reading inputs and outputs for the function *f* from a table of values and showed understanding of composition.
- Part A (ii): Most responses showed an understanding of inverse functions and the notation used for inverse functions.
- Part B (i): Responses generally showed an understanding of using technology to solve an equation with multiple solutions.
- Part B (ii): Most responses had correct end behavior with correct limit notation.
- Part C (i): Most responses correctly identified that an exponential function would be the most appropriate model.
- Part C (ii): Some responses had a complete and correct explanation of why an exponential function best models *f*. These responses included an indication of the equal-length input-value intervals with corresponding proportional outputs with a ratio of 0.5. To earn the point, a response was required to include specific reference to the values of *f*.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• Part A (i): Presenting an answer of 0.333 without the necessary supporting work	• $g(f(1)) = g(1.75) = 0.333$ Note that some indication that $f(1) = 1.75$ was required to earn the point.
• Part A (ii): Presenting both an answer of 0 and "no solution"	• Identifying that $f^{-1}(3.5) = 0$
• Part B (i): Not presenting an answer that is accurate to three places after the decimal point, not having all three solutions, or presenting answers that are the negatives of the correct answers	• $x = -1.233$, $x = 1.578$, $x = 5.643$
• Part B (ii): Presenting incorrect limit notation	• $\lim\limits_{x\to\infty} g(x) = -\infty$
• Part C (ii): Justifying that an exponential function best models f by including information both about the equal-length input-value intervals and the corresponding proportional output values with proportion 0.5	• Input-value intervals all have length 1 and $\frac{f(-1)}{f(-2)} = \frac{f(0)}{f(-1)} = \frac{f(1)}{f(0)} = \frac{f(2)}{f(1)} = 0.5$

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve student performance on the exam?

- Students need facility with composition of functions where the functions are represented in different ways (i.e., graphically, numerically, analytically).
- Students need to understand that for an invertible function, inputs and outputs swap when moving to the inverse function.
- Students need to develop skills with using their graphing calculators to evaluate functions and to find numerical solutions to equations in one variable. This includes functions that may require students to adjust the viewing window to find all solutions. Emphasize that the part A instructions for the free-response section indicate that "any decimal approximations reported in your work should be accurate to three places after the decimal point."
- Exposure to and practice with limit notation is crucial (Topics 1.6, 1.7, 1.9, 1.10, 2.3, and 2.11), both for performance on the AP Precalculus Exam and as preparation for calculus.
- Students need to understand the relationship between the change in output values and change in input values for various families of functions, including linear, quadratic, cubic, exponential, and logarithmic functions.
- Students need practice with writing clear, unambiguous responses to questions asking for explanations or reasoning (such as in part C).

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- AP Daily videos and AP Classroom topic questions from Topics 2.7 and 2.8 can be used to reinforce concepts related to composition of functions and inverse functions. Teachers may wish to revisit these concepts prior to introducing Topic 3.9: Inverse Trigonometric Functions.
- Limit notation is found in Topics 1.6, 1.7, 1.9, 1.10, 2.3, and 2.11. Although this notation is initially challenging, students will become more comfortable with the notation by the end of Unit 2. Using AP Daily videos that emphasize limit notation can help to reinforce the concept of a limit and its notation.
- Teachers can use the question bank to assign additional questions from LOs 1.3.A, 2.5.A, and 2.10.A, which focus on the language necessary to describe relationships between input and output values for various function models.

Question 2

Topic: Modeling a Non-Periodic Context

	Max Points:	Mean Score:
Point A1	1	0.53
Point A2	1	0.48
Point B1	1	0.63
Point B2	1	0.41
Point B3	1	0.06
Point C1	1	0.05

Overall Mean Score: 2.17

What were the responses to this question expected to demonstrate?

This question assesses several skills and essential knowledge statements from the course framework in a non-periodic context. The total number of plays for the song is given at three different times: first, when the musician began using an app that counts the number of plays at $t = 0$, then at $t = 2$ months, and at $t = 4$ months. A quadratic function of the form $D(t) = at^2 + bt + c$ is used to model the total number of plays, in thousands, for the song.

