Chief Reader Report on Student Responses:

2025 AP® Calculus Free-Response Questions

• Number of Readers (Calculus AB/BC)	2201		
Calculus AB			
• Number of Students Scored	286,722		
• Score Distribution	Exam Score	N	%At
	5	58,174	20.3
	4	82,970	28.9
	3	42,896	15.0
	2	65,405	22.8
	1	37,277	13.0
• Global Mean	3.21		
Calculus BC			
• Number of Students Scored	160,954		
• Score Distribution	Exam Score	N	%At
	5	70,694	43.9
	4	35,154	21.8
	3	20,610	12.8
	2	24,514	15.2
	1	9,982	6.2
• Global Mean	3.82		
Calculus AB Subscore			
Number of Students Scored	160,953		
Score Distribution	Exam Score	N	%At
	5	76,764	47.7
	4	49,126	30.5
	3	15,701	9.8
	2	14,481	9.0
	1	4,881	3.0
Global Mean	4.11		

The following comments on the 2025 free-response questions for AP® Calculus were written by the Chief Reader, Sharon Taylor, Professor of Mathematics, Georgia Southern University. They give an overview of each free-response question and of how students performed on the question, including typical student errors. General comments regarding the skills and content that students frequently have the most problems with are included. Some suggestions for improving student preparation in these areas are also provided. Teachers are encouraged to attend a College Board workshop to learn strategies for improving student performance in specific areas.

Question AB1/BC1

Task: Average Value, Instantaneous Rate of Change, Average Rate of Change, End Behavior, Extreme Value Theorem
Topic: Modeling

	Max Points:	Mean Score:
AB1 P1	1	0.36
AB1 P2	1	0.32
AB1 P3	1	0.57
AB1 P4	1	0.38
AB1 P5	1	0.64
AB1 P6	1	0.38
AB1 P7	1	0.34
AB1 P8	1	0.07
AB1 P9	1	0.19

AB1 Overall Mean Score: 3.26

	Max Points:	Mean Score:
BC1 P1	1	0.62
BC1 P2	1	0.57
BC1 P3	1	0.77
BC1 P4	1	0.62
BC1 P5	1	0.84
BC1 P6	1	0.62
BC1 P7	1	0.62
BC1 P8	1	0.18
BC1 P9	1	0.37

BC1 Overall Mean Score: 5.22

What were the responses to this question expected to demonstrate?

In the stem of the problem, students were told of an invasive species appearing in a fruit grove. Students were given a function $C(t) = 7.5\arctan(0.2t)$, where C is measured in acres and t is measured in weeks, as well as an analytical expression for $C'(t)$, the derivative of the function C.

In part A students were expected to find the average number of acres affected by the invasive species over a given time interval. A correct response presents a definite integral of $C(t)$ over the given interval and divides by the difference of the limits of integration. A graphing calculator was required. **P1** was earned for using the average value formula, while **P2** was earned for the correct numerical answer.

In part B students needed to find the average rate of change from $t = 0$ to $t = 4$ and then find the time on that interval when the derivative was equal to the average rate of change. Students needed a calculator to find the average rate of change and to solve for time. **P3** was earned for the correct average rate of change expression or value, and **P4** was earned for the correct time, supported by the appropriate equation.

Part C required students to write and evaluate a limit expression describing the end behavior of the rate of change in the number of acres affected by the species. **P5** was earned for the limit expressed symbolically, and **P6** was earned for the correct answer of 0.

In part D students were given a new function A and asked to find the time on the interval $4 \le t \le 36$ when A attains its maximum value. This required students to set the derivative of A equal to zero and then use the graphing calculator to solve for the critical point. **P7** was earned for considering where $A'(t) = 0$. **P8** was earned by providing a global argument justifying the correct conclusion. **P9** was earned by providing the time when the maximum number of acres was affected, along with supporting work. **P9** could be earned without having earned **P8**.

How well did the responses address the course content related to this question? How well did the responses integrate the skill(s) required on this question?

Part A: Responses earned **P1** by presenting an average value expression; although some presented an incorrect integral expression involving the function $C'(t)$. Many responses earned **P2** by using their calculator to evaluate the correct integral and divide that value by 4.

Part B: Most responses earned **P3** for presenting the average rate of change expression or the value for that expression. Somewhat fewer responses earned **P4** for presenting the equation $C'(t) = 1.282008$ and solving that equation for t.

Part C: Most responses earned **P5** for representing the verbal description involving a limit in symbolic form. **P6** was not earned for the answer to an incorrect limit expression. Of those who wrote the correct limit expression, however, most earned **P6** with the correct value of 0.

Part D: Some responses earned **P7** by presenting $A'(t) = 0$ or $C'(t) - 0.1\ln(t) = 0$ or by saying that the function $A'(t)$ changed signs at the critical point. **P8** was earned for a correct justification to support the conclusion that the global maximum occurs at $t = 11.4417$, with many attempting to use the candidates test. However, the number of responses earning **P8** was low, either because of errors in an attempt at a candidates test or because of stopping short of a global argument using other approaches. **P9** was earned by making the correct conclusion about the time when the largest value of A was attained, provided supporting work was also given.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Part A: The two most common misconceptions were using $C'(t)$ instead of $C(t)$ as the integrand and finding average rate of change instead of average value.

Part B: The most common error was presenting the equation $C'(t) = \dfrac{C(4) - C(0)}{4 - 0}$ but not understanding how to solve that equation.

Part C: The most common misconception was setting up the limit so that it approached 0 instead of infinity.

Part D: The most common error was in **P8** for justification. Many responses evaluated $A(4)$, $A(11.442)$, or $A(36)$ incorrectly.

<table>
<tr><th>Common Misconceptions/Knowledge Gaps</th><th>Responses that Demonstrate Understanding</th></tr>
<tr><td>Part A

• Using $\frac{1}{4}\int_0^4 C'(t)\,dt$ or $\frac{C(4)-C(0)}{4-0}$ to find the average value of C over the four-week interval.</td><td>• $\frac{1}{4}\int_0^4 C(t)\,dt$</td></tr>
<tr><td>Part B

• Many students interpreted the question to mean $C'(t) = \frac{C(4)-C(0)}{4-0}$ and then did not know how to solve that equation.</td><td>• $\frac{C(4)-C(0)}{4-0} = 1.282$

Solve $C'(t) = 1.282$ using a graphing calculator to find $t = 2.154$.</td></tr>
<tr><td>Part C

• Some responses incorrectly represented the end behavior of C' as $\lim_{t\to 0} C'(t)$.</td><td>• $\lim_{t\to\infty} C'(t)$</td></tr>
<tr><td>Part D

• Some responses attempting to evaluate $A(4)$, $A(11.4417)$, and $A(36)$ as part of a justification using the candidates test did not earn P8 because they did not correctly determine all of these values. Some made apparent errors in calculator entry for one or more of these values. Others used the wrong function, evaluating $A'(4)$, $A'(11.4417)$, and $A'(36)$, for example.

• Many responses attempting to justify a conclusion using methods other than the candidates test presented local arguments that did not adequately explain the behavior of $A(t)$ over the entire interval. For example, “because the sign of $A'(t)$ changes from positive to negative at $t = 11.442$.”</td><td>• Candidates test:

<table><tr><th>t</th><th>$A(t)$</th></tr><tr><td>4</td><td>5.128</td></tr><tr><td>11.442</td><td>7.317</td></tr><tr><td>36</td><td>1.743</td></tr></table>
Therefore, $A(t)$ attains its maximum value at $t = 11.442$.

• $A'(t) > 0$ when $4 < t < 11.443$ and $A'(t) < 0$ when $11.442 < t < 36$, which means $t = 11.442$ is the location of an absolute maximum of A on $4 \le t \le 36$.

OR

Because the sign of $A'(t)$ changes from positive to negative at $t = 11.442$ and $t = 11.442$ is the only critical point of $A(t)$ on $4 < t < 36$, $A(t)$ attains its maximum value at $t = 11.442$.</td></tr>
</table>

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve student performance on the exam?

- Student responses offered evidence that teachers are doing a great job guiding students in the use of calculators. However, some responses would have benefited from a better understanding of which functions should be used for derivatives, integrals, and evaluations. For example, to find the average value of C over an interval of time in part A, the integrand should be $C(t)$. Help students to understand that $\frac{1}{4}\int_0^4 C'(t)\,dt$ is not the correct setup to answer this question, but that it does answer a different question: $\frac{1}{4}\int_0^4 C'(t)\,dt = \frac{1}{4}(C(4) - C(0))$, which is the average rate of change of C over the time interval $0 \le t \le 4$. That is, $\frac{1}{4}\int_0^4 C'(t)\,dt$ gives the average value for $C'(t)$ over the time interval $0 \le t \le 4$. Encouraging students to take the time to think about the meaning of their work helps them to develop genuine understanding.
- Calculus offers many opportunities for just-in-time review of precalculus topics. For example, emphasizing that end behavior of a function is describing what happens as the independent variable approaches positive or negative infinity is an opportunity to help students to make connections with past learning about horizontal asymptotes.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

Reasoning and justification involving differentiation were important skills in part D of this question. Unit 5 Student Practice assignments are excellent resources to help students to develop these skills. Found on the AP Classroom Unit 5 Course Guide and available to students without teacher assignment, Unit 5 Student Practice provides students with step-by-step tips and video explanations to guide their work on reasoning and justification involving differentiation.

- To develop emerging understanding, students can work through the Level 1 assignment, which includes instruction and practice on finding and reasoning about critical points, intervals on which a function is increasing or decreasing, relative extrema, and intervals on which the graph of a function is concave up or down, along with writing justifications using the Mean Value Theorem.
- Grounded with these foundational understandings and skills, a student progresses to the Level 2 assignment, which applies calculus concepts in context and includes justification for an absolute (or global) maximum. This Level 2 assignment would have been very helpful to a student preparing for the AP Exam in 2025.
- The Level 3 assignment is appropriate for students as they pull many concepts together to develop a complete and integrated understanding of the differentiation topics in the course.

Similarly, Unit 8 Student Practice: Level 1 provides excellent scaffolded preparation for other parts of this question.