- In part A (i) a response should display three equations that use the data from the question stem and that can be used to find *a*, *b*, and *c*. The equations come directly from $D(0) = 25$, $D(2) = 30$, and $D(4) = 34$.
- In part A (ii) a response should give the values of *a*, *b*, and *c*, accurate to three places after the decimal point. Both points in part A assess Skill 1.C, LO 1.14.A, and EK 1.14.A.1.
- In part B (i) a response should use the data from the question stem to demonstrate the computation of the average rate of change of the total number of plays for the song, in thousands per month, over the interval from $t = 0$ to $t = 4$ (Skill 1.B, LO 1.3.A, and EK 1.3.A.3). A decimal approximation to this average rate of change should be presented.
- In part B (ii) the average rate of change computed in part B (i) is used to estimate the total number of plays for the song, in thousands, for $t = 1.5$ months. This is a time that is in the interior of the interval $[0, 4]$ (Skill 3.B, LO 1.14.C, and EK 1.14.C.1).
- In part B (iii) a response must explain why, for time between 0 and 4 months, estimates of the total number of plays for the song using the average rate of change found in part B (i) are all less than what the model *D* predicts. A response is expected to indicate that the estimates come from the secant line on the interval and that the graph of function *D* is concave down on the interval (Skill 3.C, LO 1.14.C, and EK 1.14.C.1).
- In part C information is given that the quadratic model *D* has exactly one absolute minimum or one absolute maximum. A response must explain, in the context of the problem, how this can be used to determine a boundary for the domain (Skill 3.C, LO 1.13.B, and EK 1.13.B.3). In explaining how a right boundary of the domain can be determined, this explanation should include an indication that the total number of plays can never decrease and that after the function *D* achieves its maximum value, the function *D* decreases. If a response focuses on the left boundary, the response should include that the total number of plays is nonnegative and that the left boundary of the domain of the model will be the smallest value of *t* at which $D(t)$ changes from negative to positive. Note that at this time *t*, the model *D* has its absolute minimum value of 0.

How well did the responses address the course content related to this question? How well did the responses integrate the skill(s) required on this question?

- Part A (i): Most responses presented correct equations. Some responses incorporated the units of thousands and had correct equations with these values (i.e., 25,000, 30,000, and 34,000).
- Part A (ii): Many responses gave correct values of a, b, and c as decimal approximations accurate to three decimal places. Many responses that had a correct translation to thousands in part A (i) were able to find corresponding correct values of a, b, and c.
- Part B (i): Most responses showed the setup for the requested average rate of change and presented a correct decimal approximation for this value.
- Part B (ii): Many responses showed complete work, finding the y-coordinate of the point with t-coordinate 1.5 along the secant line from the point $(0, 25)$ to the point $(4, 34)$.
- Part B (iii): Very few responses communicated an understanding that because the graph of $y = D(t)$ is concave down on the interval $0 < t < 4$, the secant line that passes through the point $(0, D(0))$ and the point $(4, D(4))$ is below the graph of $y = D(t)$ on this interval.
- Part C: Very few responses gave a complete explanation of how the maximum value of $D(t)$ gives an upper bound of the use of D. Few responses noted that function D decreases after $t = 11$, while the context, the total number of plays of the song, is always nondecreasing.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• Part A (ii): Difficulty algebraically solving a system of linear equations. However, work was not required in this part because the system can be solved using technology without algebraic manipulation. Also, not presenting an answer in decimal form that is accurate to three places after the decimal point.	• Values of $a = -0.125$, $b = 2.75$, $c = 25$
• Part B (i): Not presenting the required computations that lead to the correct answer	• Average rate of change $= \frac{34 - 25}{4 - 0} = 2.25$
• Part B (ii): Using a line with the appropriate slope but through the point $(2, 30)$, a point not on the secant line	• $y = 25 + 2.25 \cdot 1.5 = 28.375$
• Part B (iii): Difficulty clearly explaining that A_t is a y-coordinate on the secant line through the points $(0, D(0))$ and $(4, D(4))$, that the graph of $y = D(t)$ is concave down on the interval $0 < t < 4$, and so $A_t < D(t)$ on $0 < t < 4$.	• The graph of $y = D(t)$ is concave down on $0 < t < 4$, so the secant line on this interval is below the graph of D. Therefore, A_t, the y-coordinate of a point on the secant line, is less than $D(t)$ on $0 < t < 4$.
• Part C: Difficulty identifying specific properties of the context and the model that imply the following: the location of the absolute maximum value of D provides an upper bound for D as a model for this context.	• The function D has an absolute maximum value at $t = 11$. After this time, D decreases. However, the total number of plays must always be nondecreasing. Therefore the domain of D as a model is bounded above by $t = 11$.

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve student performance on the exam?

- Students should be required to write equations from a given data set for a variety of functions. Emphasize correct use of the units in the model and represented by the data.
- Students should understand how the average rate of change of a function over an interval is the same as the slope of the secant line through the two corresponding points on the graph of the function.
- Students must have practice following the part A instructions and question-specific instructions on the free-response section of the exam. This includes presenting answers in decimal form when asked to do so.
- Precise use of mathematical notation is crucial. Students should have practice seeing correct answers that are accompanied by work that contains imprecise use of notation, such as expressions connected with "=" when the expressions are not truly equal to each other. This will help students recognize characteristics of using precise notation.
- Students should have practice writing clear, unambiguous responses to questions that involve explanations or reasoning. Teach students to be precise with their mathematical language and to model using the language in the course framework throughout the course. This practice can include exposure to a collection of responses, some that are vague, some that are logically confusing, and some that are correct. This will help students recognize characteristics of good responses.
- Students should develop skills with using technology. These include producing graphs and tables, solving equations, calculating regressions (when applicable), and performing computations. Emphasize that the part A instructions for the free-response section indicated that "any decimal approximations reported in your work should be accurate to three places after the decimal point." Students should have practice using their graphing calculator or Desmos to store/save information such as computed values for constants, functions they are using, and any intermediate values. Computations with technology that use the stored/saved information help to maintain as much precision as possible and ensure the desired accuracy in final answers.
- Students should experience how rounding intermediate computations from their graphing calculator can change the accuracy of a final answer. Emphasize to students to avoid rounding intermediate computations on the way to the final answer.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- Answering questions that require reasoning supported by mathematics can be challenging for students. To help students become familiar with well-written mathematical reasoning, teachers can create custom quizzes by using the AP Classroom question bank and selecting the multiple-choice questions that focus on Skills 3.A and 3.C. As students review the options given, ask them to describe what makes a well-written mathematical explanation.
- When using practice FRQ 2 items from AP Classroom, have students write their responses on paper for parts B (iii) and C. Then have students work with a partner to combine their responses to make a revised explanation. Finally, share the model solution and have the students compare it to their own responses.
- Use the task verbs from page 151 of the *AP Precalculus Course and Exam Description* in your daily instruction. Free-response questions commonly use these task verbs: Explain/Give a reason/Provide a rationale/Justify. Give students a response that does not have sufficient reasoning and ask them to improve the response.

Question 3

Topic: Modeling a Periodic Context

	Max Points:	Mean Score:
Point A1	1	0.77
Point A2	1	0.34
Point B1	1	0.67
Point B2	1	0.18
Point C1	1	0.67
Point C2	1	0.51

Overall Mean Score: 3.13

What were the responses to this question expected to demonstrate?

This question assesses several skills and essential knowledge statements from the course framework in the periodic context of a vibrating guitar string. A sinusoidal function h is used to model the position of a fixed point, X, on the string from its resting position. The point X is at position $+2$ at time $t = 0$, moves to position -2, and then back to position $+2$ with this motion happening 200 times in 1 second.

- In part A a generic sinusoidal graph is given, without scale or axes. Five points are labeled with letters. The response should give appropriate t-coordinates (Skill 2.B, LO 3.7.A, EK 3.7.A.1, and EK 3.7.A.3) and $h(t)$-coordinates (Skill 2.B, LO 3.7.A, and EK 3.7.A.2) for these points based on the context.
- In part B it is indicated that $h(t) = a\sin(b(t+c)) + d$. The response should present valid values of the four parameters: a, b, c, and d (Skill 1.C, LO 3.6.A, and EK 3.6.A.6). Finding these values demonstrates an understanding of amplitude, period, phase shift, and vertical shift.
- In part C (i) a response should indicate which of four choices accurately describes the behavior of the function h on an interval between two specific points from the graph in part A (Skill 2.A, LO 1.1.A, and EK 1.1.A.4). On this interval, the function h is negative and decreasing.
- In part C (ii) a response should describe the concavity of the graph of h on the same interval as in part C (i) and state whether the rate of change of h is increasing or decreasing. On this interval, the graph of h is concave up and the rate of change of h is increasing (Skill 3.A, LO 1.1.B, and EK 1.1.B.3).