Question AB2

Task: Area, Volume, Slope of Tangent Lines
Topic: Area/Volume

	Max Points:	Mean Score:
AB2 P1	1	0.75
AB2 P2	1	0.66
AB2 P3	1	0.44
AB2 P4	1	0.26
AB2 P5	1	0.44
AB2 P6	1	0.35
AB2 P7	1	0.33
AB2 P8	1	0.45
AB2 P9	1	0.35

AB2 Overall Mean Score: 4.06

What were the responses to this question expected to demonstrate?

A graph of two functions *f* and *g* and their points of intersection was provided, along with analytical representations for both functions. The region bounded by *f* and *g* was shaded and labeled *R*.

Part A required students to compute the area of the shaded region. **P1** was awarded for the correct form of the integrand, which was $\int_0^3 (g(x) - f(x))\,dx$. Also required to earn **P1**, the integrand had to be shown in a definite integral. **P2** was earned for the correct answer, with or without supporting work. Although the answer could be found without a calculator, most students used one.

Part B stated that the region *R* was the base of a solid where cross sections perpendicular to the *x*-axis were rectangles with height *x* and base in region *R*. **P3** was awarded for a definite integral with an integrand presented as a product of two nonconstant factors with one of the factors equal to *x*, $g - f$, or $f - g$. **P4** was earned for the correct answer.

Part C required students to write, but not evaluate, an integral expression for the volume of the solid generated when the region *R* was rotated about the horizontal line $y = -2$. The expected response for **P5** was to present the form of an integrand that consisted of the outer radius squared minus the inner radius squared. A response that earned **P5** often did not earn **P6** for the correct integrand due to parentheses, arithmetic, or algebra errors. The limits of integration, constant π, and the differential dx were assessed in **P7**, provided **P5** had been earned.

Part D provided the derivative of $g(x)$ as $g'(x) = 1 + \pi\cos(\pi x)$ and asked students to find the value of *x* for which the line tangent to the graph of *f* was parallel to the line tangent to the graph of *g*. To earn **P8**, a response had to show $f'(x) = g'(x)$. **P9** was earned by a correct answer, with or without supporting work.

How well did the responses address the course content related to this question? How well did the responses integrate the skill(s) required on this question?

Part A: Responses that earned **P1** with a definite integral and correct integrand also tended to earn **P2** for the correct answer.

Part B: Earning **P3** and **P4** in this part proved challenging. There were errors involving parentheses as well as indefinite integrals. Students who earned **P3** and **P4** usually did so with a minimal number of steps. The more a response tried to simplify the problem, the more likely there would be an error.

Part C: Most responses indicated a conceptual understanding of how to use the washer method for this problem. Many responses earned **P5** for presenting an eligible attempt at the difference of the squared radii. **P6** was more difficult to earn because of errors in representing the outer and inner radii, often because of incorrect attempts at simplification. A response that earned **P5** was eligible to earn **P7** for the correct limits of integration, constant, and differential and many students were successful in earning **P7**.

Part D: For **P8**, there was a basic understanding that parallel lines have the same slope, and many responses indicated $f'(x) = g'(x)$ to earn **P8**. **P9** required the use of technology to solve the equation, and many responses were successful in solving $f'(x) = g'(x)$.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Part A: Some responses presented an integrand of $(g - f)^2$ instead of $(g - f)$, suggesting misconceptions about construction of an appropriate integral to find an area or volume. Another common error involved missing parentheses in the integrand. This usually happened when a response substituted the given function for *f* and *g* rather than leaving the integrand in *f* and *g* notation. Such a response did not earn **P1** but could earn **P2** for presenting the correct answer from the calculator. Unfortunately, some answers obtained using technology were not rounded appropriately and did not earn **P2**.

Part B: Many responses presented $(g - f)^2$, indicating a square cross section rather than a rectangular cross section. This integrand earned **P3** for the form of the integrand but would not lead to the correct answer, as required to earn **P4**. Another incorrect solution was using an integrand of $x(g - f)^2$ or another form of three factors in the integrand. Such a response did not earn **P3** or **P4**. Responses that used an indefinite integral rather than a definite integral did not earn **P3** but could have earned **P4** with the correct answer. **P4** could be earned with or without supporting work.

Part C: Many responses revolved the region about the line $y = 0$. While some incorrect constants were allowed, zero was not one of those and the response did not earn **P5**, **P6**, or **P7**. Some responses were on the right track and used $\int_0^3 \left(R^2 - r^2\right) dx$ but did not go further to specify *R* or *r*. Those responses did not earn **P5**, **P6**, or **P7**.

Part D: Some responses presented second derivatives equal to 0 or only $g' = 0$.

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
Part A • $\int_0^3 (g - f)^2\, dx$ The response used an integrand to find volume instead of area. • $\int_0^3 \left((x + \sin(\pi x)) - x^2 - 2x\right) dx$ When a response substituted $x + \sin(\pi x)$ for *g* and $x^2 - 2x$ for *f*, there was a tendency to lose parentheses. In such cases, the response did not earn **P1** but could earn **P2** with the correct answer.	• $\int_0^3 (g - f)\, dx$ The correct response finds area by subtracting the lower function from the upper function. • $\int_0^3 \left((x + \sin(\pi x)) - \left(x^2 - 2x\right)\right) dx$ Parentheses need to clearly separate the two functions as well as set off the integrand from the differential.

• A presented answer of 5.1 or 5.13 did not earn **P2** due to the decimal presentation error.	• 5.137 (or 5.316) The presented decimal must be accurate to three places after the decimal, rounded or truncated.
Part B • $\int_0^3 (g-f)^2\,dx$ This response treated the solid as a square instead of a rectangle. • $\int_0^3 x(g-f)^2\,dx$ Responses such as this indicate three factors were needed as the integrand instead of only two. • $\int x(g-f)\,dx$ Although the integrand is correct, it was not presented as part of a definite integral and did not earn **P3**.	• $\int_0^3 x(g-f)\,dx$ The correct response indicates the two sides of a rectangular cross section are $g-f$ and x. • $\int_0^3 x(g-f)\,dx$ Finding the area of a rectangular cross section only requires two factors. • $\int_0^3 x(g-f)\,dx$
Part C • A response that presented $\pi\int_0^3 \left(g^2-f^2\right)dx$ is revolving region R around $y=0$ instead of $y=-2$. • A response of $\pi\int_0^3 \left(R^2-r^2\right)dx$ identifies an appropriate formula but must go on to correctly define R and r, typically by writing a complete integrand, to have earned **P5**, **P6**, and **P7**.	• $\pi\int_0^3 \left((g(x)+2)^2-(f(x)+2)^2\right)dx$ • $\pi\int_0^3 \left((g(x)+2)^2-(f(x)+2)^2\right)dx$
Part D • A response of $g'(x)=0$ or $f''(x)=g''(x)$ did not earn **P8**.	• $f'(x)=g'(x)$

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve student performance on the exam?

- When a question is presented with named functions, students should use those function names in their responses. The chances of making an error are much higher when a response uses the function expression instead of the given function name. Emphasize to students the likelihood of making an error if not using f and g (or some other function name). For example: Use $\int_0^3 (g(x)-f(x))\,dx$, instead of attempting to use $\int_0^3 \left((x+\sin(\pi x))-\left(x^2-2x\right)\right)dx$.

- Students need help distinguishing among questions that ask about volumes.
  - Questions that use the given region as a base of a solid with known cross sections require using some type of area formula—in this problem, a rectangle. Students should also be familiar with squares, triangles, and semicircles.
  - Questions that ask about revolving a region around an axis or line often require an integrand that involves outer radius squared minus inner radius squared. However, only presenting a formula does not usually earn any points. The outer and inner radii must be explicitly defined in the context of the problem.
  - When revolving a region around an axis or line, students should multiply the integrand (or integral) by π, use the correct limits of integration, and include the differential.
- Once an appropriate equation is presented (e.g., $f'(x) = g'(x)$), students must use technology carefully and correctly to solve the equation in a calculator-active question.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

Found on the AP Classroom Unit 8 Course Guide and available to students without teacher assignment, Unit 8 Student Practice provides students with step-by-step tips and video explanations to help them to understand the distinctions among the many variations on area and volume problems found in Topics 8.4–8.12. While Unit 8 Practice: Level 1 focuses on other applications of integration, Levels 2 and 3 fully develop student proficiency with categorizing area/volume questions and help them to identify and skillfully apply appropriate solution strategies. Unit 8 Practice: Level 2 develops a strong foundation for success and Unit 8 Practice: Level 3 develops the deeper understanding we would hope to see by the end of the year.

Question AB3/BC3

Task: Average Rate of Change, Intermediate Value Theorem, Trapezoidal Sum, Definite Integral
Topic: Modeling (Tabular Stem)

	Max Points:	Mean Score:
AB3 P1	1	0.80
AB3 P2	1	0.54
AB3 P3	1	0.27
AB3 P4	1	0.28
AB3 P5	1	0.56
AB3 P6	1	0.52
AB3 P7	1	0.71
AB3 P8	1	0.61
AB3 P9	1	0.54

AB3 Overall Mean Score: 4.82

	Max Points:	Mean Score:
BC3 P1	1	0.88
BC3 P2	1	0.66
BC3 P3	1	0.38
BC3 P4	1	0.43
BC3 P5	1	0.73
BC3 P6	1	0.68
BC3 P7	1	0.90
BC3 P8	1	0.84
BC3 P9	1	0.77

BC3 Overall Mean Score: 6.27

What were the responses to this question expected to demonstrate?

The stem of the problem presented data in tabular form. The data represented the reading rate of a student as $R(t)$ at various times over a 10-minute interval.

In part A students were asked to approximate $R'(1)$ using the average rate of change of R over the time interval $0 \le t \le 2$. **P1** was earned for the correct answer with setup, meaning a response needed to show a difference quotient to earn the point. **P2** was awarded for correct units for the average rate of change, which are words per minute per minute. A response could earn **P2** without having earned **P1**.

Part B asked students to justify whether there was a value c in the interval $0 < c < 10$, such that $R(c) = 155$. Responses needed to state that because R is differentiable, it is continuous. That statement earned **P3**. To earn **P4**, a response had to answer "yes," because $R(0) < 155$ (or $R(2) < 155$ or $R(8) < 155$), $R(10) > 155$, and R is continuous.