How well did the responses address the course content related to this question? How well did the responses integrate the skill(s) required on this question?

- Part A: Most responses indicated an understanding of the displacement of the guitar string but fewer were able to correctly determine accurate times at which these displacements occurred.
- Part B (i): Most responses that earned the point for $h(t)$-coordinates in part A (i) went on to present correct values for the amplitude and vertical shift (values of a and d).
- Part B (ii): Few responses interpreted the period and phase shift to present correct values for b and c.
- Part C (i): Most responses identified the correct behavior of the graph of h on the specified interval.
- Part C (ii): Many responses indicated the correct concavity of the graph of $y = h(t)$ on the interval and an understanding of how the rate of change of h was changing on the specified interval.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• Part A: Difficulty identifying mathematical information from a verbal context	• Providing correct coordinates for the five points labeled on the graph
• Part B (ii): Difficulty translating period and phase shift in a context to constants in a presentation of a sinusoidal model	• Correct forms include $h(t) = -2\sin\left(400\pi\left(t - \frac{1}{800}\right)\right)$ and $h(t) = 2\sin\left(400\pi\left(t + \frac{1}{800}\right)\right)$.
• Part C (ii): Difficulty clearly expressing the concavity of the graph of h and the behavior of the rate of change of h	• The graph of $y = h(t)$ is concave up, and the rate of change of h is increasing on the interval.

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve student performance on the exam?

- Students need extensive practice with contextual scenarios throughout the AP Precalculus course.
- Students should have practice sketching and labeling points on a single cycle of a sinusoidal function based on contextual scenarios.
- Identifying amplitude, midline/vertical shift, period, and phase shift within a periodic context is important.
- Students need an understanding that the period of a sinusoidal function is the reciprocal of the frequency.
- Students need understanding of and practice with the relationship between the properties of the rate of change of a function and aspects of the graph of the function. For example, students should know that if the rate of change of a function is positive on an interval, then the function is increasing on the interval.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- Teachers now have the flexibility to use individual questions from progress checks in AP Classroom. Using some of the questions from the Unit 3 Progress Check: FRQ Part A will give students additional practice in sinusoidal modeling. Students can also gain additional practice by using multiple-choice questions in the question bank on Topic 3.7: Sinusoidal Function Context and Data Modeling.
- Students are introduced to the concepts related to function behavior in Unit 1, and it is reinforced in units 2 and 3. To have students practice Skill 3.A, specifically describing function behavior as it pertains to rates of change and concavity, search for items in the AP Classroom question bank by using keyword "concavity" and create an assignment with those items.
- Use the task verbs from page 151 of the *AP Precalculus Course and Exam Description* in your daily instruction. Free-response questions commonly use these task verbs: Explain/Give a reason/Provide a rationale/Justify. Give students a response that does not have sufficient reasoning and ask them to improve the response.

Question 4

Topic: Symbolic Manipulations

	Max Points:	Mean Score:
Point A1	1	0.53
Point A2	1	0.43
Point B1	1	0.45
Point B2	1	0.28
Point C1	1	0.14
Point C2	1	0.10

Overall Mean Score: 1.93

What were the responses to this question expected to demonstrate?

This question assesses facility with symbolic manipulation of exponential, logarithmic, trigonometric, and inverse trigonometric functions. Symbolic manipulation is an important theme in the course framework.

- In part A a logarithmic function and a trigonometric function are given analytically. Each of these functions is used in an equation that is to be solved, one in part (i) and the other in part (ii). In part A (i) a response should present the work and solution to an equation involving a logarithmic expression (Skill 1.A, LO 2.13.A, and EK 2.13.A.1). In part A (ii) a response should present the work and solution to an equation involving a trigonometric expression (Skill 1.A, LO 3.10.A, and EK 3.10.A.1).
- In part B two functions are given, function j that involves two terms with logarithm base 2 and function k that involves trigonometric expressions. A response should rewrite the expression for each function in a specified way. In part B (i) a response is to use properties of logarithms to rewrite $j(x)$ so that its expression involves only one term of the form $\log_2(\text{expression})$ (Skill 1.B, LO 2.12.A, EK 2.12.A.1, and EK 2.12.A.2). In part B (ii) a response is to use trigonometric identities to rewrite $k(x)$ so that it is an expression with $\tan x$ appearing once and no other trigonometric functions are involved (Skill 1.B, LO 3.12.A, and EK 3.12.A.2).
- In part C a function whose expression involves exponentials with base e is given: $m(x) = e^{2x} - e^x - 12$. A response, showing the work leading to the answer, is to determine all inputs to m that yield an output of 0. Both points of this part assess Skill 1.A, LO 2.13.A, and EK 2.13.A.1. A response earned the first point by writing the expression for $m(x)$ in a quadratic form with e^x. A response earned the second point with the correct answer and supporting work.