In part C students were asked to approximate the value of $\int_0^{10} R(t)\,dt$ using a trapezoidal sum. **P5** was earned by presenting the form of a trapezoidal sum, and **P6** was earned for the correct answer with supporting work.

Part D presented a new function describing a teacher reading over the time interval $0 \le t \le 10$. Based on that model, students were asked how many words were read over that time interval and to show the work leading to their answer. **P7** was earned for a correct integrand, **P8** for the correct antiderivative, and **P9** for the numerical answer, provided **P8** had been earned.

How well did the responses address the course content related to this question? How well did the responses integrate the skill(s) required on this question?

Part A: Most responses were able to earn **P1** by finding the average rate of change of R over the time interval $0 \le t \le 2$ and showing the supporting work of a difference quotient. Most responses that simplified the difference quotient did so correctly to get an approximation of 5. Although many responses did not provide units of measure at all, most of those that did gave correct units of measure and earned **P2**.

Part B: Some responses earned **P3** by stating that R is differentiable and therefore continuous. Some earned **P4** by answering "yes" in some way, because R is continuous and 155 is between either $R(0)$, $R(2)$, or $R(8)$ and $R(10)$. Of the 9 points in this question, **P3** and **P4** were earned by the smallest proportion of responses.

Part C: Many responses were able to earn **P5** by providing an expression for the form of the trapezoidal sum, although there were responses that used an incorrect formula for the area of a trapezoid. Most responses were then able to go on and provide a correct answer to earn **P6**.

Part D: Most responses were able to earn **P7** by providing $\int_0^{10} W(t)\, dt$. Most responses that earned **P7** were also able to earn **P8** with a correct antiderivative. Nearly all responses that earned **P8** also earned **P9** for the correct numerical answer.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Part A: Some responses used the average rate of change in a tangent line approximation and did not earn **P1**. However, these responses were eligible for **P2**. Other than responses that provided no answer at all for units, the main difficulty was responses that provided units for $R(t)$ instead of $R'(t)$.

Part B: Some responses simply stated that R was continuous without justification and did not earn **P3**. The most common error in earning **P4** was not addressing the continuity of R.

Part C: A few responses wrote a sum of three terms but did not provide sufficient correct work to earn **P5**. Responses that had only one error in the sum earned **P5** but were not eligible for **P6**.

Part D: Some responses did not present an integral expression or an antiderivative of $W(t)$. A few responses attempted to evaluate $W(10)$ as the amount read at the end of the 10-minute interval.

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
Part A • Some responses correctly found an average rate of change of $\frac{100-90}{2-0}$ but went on to incorrectly attempt to calculate a tangent line approximation, presenting the statement $R'(1) \approx 90+5=95$. Such responses did not earn **P1**.	• $R'(1) \approx \frac{R(2)-R(0)}{2-0} = \frac{100-90}{2}$
• A few responses stated that $R'(1) \approx 5$ without the required supporting work and did not earn **P1**.	
• Many responses indicated that the units for $R'(t)$ were words per minute. • Some responses did not provide any units of measure.	• 5 words per minute per minute OR 5 words/minute/minute
Part B • Many responses simply stated that R is continuous and differentiable without sufficient justification.	• R is continuous because R is differentiable. OR R is differentiable; therefore R is continuous. OR R is differentiable implies R is continuous.
• Many responses did not show that the hypotheses for the Intermediate Value Theorem were met or tried to apply the wrong theorem: ○ Some responses listed values that were presented in the given table but did not clearly indicate that $R(c)=155$ is between the listed values. ○ Some responses did not state that R is continuous. ○ A few responses incorrectly attempted to apply the Mean Value Theorem in their justification.	• Because R is continuous and $R(0)=90<155<R(10)=162$, the Intermediate Value Theorem guarantees that there must be a value c, $0<c<10$, such that $R(c)=155$.

Part C • Some responses wrote an expression using an incorrect formula for the area of a trapezoid: $\frac{1}{2}[(100-90)\cdot 2+(150-100)\cdot 6+(162-150)\cdot 2]$.	• $\frac{1}{2}[(100+90)\cdot 2+(150+100)\cdot 6+(162+150)\cdot 2]$
• A few responses did not show sufficient evidence of the trapezoidal form when presenting their approximation: $95(2)+125(6)+156(2)$. This response is a special case that earned **P6** without having earned **P5**.	• $\frac{190}{2}(2)+\frac{250}{2}(6)+\frac{312}{2}(2)$ earned **P5** and was sufficient to earn **P6**.
• Some responses presented an expression for a trapezoidal sum with a single copy error: $\frac{(90+100)(2)}{2}+\frac{(100+150)(2)}{2}+\frac{(150+162)(2)}{2}$, which earned **P5** but not **P6**.	• $\frac{(90+100)(2)}{2}+\frac{(100+150)(6)}{2}+\frac{(150+162)(2)}{2}$ earned **P5** and was sufficient to earn **P6**.
Part D • Some responses used inappropriate strategies to answer the question. ○ Some evaluated the function W without first antidifferentiating: $W(10)=-\frac{3}{10}(10)^2+8(10)+100=150$. ○ Some approximated the number of pages read using a constant reading rate over the 10-minute interval: $150(10)=1500$ words.	• The number of pages read by the end of the first ten minutes is given by $\int_0^{10} W(t)\,dt$.
• Some responses made an error when presenting the coefficients for their antiderivative. For example: ○ $\int_0^{10} W(t)\,dt=-\frac{1}{3}t^3+4t^2+100t$ ○ $\int_0^{10} W(t)\,dt=-\frac{1}{10}t^3+16t^2+100t$	• $\int_0^{10}\left(-\frac{3}{10}t^2+8t+100\right)dt$ $=\left(-\frac{1}{10}t^3+4t^2+100t\right)\Big\|_0^{10}$
• A few responses attempted to apply the FTC without finding an antiderivative: $-\frac{3}{10}(10)^2+8(10)+100-\left(-\frac{3}{10}(0)^2+8(0)+100\right)$	• $\left(-\frac{1}{10}(10)^3+4(10)^2+100(10)\right)$ $-\left(-\frac{1}{10}(0)^3+4(0)^2+100(0)\right)$

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve student performance on the exam?

- In part A some responses did not recognize that the average rate of change of R is the approximation of $R'(1)$. These responses used the average rate of change found in part A and applied this value to a tangent line approximation for $R(1)$.
  - Help students to make connections among derivatives, approximations of a derivative, and the average rate of change of a given function. Additionally, help to develop math fluency by reinforcing and using proper vocabulary in the classroom consistently during the year.
  - Provide students with ample opportunities to work with contextual problems that involve units of measure. It is recommended to spend class time specifically aimed at helping students understand how units of measure are affected when differentiating (and later when integrating) a given function.
- Spend time working with theorems, specifically the existence theorems: the Mean Value Theorem (MVT), Intermediate Value Theorem (IVT), and Extreme Value Theorem (EVT). Provide students with opportunities to connect hypotheses and conclusions when attempting to apply theorems. Many students attempted to apply the conclusion of the IVT without first explicitly checking and/or stating the required conditions.
- In part D many students were unable to connect accumulation to the integral of a given rate function. Students need instruction and practice that explicitly connects these two concepts, especially in problems involving a real-world context.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

In part B of this question, fewer than half of students earned **P3** for establishing continuity based on differentiability and a similar proportion earned **P4** for applying the Intermediate Value Theorem. Students need resources to help them develop proficiency with establishing continuity and applying the Intermediate Value Theorem and other existence theorems.

In Unit 5 on AP Classroom, as part of the "Unit 5 More Resources," under the "More" tab, there are several lesson plans with student handouts focused on writing mathematical justifications. Most relevant to part B of this question is the lesson for Topic 5.2: Establishing Continuity for EVT and IVT. Taken together with the lesson for Topic 5.1: Establishing Differentiability for MVT, students can learn to develop understanding of which theorem to apply in a particular situation and the skills they need to write good justifications using existence theorems.

Question AB4/BC4

Task: Fundamental Theorem of Calculus, Points of Inflection, Definite Integrals, Extreme Value Theorem
Topic: Graphical Analysis

	Max Points:	Mean Score:
AB4 P1	1	0.62
AB4 P2	1	0.70
AB4 P3	1	0.27
AB4 P4	1	0.18
AB4 P5	1	0.65
AB4 P6	1	0.48
AB4 P7	1	0.39
AB4 P8	1	0.15
AB4 P9	1	0.47

AB4 Overall Mean Score: 3.91

	Max Points:	Mean Score:
BC4 P1	1	0.79
BC4 P2	1	0.88
BC4 P3	1	0.38
BC4 P4	1	0.24
BC4 P5	1	0.87
BC4 P6	1	0.71
BC4 P7	1	0.62
BC4 P8	1	0.29
BC4 P9	1	0.69

BC4 Overall Mean Score: 5.46

What were the responses to this question expected to demonstrate?

The stem of the question presented a graph of the continuous function *f*, which consisted of two semicircles and a line segment on the interval $-6 \le x \le 12$. The function *g* was defined by $g(x) = \int_6^x f(t)\,dt$.

In part A students were asked to find $g'(8)$. Responses were expected to apply the Fundamental Theorem of Calculus to conclude that $g'(x) = f(x)$, which earned **P1**. To earn **P2**, a response needed to identify $f(8) = 1$ from the graph and report $g'(8) = 1$.

Part B asked students to find all values of *x* on the open interval $-6 < x < 12$ at which the graph of *g* has a point of inflection and to give a reason for their answer. A response earned **P3** by identifying $x = -3$, $x = 3$, and $x = 6$ as the locations of the points of inflection in the graph of *g* on the given interval (and no other values). To earn **P4**, the response had to provide a reason for this answer that follows from behaviors in the graph of *f* at those three values of *x*.

In part C the question asked for two values, $g(12)$ and $g(0)$. **P5** was awarded for the correct value of $g(12) = 9$, while **P6** was earned for presenting $g(0) = -\frac{9\pi}{2}$. These points could be earned with or without supporting work.

Part D required using the Extreme Value Theorem to find the value of x where an absolute minimum value of g occurs on the given interval. **P7** was earned for considering $g'(x) = 0$. A response earned **P8** for presenting a correct global justification of where the absolute minimum occurs. **P9** was earned by drawing the correct conclusion that the absolute minimum value occurred at $x = 0$.

How well did the responses address the course content related to this question? How well did the responses integrate the skill(s) required on this question?

Part A: Many responses earned both **P1** and **P2** by stating $g'(8) = f(8) = 1$.

Part B: Less than half of responses earned **P3** for identifying the three values of x. Less than a quarter of responses earned **P4** for providing a reason for their answer.

Part C: **P5** and **P6** could be earned with or without supporting work. Most responses earned **P5** and slightly fewer earned **P6**. To earn **P6**, a response had to correctly handle the reversal in the limits of integration to find that $g(0) = -\frac{9\pi}{2}$. Although showing calculations was not necessary, some responses showed their work to find the area of the triangle over the interval $6 \le x \le 12$ or the area of the semicircle on the interval $0 \le x \le 6$.

Part D: Many responses earned **P7** for considering $g'(x) = 0$. Most responses chose to use the candidates test as the justification to earn **P8**. Some were successful and earned **P9** by presenting the correct answer of $x = 0$.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
Part A • Some responses attempted to apply the Fundamental Theorem of Calculus but incorrectly handled the limits of integration, stating that $g'(x) = f(x) - f(6)$. This was the most common error seen for responses that attempted to answer this part. • Some responses stated that $g'(x) = f(x)\,dx$ or $f(t)\,dt$. If they went on to successfully find $g'(8)$, this presentation error was considered to be resolved and the response earned **P2**. If not, the response did not earn **P2**. • Some responses found $g'(8) = 1$ incorrectly, by finding the area between f and the x-axis from $x = 6$ to $x = 8$. This is the value of $g(8)$, and happens to also be equal to 1. **P2** was not awarded if it was clear that this was how the value was determined.	• $g'(x) = f(x)$ $g'(8) = f(8) = 1$

• Some responses found the slope of *f* at $x = 8$ (interpreting the given graph as *g,* not *f*) and reported $g'(8) = \frac{1}{2}$.	
Part B • Many responses included $x = 0$ as a point of inflection (either by itself, or with other values). These responses were not eligible to earn either **P3** or **P4**. • Many responses presented only two of the three *x*-values. Most often, these were $x = 3$ and $x = -3$, omitting $x = 6$ from consideration. This did not earn **P3** but was eligible to earn **P4**. • Many responses tied the reasoning to the behavior of g'' or f' and did not earn **P4**. • Most responses that presented only two of the three *x*-values were unable to provide the correct reasoning tied to the graph of *f*. • Some of the responses that attempted to tie the reasoning to the graph of *f* stated only that these were locations where $f'(x) = 0$, which did not earn **P4**.	• The graph of *g* has a point of inflection at $x = -3$, $x = 3$, and $x = 6$. • $x = -3$, $x = 3$, and $x = 6$ (or at least two of these three and no others) because *f* changes from increasing to decreasing, or vice versa at these points. OR … because the slope of *f* changes from positive to negative, or vice versa, at these points. OR … because *f* attains relative extrema at these points.

Part C
• Some responses reported the value for $f(12)$ (which was 3) as $g(12)$, interpreting the graph of *f* as *g*.
• Several misconceptions were evident in responses for $g(0)$:
  ○ Reporting the value for $f(0)$ (which was 0) as $g(0)$
  ○ Reporting the value of $g(0)$ as -9π, neglecting to divide by 2
  ○ Demonstrating an understanding of accumulation, but presenting $g(0) = \frac{9\pi}{2}$, rather than $-\frac{9\pi}{2}$
	• $g(12) = \frac{1}{2}(6)(3) = 9$
	• The area of the relevant semicircle is $\frac{1}{2}\pi(3)^2$.
	$g(0) = -\frac{1}{2}\pi(3)^2 = -\frac{9\pi}{2}$

<table>
<tr><td colspan="2">Part D</td></tr>
<tr><td>

- Many responses identified $x = 0$ and $x = 6$ either in a table or a list but never justified why those were candidates.

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- $g'(x) = f(x) = 0$ when $x = 0$ and $x = 6$.

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- Most responses that mentioned “critical values” or “critical points” did not identify that these were values or points for g. Some responses attempted to present a geometric argument based on areas, but were unable to clearly articulate how these areas represented values for g.

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- $x = 0$ and $x = 6$ are critical points of g.

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- Many responses presented incomplete candidates tests, either leaving out consideration of $x = 6$ or $x = 12$. If one of these was left out of the table, the response needed to present a clear argument as to why they cannot be locations of the absolute minimum, which most responses in this situation failed to do.

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- Absolute minima occur where $g'(x) = 0$ and is changing from negative to positive values, that is at $x = 0$, or at endpoints of the interval.

<table>
<tr><th>x</th><th>$g(x)$</th></tr>
<tr><td>-6</td><td>0</td></tr>
<tr><td>0</td><td>$-\frac{9\pi}{2}$</td></tr>
<tr><td>12</td><td>9</td></tr>
</table>

</td></tr>
<tr><td>

- Many responses presented only a local argument.
- For responses that did not earn **P9**, the most common error was giving the location of the absolute minimum for f (which was $x = -3$) rather than the location of the absolute minimum for g.
- Most responses that correctly identified $x = 0$ provided some support for this statement and were able to earn **P9**.
- Some responses attempted to identify an absolute maximum instead of minimum.

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- Alternate justification and answer that provide a global argument: Because $g'(x) \le 0$ for $-6 \le x < 0$ and $g'(x) \ge 0$ for $0 < x \le 12$, the absolute minimum of g occurs at $x = 0$.

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</table>

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve student performance on the exam?

- Remind students that in graphical interpretation problems, their reasoning (in general) must be tied to the given graph.
- Some students continue to provide vague or ambiguous reasoning (e.g., “the graph” or “the function” or “the slope”), which in most cases will not earn the point. Provide many opportunities in the classroom to reinforce the importance of using precise language to explain and justify work.

- Many students drew number line sign charts to help with their analysis for relative minimum and points of inflection. These charts can be helpful for students to organize their thinking but do not by themselves provide justification. Continue to remind students that they must interpret and explain the information that the sign chart is providing to them.
- To determine locations of absolute extrema, the candidates test is often the most straightforward way to justify the result. Many students that attempted to justify without using the candidates test were unsuccessful.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

To apply their analytical skills to justify conclusions about accumulation functions, students require a strong understanding of both how to justify conclusions about the behavior of functions based on the behaviors of their derivatives and how to evaluate and graph accumulation functions.

- Under "More Resources" in Topic 6.5 on AP Classroom, there is a lesson and student handout to help students to develop foundational understandings and skills. The student handout begins with the basics, such as determining values of $g(x) = \int_{-1}^{x} f(t)\,dt$ based on the graph of $y = f(t)$ and exploring the graph of $y = g(x)$.
- Unit 6 Practice: Level 3 also provides instruction and practice with these important concepts and skills.

Almost every AP Calculus Exam in recent years includes questions similar to AB/BC question 4 in the 2025 exam. As students' understanding and skills advance, these past AP Exam questions offer many opportunities for students to perfect their understanding and skills. Peer-feedback activities using published scoring guidelines, available on AP Classroom, is helpful to students who are giving and receiving feedback.

Question AB5

Task: Velocity, Direction of Motion, Speed of a Particle Increasing or Decreasing at a Point, Position
Topic: Particle Motion

	Max Points:	Mean Score:
AB5 P1	1	0.77
AB5 P2	1	0.55
AB5 P3	1	0.42
AB5 P4	1	0.04
AB5 P5	1	0.03
AB5 P6	1	0.43
AB5 P7	1	0.69
AB5 P8	1	0.44
AB5 P9	1	0.23

AB5 Overall Mean Score: 3.60

What were the responses to this question expected to demonstrate?

The stem of the problem presented information about two particles, *H* and *J*, moving along the *x*-axis. The position of particle *H* at time *t* was given by $x_H(t) = e^{t^2-4t}$, and the velocity of particle *J* at time *t* was given by $v_J(t) = 2t\left(t^2-1\right)^3$.

In part A students were asked to find the velocity of particle *H* at time $t = 1$. A response earned **P1** by considering the derivative of the position function for particle *H*, $x_H'(t)$. **P2** was earned for the correct value $x_H'(1) = -2e^{-3}$.

Part B asked students to find intervals of time, for $0 < t < 5$, when the particles *H* and *J* were moving in opposite directions. A response could earn **P3** by considering the sign of $x_H'(t)$ or $v_J(t)$. **P4** could be earned by the analysis of signs of velocity or direction of motion on the interval $0 < t < 5$ for one of the particles. To earn **P5**, a response had to provide such analysis for both particles and determine when the particles traveled in the opposite directions, which was over the interval $1 < t < 2$.

In part C the question provided the information that $v_J'(2) > 0$ and asked if the speed of particle *J* was increasing, decreasing, or neither at $t = 2$. To earn **P6**, the response needed to state that $v_J(2) > 0$ and draw the correct conclusion that the speed of particle *J* was increasing.

In part D the position of particle *J* at time $t = 0$ was given as $x = 7$. Students were asked to find the position of particle *J* at time $t = 2$ and to show the work that leads to their answer. **P7** was earned with a correct integrand of $v_J(t)$, and **P8** was earned for the correct antiderivative of $v_J(t)$, which was $\frac{1}{4}\left(t^2-1\right)^4$. **P9** was earned for correctly incorporating the initial condition with the value of the correct definite integral and giving the correct numerical answer of 27.

How well did the responses address the course content related to this question? How well did the responses integrate the skill(s) required on this question?

Part A: **P1** was earned by most students for considering $x_H{}'(t)$ in any of the following forms: $x_H{}'(t)$, $x_H{}'(1)$, $x'(t)$, $x'(1)$, $(2t-4)e^{t^2-4t}$, or $(2\cdot 1-4)e^{1^2-4\cdot 1}$. Most responses that earned **P1** went on to earn **P2** with the correct answer of $-2e^{-3}$.

Part B: Many responses earned **P3** because a response only had to consider the sign of the velocity of one of the two particles. However, very few responses earned **P4** and were therefore ineligible to earn **P5**.

Part C: Somewhat less than half of responses earned **P6** by presenting $v_J(2) > 0$ and concluding that the speed of the particle was therefore increasing.