How well did the responses address the course content related to this question? How well did the responses integrate the skill(s) required on this question?

- Part A (i): Many responses included sufficient work and the correct answer. Some responses rewrote the equation using the power rule for logarithms as $\log_3 x^2 = 4$. If these responses listed only $x = 9$ (rather than the additional $x = -9$), the response still earned the point.
- Part A (ii): Many responses included sufficient work and the correct answer. In addition to the correct answer, some responses included values for x that were outside the interval $\left[0, \frac{\pi}{2}\right)$. If these values for x satisfied the equation, the response still earned the point.
- Part B (i): Many responses correctly used properties of logarithms and rewrote the expression as instructed in the question and in the FRQ 4 directions.

- Part B (ii): Some responses made use of the Pythagorean identity $\csc^2 x - 1 = \cot^2 x$ and correctly rewrote $k(x)$ as an expression in which $\tan x$ appears exactly once and no other trigonometric functions are present.
- Part C: Few responses rewrote the equation in a manner indicating a recognition that the equation was quadratic in e^x. Some responses went on to solve the equation correctly, arriving at $x = \ln 4$ and eliminating the possibility that $e^x = -3$.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• Part A (i): Misuse of function notation can indicate a misunderstanding of functions mapping inputs to outputs.	• $\log_3 x = 2$ $x = 9$
• Part A (ii): Lack of knowledge of the inverse trigonometric functions	• $\cos^2 x = \frac{3}{4}$ $\cos x = \pm\frac{\sqrt{3}}{2}$ $x = \frac{\pi}{6}$
• Part B (i): Difficulty working with algebraic properties of logarithmic expressions	• $j(x) = \log_2 x + \log_2\left(2^3\right) = \log_2(8x)$
• Part B (ii): Lack of facility with trigonometric identities and algebraic manipulation of trigonometric expressions	• $k(x) = \frac{6}{\tan x \cdot \cot^2 x} = \frac{6}{\cot x} = 6\tan x$
• Part C: No recognition of equation as quadratic in e^x and not eliminating the possibility that $e^x = -3$	• $\left(e^x\right)^2 - e^x - 12 = 0$ $\left(e^x - 4\right)\left(e^x + 3\right) = 0$ $e^x = 4$ or $e^x = -3$ $x = \ln 4$

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve student performance on the exam?

- Symbolic manipulation is an important part of the AP Precalculus course. Students need practice using the properties of exponents, properties of logarithms, definitions of trigonometric functions, and trigonometric identities to rewrite expressions in mathematically equivalent forms. Without the help of technology, students need practice solving equations with exponential functions, logarithmic functions, trigonometric functions, and inverse trigonometric functions.
- Students should practice showing *all* the steps that lead to their answer when solving an equation or when rewriting an expression.
- Students need time and much practice to understand inverse trigonometric functions.
- Teachers should share the FRQ 4 directions with students in advance. Students should practice rewriting numerical and algebraic expressions based on the requirements of these directions. The directions are included in the *AP Precalculus Course and Exam Description*, the AP Precalculus Practice Exams, and in the free-response questions on AP Central and in AP Classroom.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- The skills of procedural and symbolic fluency require extensive practice. When doing multiple-choice questions that require Skill 1.A, students may want to use a guess-and-check approach to get the answer. However, that prevents them from having additional practice that is necessary to demonstrate understanding when answering FRQ 4. Extra practice can be created by giving students a topic question from AP Classroom without the multiple-choice options.
- Use the AP Classroom video "AP Practice Session 8" to reinforce Skills 1.A and 1.B. Prior to watching the video, have students solve the problems that will be presented in the video. Discuss any other solution strategies students may have used.