Part D: Most responses earned **P7** with the correct integrand. Fewer responses earned **P8** for the antiderivative, owing to errors in trying to expand the original integrand. When a response earned **P8**, it was likely to earn **P9** for the correct answer of 27, which included the initial condition.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Part A: Responses that did not earn **P1** or **P2** usually had difficulty with parentheses or finding the derivative of $x_H{}'(t)$.

Part B: The most common reason responses did not earn **P4** was incomplete analysis or communication. Many responses presented a number line analysis that did not clearly communicate the sign of the velocity or the direction of motion for a particle over the entire interval $0 < t < 5$.

Part C: The most common way to not earn **P6** was to find an incorrect value for $v_J(2)$ or to make an incorrect attempt to find a value or sign for $v_J{}'(2)$, even though that information was given in this part.

Part D: While most responses earned **P7** with an integrand of $v_J(t)$ or $2t\left(t^2-1\right)^3$, difficulties in earning **P8** and **P9** were encountered when using *u*-substitution, expanding the given expression for $v_J(t)$ before finding the antiderivative, or expanding the expression for the antiderivative before substituting limits of integration.

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
Part A • Many responses renamed $x_H{}'(t)$ (as $H'(t)$, for example) without defining the new notation. • Many responses attempted to differentiate e^u as $u\cdot e^u$ or $u\cdot e^u\cdot du$. • Many responses did not use parentheses when writing the product of a binomial and an exponential expression, as in $2t-4\cdot e^{t^2-4t}$.	• $x_H{}'(t) = (2t-4)e^{t^2-4t}$

Part B • Responses evaluated one or both velocity equations with all integers on the given interval instead of solving the equation $v(t) = 0$. If a zero of the velocity function were not an integer, the zero would not have been found. • Many responses attempted to analyze the signs of velocity in a number-line chart but failed to communicate what they learned about the signs over the interval $0 < t < 5$.	• $v_H(t)$ is negative from $t = 0$ to $t = 2$ and positive from $t = 2$ to $t = 5$. $v_J(t)$ is negative from $t = 0$ to $t = 1$ and positive from $t = 1$ to $t = 5$. On $1 < t < 2$, the velocities of the two particles are opposite in sign. Therefore, *H* and *J* are moving in opposite directions on $1 < t < 2$.
Part C • Some responses only addressed the sign of $v_J'(2)$ and did not demonstrate an understanding that the determination of whether speed increases or decreases depends on the signs of both acceleration and velocity. • Responses found values for $v_J(2)$ and $v_J'(2)$, although doing so was not necessary. The sign of $v_J'(2)$ was given, and work in part B could substantiate that $v_J(2) > 0$. Those who attempted to calculate $v_J'(2)$ often struggled with applying the product rule correctly.	• $v_J(2) > 0$ and $v_J'(2) > 0$. Because the signs of velocity and acceleration at time $t = 2$ are the same, the particle is speeding up. • $v_J'(t) = (2t) \cdot 3\left(t^2 - 1\right)^2 \cdot 2t + \left(t^2 - 1\right)^3 \cdot 2$ $v_J'(2) = (4) \cdot 3(4-1)^2 \cdot 4 + (4-1)^3 \cdot 2 = 486 > 0$ $v_J(2) = 2 \cdot 2\left(2^2 - 1\right)^3 = 108$
Part D • Some responses did not use *u*-substitution to integrate $\int v_J(t)\,dt$, attempting instead to expand to a polynomial expression, which many responses were unable to do without error. • Some responses used *u*-substitution, but the bounds of integration were not changed to the corresponding values of *u*. • Responses demonstrated poor notation, such as writing an integral symbol in front of the antiderivative, not writing the differential when writing an integral expression, and writing an equal sign between unequal quantities.	• $\int_0^2 v_J(t)\,dt = \int_0^2 2t\left(t^2 - 1\right)^3 dt = \int_{-1}^3 u^3\,du$, for $u = t^2 - 1$. $\int_{-1}^3 u^3\,du = \left.\frac{u^4}{4}\right\|_{-1}^3 = \frac{3^4}{4} - \frac{(-1)^4}{4} = 20$ $7 + 20 = 27$

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve student performance on the exam?

- Although **P1** was designed to be an entry point, there were many responses that presented the derivative without parentheses. This poor notation often led to an incorrect answer, such as $2 - 4e^{-3}$, which did not earn **P2**. Encourage students to develop strong communication and notation skills.
- Every point in part B was associated with skillful analysis of the signs of a function over an interval of time and clear communication of reasoning. Whether presenting an algebraic argument leading to conclusions about the signs of velocities or organizing findings in a tabular representation, it is imperative that the entire interval given in the prompt be addressed in such analysis. There were many responses that did not clearly communicate their analysis for the entire interval $0 < t < 5$ and thus were unable to earn **P4** and **P5**. Peer-feedback activities might be helpful to students as they develop these reasoning and communication skills.
- Help students to understand that they need to provide more than a formulaic answer like "When the signs of velocity and acceleration are the same, the particle is speeding up." They need to explicitly apply this idea to a particular problem. In addition, remind students to avoid unnecessary simplification and calculations that need not be performed. An evaluation of $v_J(2)$ or $v_J'(2)$ was not required, for example, but if presented, it needed to be correct.
- The differential dt (or du) was not required to earn **P7**, but not including a differential could lead to not earning **P7** or **P8** because of mishandling of the constant 7. It is always important to stress proper notation in student work.
- For many integrals, the only available integration technique is u-substitution, so help students to recognize opportunities to use u-substitution, particularly with the inclusion of $2t\ dt$ in the integrand, as in part D. If the integrand in part D had had a much larger exponent on $\left(t^2 - 1\right)$, for example, very few students who chose to expand the binomial and distribute the $2t$ would have been successful.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

To help students to develop proficiency with communicating reasoning about the relationship between the sign of velocity on an interval of time and the direction of motion over that interval of time, encourage them to watch AP Video 1 for Topic 4.2 and to pay particular attention to what they need to say to communicate reasoning from a sign chart in Example 2 part b. Note that the presenter answers that the particle is moving to the left on the interval $(1, 4)$ because $v(t) < 0$ on that interval.

Question AB6

Task: Verify First Derivative, Tangent Line Approximation, Vertical Tangent Line, Related Rates
Topic: Implicit Differentiation

	Max Points:	Mean Score:
AB6 P1	1	0.70
AB6 P2	1	0.67
AB6 P3	1	0.53
AB6 P4	1	0.43
AB6 P5	1	0.47
AB6 P6	1	0.36
AB6 P7	1	0.42
AB6 P8	1	0.29
AB6 P9	1	0.23

AB6 Overall Mean Score: 4.12

What were the responses to this question expected to demonstrate?

The stem of the problem presents a curve that is defined implicitly.

Part A asked students to verify the first derivative of the given curve. **P1** was earned for the implicit differentiation, and **P2** was earned for verifying the given expression for $\frac{dy}{dx}$.

In part B students were given the x-coordinate of a point P on the curve and asked to use the line tangent to the curve at the point $(2, -1)$ to approximate the y-coordinate of P. **P3** was awarded for finding the correct slope of the line. **P4** was earned with the correct tangent line approximation value.

In part C students were told a point existed on the curve where the tangent line was vertical and asked to find the y-coordinate of that point. To earn **P5**, students had to set the denominator of the expression for $\frac{dy}{dx}$ from part A equal to zero. The solution to this equation resulted in two possible answers and students were required to choose the correct answer of $y = 1$.

In part D students were given another curve, also defined implicitly, that describes the motion of a particle along the curve. The question went on to give a point and a value for $\frac{dx}{dt}$ and ask students to find $\frac{dy}{dt}$ at the instant when the particle was at the given point. **P7** required an eligible attempt at implicit differentiation with respect to t, while **P8** required completely correct implicit differentiation. **P9** was earned for the correct numerical value for $\frac{dy}{dt}$ at that instant.

How well did the responses address the course content related to this question? How well did the responses integrate the skill(s) required on this question?

Part A: Most responses entered the problem with a successful attempt at implicit differentiation, and most were able to go on to earn **P2** with algebraic verification of the given expression.

Part B: Most responses attempted to evaluate $\frac{dy}{dx}$ at $(2, -1)$, with more than half earning **P3** for reporting the correct value of the slope of $-\frac{1}{4}$. Errors made in the evaluation are discussed in the next section. A response that did not earn **P3** could earn **P4** using an incorrect slope and the point $(2, -1)$ for their tangent line approximation.

Part C: Most responses earned **P5** by correctly setting the denominator of $\frac{dy}{dx}$ equal to 0. Because both *x* and *y* had to be positive, many responses also earned **P6** by eliminating the negative *y*-value to find $y = 1$.

Part D: Many responses earned **P7** because the point could be awarded for an implicit derivative with respect to *t* with at most one error. Fewer responses earned **P8** for the completely correct derivative. A response had to have earned **P7** and **P8** to earn **P9** and most responses that earned **P7** and **P8** went on to earn **P9**.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Part A: Some responses struggled with notation for the implicit function while others forgot the "$= 0$" on the right side of the equation. In these instances, most responses were unable to recover from the errors and did not earn **P1** or **P2**.

Part B: While most responses set up the equation of the tangent line in point-slope form, many had multiple sign and parentheses errors when trying to isolate *y*, even though isolating *y* was not required.

Part C: Some responses set both the numerator and denominator equal to zero and correctly solved for *y*, but a response could earn both points without showing that the numerator is not also equal to 0 when $y = 1$.

Part D: Responses could earn **P7** and **P8** using notation such as x', y', dx, and dy, provided it was clear that they were differentiating with respect to *t*. The most common reason for not earning **P9** was a sign error.

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
Part A • Using inappropriate notation, such as using dy in place of $\frac{dy}{dx}$ or $\frac{dy}{dx}$ in place of $\frac{d}{dx}$ • Dropping the "$= 0$" that should appear on the right side of the equation after implicit differentiation • Omitting appropriate parenthesis, as in $3y^2 - 2y - 1 \cdot \frac{dy}{dx} = -\frac{1}{2}x$ • Using separation of variables instead of implicit differentiation	• $\frac{d}{dx}\left(y^3 - y^2 - y + \frac{1}{4}x^2\right) = \frac{d}{dx}(0)$ $3y^2 \cdot \frac{dy}{dx} - 2y \cdot \frac{dy}{dx} - \frac{dy}{dx} + \frac{x}{2} = 0$ $\left(3y^2 - 2y - 1\right)\frac{dy}{dx} = -\frac{x}{2}$ $\frac{dy}{dx} = -\frac{x}{2\left(3y^2 - 2y - 1\right)}$

Part B
• Various errors in arithmetic and algebra:
  ○ Sign and parentheses errors, such as squaring -1 and getting -1, losing the negative sign in the numerator
  ○ Errors in evaluation using the order of operations
  ○ Errors in attempts to simplify the fraction even with appropriate signs and parentheses
  ○ Errors in attempting to solve for y
	• $\left.\frac{dy}{dx}\right\|_{(x,\,y)=(2,\,-1)} = \frac{-2}{2\left(3(-1)^2 - 2(-1) - 1\right)}$
	$= \frac{-2}{2(3+2-1)} = -\frac{1}{4}$
	• $y = -1 - \frac{1}{4}(1.6 - 2)$
	Note that when using the form of the equation of a line $y = y_0 + m(x - x_0)$ for tangent line approximations, there is no need to simplify.

Part C • Setting $\frac{dy}{dx} = 0$ or the numerator of $\frac{dy}{dx}$ equal to 0 to find coordinates of a point on a curve where the curve has a vertical tangent line • Copy errors from the given derivative • Algebra errors, such as dropping the "$= 0$" while solving the equation, or difficulty solving a straightforward quadratic equation by factoring • Not clearly stating that $y = 1$ is the answer	• The curve G has a vertical tangent line when the denominator of $\frac{dy}{dx}$ is equal to 0. $2\left(3y^2 - 2y - 1\right) = 0 \Rightarrow 2(3y+1)(y-1) = 0$ Because $y > 0$, $y = 1$. Note: Because the numerator of $\frac{dy}{dx}$ is $-x$ and $x > 0$, the numerator of $\frac{dy}{dx}$ cannot also equal 0 when $y = 1$.

Part D
• Differentiating with respect to x and never connecting that with t
• Inappropriate notation, such as dx, dy, or x', sometimes mixed together with $\frac{dy}{dt}$ or $\frac{dx}{dt}$
• Single errors that earned **P7** but not **P8**:
  ○ Product rule error: Writing the derivative of $2xy$ as $2\frac{dx}{dt}\frac{dy}{dt}$
  ○ Chain rule error: Writing $\frac{d}{dt}(\ln y)$ as $\frac{1}{y}$ or $\frac{1}{dy/dt}$
  ○ Forgetting to differentiate the 8
• Sign errors and simple arithmetic errors
• Simplification errors on the final fraction
	• $\frac{d}{dt}(2xy + \ln y) = \frac{d}{dt}(8)$
	$\Rightarrow 2\frac{dx}{dt}y + 2x\frac{dy}{dt} + \frac{1}{y}\frac{dy}{dt} = 0$
	$\Rightarrow 2(3)(1) + 2(4)\frac{dy}{dt} + \frac{1}{1}\frac{dy}{dt} = 0$
	$\Rightarrow 6 + 9\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = -\frac{2}{3}$

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve student performance on the exam?

- Teach students to pay attention to using appropriate notation for the derivative. Provide students with feedback so that they know when they're using notation inappropriately and can develop greater proficiency with that skill.
- Provide extra practice for students who are prone to drop the " $= 0$ " part of an equation when differentiating a constant on one side of the equation or solving algebraically.
- Ensure that students understand that a vertical line has a slope that is undefined. If a derivative is given as a fraction, the slope will be undefined when the denominator equals zero (and the numerator does not).
- Practice algebra skills like factoring quadratics with a leading coefficient other than 1. If a student is going to use a strategy such as the "slide and divide" method, which creates a quadratic that is temporarily not equivalent to the original quadratic, make sure they understand why it works and its limitations.
- Continue practicing the product rule when completing implicit differentiation.
- Emphasize distinctions between derivatives taken with respect to x and those taken with respect to t.
- Remind students that they do not need to simplify numerical expressions for full credit on the AP Exam. Even though the AP Exam does not require simplification, practicing arithmetic and algebraic skills needed for simplification is still important. Simplification of numerical values can make questions easier to solve and understand.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

Every AP Calculus topic in AP Classroom has AP Videos and topic questions. For formative assessment and practice, assign the 12 topic questions that go along with Topic 5.12: Exploring Behaviors of Implicit Relations. Students appreciate the opportunity to practice and get quick feedback, complete with rationales for why the right answer is right and what misconceptions might have led to incorrect options. Teachers value the insights they can gain into their students' areas of strength and areas where they might need additional instruction and practice.

Question BC2

Task: Rate of Change, Area of a Region Bounded by Polar Curves, Maximum Distance from the y-axis, Rate of Change with Respect to Time
Topic: Polar curves

	Max Points:	Mean Score:
BC2 P1	1	0.63
BC2 P2	1	0.60
BC2 P3	1	0.52
BC2 P4	1	0.16
BC2 P5	1	0.49
BC2 P6	1	0.06
BC2 P7	1	0.17
BC2 P8	1	0.24
BC2 P9	1	0.21

BC2 Overall Mean Score: 3.09

What were the responses to this question expected to demonstrate?

The stem of the problem presented a curve defined by a polar equation, $r(\theta) = 2\sin^2\theta$ for $0 \le \theta \le \pi$, as well as the polar equation for a semicircle, $r = \frac{1}{2}$. A graph was provided including both curves.

In part A students were asked to find the rate of change of r with respect to θ when $\theta = 1.3$. **P1** was awarded for a correct answer and setup.

Part B asked students to find the area of the region inside the curve and outside the semicircle. **P2** was awarded for presenting an integrand that included $(r(\theta))^2$. **P3** was earned for the correct integrand of $(r(\theta))^2 - \left(\frac{1}{2}\right)^2$ in a definite integral. **P4** was awarded for the correct answer, with or without supporting work.

In part C students were given an expression for $\frac{dx}{d\theta}$ for the curve and asked to find the value of θ that corresponds to the point on the curve that is farthest from the y-axis. **P5** was earned by considering $\frac{dx}{d\theta} = 0$. **P6** was earned by presenting a justification for the answer, and **P7** was earned for the correct value of θ with supporting work.

In part D students were told a particle was traveling along the curve so that $\frac{d\theta}{dt} = 15$ and asked to find the rate at which the particle's distance from the origin changes with respect to time when the particle is at the point where $\theta = 1.3$. A response earned **P8** for a presentation of $\frac{dr}{d\theta} \cdot \frac{d\theta}{dt}$, either symbolically or numerically. **P9** was earned for the correct answer, with or without supporting work. A response could earn **P8** and **P9** by importing an incorrect value of $\frac{dr}{d\theta}$ from part A and multiplying by 15.

How well did the responses address the course content related to this question? How well did the responses integrate the skill(s) required on this question?

Part A: Most responses earned **P1** for a correct answer with setup. When **P1** was not earned, it was usually because the setup was insufficient.

Part B: Most responses earned **P2** for an integrand involving $(r(\theta))^2$. **P3** was more difficult to earn, but most responses that earned **P2** also earned **P3**. Relatively few responses earned **P4** for the correct answer, which required correctly handling the constant factor of $\frac{1}{2}$, finding correct limits of integration, and evaluation of the definite integral.

Part C: Most responses earned **P5** by considering $\frac{dx}{d\theta} = 0$. The point could have also been earned without specific reference to $\frac{dx}{d\theta} = 0$ with language such as "the sign of $\frac{dx}{d\theta}$ is changing" or "critical point." The most common response for **P6** involved using the candidates test to present values of $x(\theta)$ for $\theta = 0.955317$ and the endpoints of the interval, and a few responses were able to earn **P6**. Common errors resulting in not earning **P6** are discussed in the next section. Most responses that earned **P6** also earned **P7** for the correct answer.

Part D: About a quarter of responses earned **P8** with a correct product of derivatives, and slightly fewer than that earned **P9** for the correct (or consistent) answer.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Part A: The most common reason a response did not earn **P1** was the lack of providing a setup. Some responses only showed an answer. Some responses with less than perfect communication earned **P1**, nonetheless.

Part B: The most common error seen on responses was failing to square $r(\theta)$, which resulted in not earning **P2**. Responses that did not earn **P2** typically did not earn **P3** or **P4**, so it is difficult to determine if there were difficulties with the constant and limits of integration.

Part C: Many responses presented incorrect, incomplete, or insufficient global arguments (which can be seen in the following section) and did not earn **P6**. There were responses that did not earn **P7** because the correct answer was presented with insufficient supporting work.

Part D: Some responses did not earn **P8** because of multiplying 15 by the wrong derivative such as $\frac{dx}{dt}$, $\frac{dr}{dt}$, or $\frac{d\theta}{dr}$. Very few responses that earned **P8** failed to earn **P9**. This was usually due to decimal presentation errors.

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
Part A • Incorrect derivatives: ○ $r'(\theta) = 4\sin\theta$ ○ $r'(\theta) = 4\cos\theta$ ○ $r'(\theta) = 4\cos^2\theta$ ○ $r'(\theta) = 4\sin\theta\cos^2\theta$ • Decimal presentation errors	• $\left.\frac{dr}{d\theta}\right\rvert_{\theta=1.3} = 1.031$ Note that it is not necessary to provide an expression for $\frac{dr}{d\theta}$ in this calculator-active question.
Part B • Solving trigonometric equations incorrectly, such as presenting $\frac{\pi}{6}$ and $\frac{7\pi}{6}$ OR $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ as solutions to the equation $\sin\theta = \frac{1}{2}$ • Copy errors and forgotten squares in the integrand	• To find θ_1 and θ_2, the radian measures corresponding to the intersection points of curve C and the semicircle, use the calculator to solve the equation $2\sin^2\theta = \frac{1}{2}$ or solve algebraically: $\theta_1 = 0.523599$ (or $\frac{\pi}{6}$), and $\theta_2 = 2.617994$ (or $\frac{5\pi}{6}$). • $\frac{1}{2}\int_{\theta_1}^{\theta_2}\left((r(\theta))^2 - \left(\frac{1}{2}\right)^2\right)d\theta$
Part C • Providing a local argument or an insufficient or incorrect global argument: ○ $\frac{dx}{d\theta}$ changes from positive to negative at $\theta = 0.955$ (a local argument) ○ $r(0) = 0$, $r(0.955) = 1.333$, and $r\left(\frac{\pi}{2}\right) = 2$ (an attempt at a candidates test with errors)	• $\frac{dx}{d\theta} > 0$ for $0 < \theta < 0.955$ and $\frac{dx}{d\theta} < 0$ for $0.955 < \theta < \frac{\pi}{2}$ (a sufficient global argument) • $r(0) = 0$, $r(0.955) = 0.7698$, and $r\left(\frac{\pi}{2}\right) = 0$ (a correct candidates test)
Part D • Working with inappropriate or incorrect derivatives: ○ $\frac{dr}{dt} = \frac{dr}{dx}\cdot\frac{dx}{dt}$ ○ $\frac{dr}{dt} = \frac{dr/d\theta}{d\theta/dt}$	• $\frac{dr}{dt} = \frac{dr}{d\theta}\cdot\frac{d\theta}{dt}$

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve student performance on the exam?

- Student performance on this question offered evidence of limited understanding of features of polar curves introduced in precalculus courses. For example, in part B, students tended to earn points associated with polar area and then missed **P4**, often because they were unable to identify the correct limits of integration. In part C many either did not know that $x(\theta) = r(\theta)\cos\theta$ or were unsuccessful in applying that knowledge. It is a good idea to presume that students require just-in-time review of content and skills you might think they would have mastered in earlier coursework.
- Even though this is a calculator-active question, responses need to show setups to earn points in parts A, B, and D and to write a justification in part C. Students need to practice writing appropriate setups for answers found using a calculator and justifications for calculus-based conclusions.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

In addition to AP Videos for each topic on AP Classroom, there are sets of AP Exam Review videos, which can be found on the "Review" link after the Practice Exams link on the bottom of the Overview section.

- Practice Session 3: FRQ (Calculator Active) from the 2024 AP Exam On-Demand Review works through precalculus and calculus concepts and skills associated with a polar curve. This video explores in detail how to determine intervals of θ needed to determine areas of various regions defined by a polar curve, as well as whether points on the polar curve are getting closer to or farther away based on the relative signs of $r(\theta)$ and $\frac{dr}{d\theta}$.
- Practice Session 2: FRQ (Calculator Active) from the 2023 AP Exam On-Demand Review is a comprehensive review of a variety of topics associated with polar curves and an excellent preparation in advance of the AP Exam.
  - Part A begins with the formula for polar area and goes on to provide useful tips for finding the limits of integration needed and for using a calculator to evaluate a definite integral to find area (e.g., use rectangular and not polar mode).
  - Part B of this FRQ models how to justify an absolute maximum distance from the origin. Recall that r gives the distance from the origin, so this optimization problem begins by finding critical points for $r(\theta)$.
  - Part C asks for the average distance from the curve to the y-axis. The video reminds us that $|x|$ gives the distance from the curve to the y-axis and that average distance is the average value of $|x|$. To complete the setup for this solution, we need to recall that $x = r(\theta)\cos\theta$, so $|x| = |r(\theta)\cos\theta|$.
  - Part D introduces a particle moving along the path of the polar curve and asks for an interpretation of an integral expression.

Question BC5

Task: Second Derivative, Taylor Polynomial, Lagrange Error Bound, Euler's Method
Topic: Differential Equations

	Max Points:	Mean Score:
BC5 P1	1	0.73
BC5 P2	1	0.75
BC5 P3	1	0.63
BC5 P4	1	0.62
BC5 P5	1	0.55
BC5 P6	1	0.48
BC5 P7	1	0.28
BC5 P8	1	0.72
BC5 P9	1	0.44

BC5 Overall Mean Score: 5.21

What were the responses to this question expected to demonstrate?

The stem of the question presented the differential equation $\frac{dy}{dx} = (3 - x)y^2$ and defined $y = f(x)$ as the particular solution to the differential equation such that $f(1) = -1$.

Part A asked for the value of $\frac{d^2y}{dx^2}$ at the point $(1, -1)$. Responses earned **P1** for using the product rule and **P2** for using the chain rule. **P3** was earned with a correct numerical value.

In part B students were asked to write a second-degree Taylor polynomial for *f* about $x = 1$. A response earned **P4** with two terms and **P5** for the remaining term.

Part C asked students to use the Lagrange error bound to show that an approximation for $f(1.1)$ found using the Taylor polynomial from part B differs from $f(1.1)$ by at most 0.01. **P6** was earned for the form of the error bound and **P7** was earned for the correct analysis.

In part D students were asked to approximate $f(1.4)$ using Euler's method with two steps of equal size, starting at $x = 1$. **P8** was earned for the first step of Euler's method. Responses earned **P9** for a correct answer with supporting work.

How well did the responses address the course content related to this question? How well did the responses integrate the skill(s) required on this question?

Part A: Many responses were able to correctly apply the product rule and the chain rule. Responses that earned **P1** were generally able to earn **P2**. Earning **P2** was needed to be eligible to earn **P3**, and many responses successfully earned **P3** for a correct answer.

Part B: Most responses earned **P4** by writing at least two correct Taylor polynomial terms. No supporting work was required to earn this point. Most responses that earned **P4** also earned **P5** for the remaining term of the Taylor polynomial.

Part C: About half of responses presented the correct value of the error bound and earned **P6**. Subsequent errors in simplification resulted in some not earning **P7**, as described in the next section. A little over a quarter of students earned **P7** for the correct analysis.

Part D: Most of the responses earned **P8** by using the correct initial condition value, correct derivative value, correct step size, and composing an equation or expression that would lead to a correct computation of $f(1.2)$. If subsequent errors were made in arithmetic, the response was not eligible to earn **P9**. Just under half of responses earned **P9** with the correct approximation.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Part A: About a quarter of responses made errors in the product rule or chain rule and did not earn either **P1** or **P2**, depending on the error. For responses that did earn **P1** and **P2**, the most common reasons for not earning **P3** were arithmetic and algebra errors.

Part B: A common reason for not earning **P4** was centering the polynomial at $x = 0$ rather than at $x = 1$. A few responses indicated their polynomial was an infinite series by including "$+ \cdots$." In this case, **P5** was not earned.

Part C: Some of the responses presented the correct form of the error bound, but many had difficulty with correctly demonstrating that they were using the maximum value of the third derivative. In the presence of the correct value of the error bound, **P6** was earned. Many responses were unable to use correct language and inequalities in their analysis. In particular, the error is not equal to 0.01, nor is the error bound strictly less than 0.01.

Part D: Some responses incorrectly simplified the expression for the first step of Euler's method. This did not impact the scoring for **P8** but made the response ineligible for **P9**.

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
Part A	
• Incorrect product rule as the product of the derivatives of the two terms: $\frac{d^2y}{dx^2} = (-1) \cdot 2y\frac{dy}{dx}$	• Correct use of the product rule (with correct use of chain rule): $\frac{d^2y}{dx^2} = -y^2 + (3-x)2y\frac{dy}{dx}$ (earned **P1** and **P2**)
• Using the distributive property to find $\frac{dy}{dx} = 3y^2 - xy^2$ and then making a sign error in applying the product rule to the second term: $\frac{d^2y}{dx^2} = 6y\frac{dy}{dx} - y^2 + x \cdot 2y \cdot \frac{dy}{dx}$	• $\frac{d^2y}{dx^2} = 6y\frac{dy}{dx} - \left(y^2 + x \cdot 2y \cdot \frac{dy}{dx}\right)$ $= 6y\frac{dy}{dx} - y^2 - x \cdot 2y \cdot \frac{dy}{dx}$
• Missing or incorrect application of the chain rule: ○ $(-1) \cdot y^2 + (3-x) \cdot 2y$ ○ $(-1) \cdot y^2\frac{dy}{dx} + (3-x) \cdot 2y\frac{dy}{dx}$ ○ $6y - y^2 - x \cdot 2y \cdot \frac{dy}{dx}$	• $(-1) \cdot y^2 + (3-x) \cdot 2y \cdot \frac{dy}{dx}$ OR $6y \cdot \frac{dy}{dx} - y^2 - x \cdot 2y \cdot \frac{dy}{dx}$

Part B • Incorrect presentation of a polynomial centered at $x = 0$, so only one Taylor polynomial term was correct: $P_2(x) = -1 + 2x - \frac{9}{2}x$	• $P_2(x) = -1 + 2(x-1) - \frac{9}{2}(x-1)^2$
• Incorrect presentation of an infinite polynomial: $P_2(x) = -1 + 2(x-1) - \frac{9}{2}(x-1)^2 + \cdots$	• $P_2(x) = -1 + 2(x-1) - \frac{9}{2}(x-1)^2$
Part C • Incomplete or incorrect form of error bound: ○ $\frac{f'''(c)(1.1-1)^3}{3!}$ ○ $\frac{60(1.1-1)^3}{4!}$	• $\frac{\max_{1 \le x \le 1.1} \lvert f'''(x) \rvert (1.1-1)^3}{3!}$ OR $\frac{60(1.1-1)^3}{3!}$
• Incorrect statements: ○ Error $= \frac{60(1.1-1)^3}{3!} = 0.01$ ○ Error $< \frac{60(1.1-1)^3}{3!} = 0.01$ ○ $\lvert f(1.1) - P_2(1.1) \rvert = \frac{60(1.1-1)^3}{3!} = 0.01$ ○ Error Bound $< \frac{60(1.1-1)^3}{3!} = 0.01$	• Error $\le \frac{60(1.1-1)^3}{3!} = 0.01$ OR $\lvert f(1.1) - P_2(1.1) \rvert \le \frac{60(1.1-1)^3}{3!} = 0.01$ OR Error Bound $= \frac{60(1.1-1)^3}{3!} = 0.01$
Part D • Errors in first step of Euler's method: ○ Incorrect form for Euler's method step: $f(1.2) \approx -1 - (0.2)(2)$ ○ Incorrect step size for Euler's method: $f(1.2) \approx -1 + (0.4)(2)$	• $f(1.2) \approx -1 + (0.2)(2)$
• Errors in second step of Euler's method: ○ Incorrect slope calculated for second step: $f(1.4) \approx -0.6 + (0.2)(3-1.4)(-0.6)^2$ ○ Incorrect step size for second step: $f(1.4) \approx -0.6 + (1.4-1)(1.8)(-0.6)^2$	• $f(1.4) \approx -0.6 + (0.2)(3-1.2)(-0.6)^2$

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve student performance on the exam?

This question required students to apply a variety of calculus topics: implicit differentiation (Topic 3.2), calculating higher-order derivatives (Topic 3.6), representing a function at a point as a Taylor polynomial (Topic 10.11), using the Lagrange error bound (Topic 10.12), and approximating solutions to an initial value problem using Euler's method (Topic 7.5).

- Continue to practice implicit differentiation as the year progresses. As students' proficiency advances, they will be more comfortable working with second-order derivatives and using Leibniz notation for second derivatives. With yearlong practice, students will be less likely to make common errors, such as the sign error that often occurs with a subtracted product.
- Help students to develop understanding of error bounds by asking them to compare and contrast formulas for the alternating series error bound (Topic 10.10) and the Lagrange error bound (Topic 10.12).
- Focus on clear communication all year, helping students to organize their steps and labels in Euler's method, for example, or to choose the most meaningful, correct way to communicate about an error bound.
- Emphasize that students do not have to simplify answers. This simplification often leads to mistakes and may result in not earning a point.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

The *AP Calculus AB and BC Course and Exam Description* (CED) includes a section on Instructional Approaches that can be used or adapted as part of instructional planning. For example, to develop application and communication skills across the curriculum, design error analysis activities using information found on page 206 of the CED.

- At the beginning of the second day on Topic 3.2: Implicit Differentiation, provide students with solutions to various implicit differentiation problems containing common errors, such as the sign error with a subtracted product, and ask them to troubleshoot. As students' skills advance over time, this activity can evolve into peer feedback and ultimately troubleshooting their own work on the exam.
- Calculating higher-order derivatives (Topic 3.6) can be suitable for students with emerging, proficient, or advanced skills. Error analysis activities can help students to engage with slightly higher levels of questions than they might be able to do on their own at that time. For example, a student who might not be quite ready to evaluate a second derivative at a point in an implicit relation might be able to develop understanding of the prerequisite concepts and skills in an error analysis activity.
- Error analysis activities with Euler's method can be used to introduce students to organizational strategies that they might want to adopt and help them to understand everything that is necessary for a complete and correct application of the method.
- Finally, an error analysis activity with incorrect statements about error bounds and the task of offering corrections for the errors will help students to understand why some of the rather complicated expressions we often see in these topics are incorrect and how to make them right.

Question BC6

Task: Ratio Test, Interval of Convergence, Terms for a Series, Convergence at a Point
Topic: Series

	Max Points:	Mean Score:
BC6 P1	1	0.77
BC6 P2	1	0.55
BC6 P3	1	0.60
BC6 P4	1	0.44
BC6 P5	1	0.14
BC6 P6	1	0.57
BC6 P7	1	0.51
BC6 P8	1	0.30
BC6 P9	1	0.38

BC6 Overall Mean Score: 4.32

What were the responses to this question expected to demonstrate?

The stem of the problem gives a Taylor series for a function f about $x = 4$, which converges to $f(x)$ on the interval of convergence of the Taylor series.

In part A responses were expected to use the ratio test to find the interval of convergence of the Taylor series about $x = 4$ and to justify their answer. **P1** and **P2** were earned by setting up the ratio and finding the limit of the ratio as n approaches infinity. A response earned **P3** with the correct interior of the interval of convergence. To earn **P4**, a response was expected to consider whether the series converges at both endpoints. **P5** was earned for the analysis at the endpoints and the correct interval of convergence.

Part B expected responses to find the first three nonzero terms of the Taylor series for f', which earned **P6**, and the general term of that series, which earned **P7**.

In part C responses were expected to use the fact that the series for f' found in part B was geometric to verify that $f'(x) = \frac{x-4}{7-x}$ for all x in the interval of convergence for f', which earned **P8**.

In part D responses were expected to use the radius of convergence of the Taylor series for f' about $x = 4$, which is the same as the radius of convergence for f about $x = 4$, to determine whether the Taylor series for f' described in part B converges at $x = 8$ to the expression given in part C. **P9** was awarded for the correct answer with reason.

How well did the responses address the course content related to this question? How well did the responses integrate the skill(s) required on this question?

Part A: Most responses set up a correct ratio, with or without the absolute value, and earned **P1**. Most responses also used correct limit notation and evaluated the limit of the ratio as n approaches infinity to earn **P2** and also correctly identified the interior of the interval of convergence to be the open interval $(1, 7)$, earning **P3**. Many responses that earned **P3** also earned **P4** for considering the endpoints of the interval, but some responses stopped after earning **P3**. Few responses earned **P5**, which required correct analysis and conclusions at both endpoints.

Part B: Most responses earned **P6** by correctly finding the first three nonzero terms of the Taylor series for f' about $x = 4$. As noted in the next section, the most common reason for not earning **P6** was incorrect simplification of the fractional coefficient of the third term. About half of responses earned **P7** with the correct general term of the series.

Part C: A response such as $\dfrac{\frac{x-4}{3}}{1-\left(\frac{x-4}{3}\right)}$, which correctly applies understanding of the behavior of a geometric series, was sufficient to earn **P8**. Even so, slightly less than a third of responses earned this point.

Part D: Slightly more than a third of responses earned **P9** by answering "no" and reasoning that $x = 8$ is outside the interval of convergence.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Part A: Arithmetic and algebra errors were the most common reasons for not earning **P3**. A common error resulting in not earning **P5** was attempting to use a comparison test with an alternating series.

Part B: Some responses did not earn **P6** because of an arithmetic mistake. For example, responses found the correct coefficient for the third term to be $\frac{4}{108}$ and then reduced it incorrectly. Some responses did not properly use parentheses in the initial presentation of the general term but later presented a correct answer.

Part C: Some responses incorrectly used 1 as the first term in the geometric series and did not earn **P8**.

Part D: Some responses never gave a definitive answer, and the statements were presented in a way that made it difficult to determine the conclusion. These responses did not earn **P9**.

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
Part A • Many responses did not present the absolute value of a ratio, which did not have an impact on scoring for **P1** or **P2** and could have been resolved with an interior interval consistent with an absolute value expression. • Some responses did not use proper limit notation or took the limit as x approaches ∞ rather than as n approaches ∞. • Some students presented intervals not centered about $x = 4$. This was most often the result of algebraic or arithmetic mistakes. For example, $-1 < \frac{x-4}{3} < 1$ was incorrectly simplified to be $-1 < x < 7$ or $\frac{11}{3} < x < \frac{14}{3}$.	• $\left\lvert \frac{(x-4)^{n+2}}{(n+2)3^{n+1}} \cdot \frac{(n+1)3^n}{(x-4)^{n+1}} \right\rvert$ earned **P1**. • $\lim_{n\to\infty} \left\lvert \frac{(x-4)(n+1)}{3(n+2)} \right\rvert = \left\lvert \frac{x-4}{3} \right\rvert$ earned **P2**. • $\left\lvert \frac{x-4}{3} \right\rvert < 1$ when $-3 < x-4 < 3 \Rightarrow 1 < x < 7$ earned **P3**.

• Many responses attempted to use an alternating series in a comparison test, which can only be used when the series being compared have positive terms. • Many students claimed $\sum_{n=1}^{\infty} \frac{3}{n+1}$ is the harmonic series or offered evidence of misconceptions regarding how to use a comparison test to justify that a series diverges.	• When $x = 1$, the series is $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot 3}{n+1}$, which converges by the alternating series test. • When $x = 7$, the series is $\sum_{n=1}^{\infty} \frac{3}{n+1}$, which diverges by comparison (direct or limit) to the harmonic series.
Part B • Simplification errors were the most common reasons for responses not earning **P6**. For example: ○ $\frac{x-4}{3} + \frac{(x-4)^2}{9} + \frac{4(x-4)^3}{27}$ ○ $1 + \frac{(x-4)^2}{9} + \frac{(x-4)^3}{27}$ • Rarely, responses indicated the general term was constructed by finding a pattern from three incorrectly presented terms. Sometimes, parentheses errors were evident in unsimplified expressions for the general term. For example, $\frac{n+1(x-4)^n}{(n+1)3^n}$.	• The first three nonzero terms of the Taylor series for f' about $x = 4$ are $\frac{x-4}{3} + \frac{(x-4)^2}{9} + \frac{(x-4)^3}{27} + \cdots$. • The general term of the Taylor series for f' about $x = 4$ is $\frac{(x-4)^n}{3^n}$.
Part C • Some responses incorrectly used 1 as the first term of the geometric series and presented an answer of the form $\frac{1}{1-r}$. For example, $\frac{1}{1-\frac{x-4}{3}} = \frac{3}{3-x+4} = \frac{3}{7-x}$.	• The first term of the Taylor series for f' about $x = 4$ is $\frac{(x-4)}{3}$ and the common ratio between terms is also $\frac{(x-4)}{3}$. Thus, $f'(x) = \frac{\frac{x-4}{3}}{1-\frac{x-4}{3}} = \frac{x-4}{7-x}$.
Part D • Many responses used imprecise language about the radius of convergence, rather than the interval of convergence. For example, “No, 8 is outside of the radius of convergence.”	• No, $x = 8$ is outside the interval of convergence.

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve student performance on the exam?

- When your class is learning to find an interval of convergence in Topic 10.13, providing practice problems that require students to use a variety of tests for convergence and divergence when testing the endpoints offers an excellent opportunity to review the tests learned in earlier topics and how to select the appropriate test for a particular situation. Challenge advanced students by asking them to explain why the ratio test would not be an appropriate way to test endpoints of an interval found using the ratio test.
- As in all units and topics, model and encourage correct limit notation and usage and precise language.
- Encourage students to check that the interval presented is centered properly. This could help them discover arithmetic mistakes (when they add or subtract the radius from the center).

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

The instructional activity suggested for Topics 10.2–10.8 on page 182 of the *AP Calculus AB and BC Course and Exam Description* is to create a graphic organizer for the series tests. For best effect, it is important for students to organize and explain the series test for themselves or in their groups, rather than doing it for them or having a single student organize and provide a graphic organizer to the class.