AP

AP® Statistics
Sample Student Responses and Scoring Commentary

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Question 3: Focus on Probability and Sampling Distributions — 4 points

General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).

- The model solution represents an ideal response to each part of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

Model Solution	Scoring
A i. $P(\text{Rock Song}) = \dfrac{100}{1,000} = 0.10$ ii. $P(\text{Both Rock Songs}) = (0.10)(0.10) = 0.01$	**Essentially correct (E)** if the response satisfies the following four components: 1. In part A (i) the response calculates the correct probability. 2. In part A (i) the response provides supporting work for the correct probability. 3. In part A (ii) the response calculates the correct probability consistent with the answer in part A (i). 4. In part A (ii) the response provides supporting work consistent with the probability calculated in part A (i). **Partially correct (P)** if the response satisfies two or three of the four components required for E. **Incorrect (I)** if the response does not meet the criteria for E or P.

Scoring Notes:

- In part A (ii) a response that does not consider independence when solving (e.g., $\left(\dfrac{100}{1000}\right)\left(\dfrac{99}{999}\right) = 0.0099$) does not satisfy component 3 but may satisfy component 4.

- If probabilities are not labeled but are given in the order they are asked, then the response may earn an E. If the probabilities are presented in a different order, there must be a label to satisfy the components. In this case, sufficient labels include probability notation, context, work, or identification of the subparts (i) and (ii).

Model Solution	Scoring
B i. Let the random variable of interest, X, represent the number of the 20 songs played in one hour that are rock songs. It is stated that any song can be replayed at any time, which establishes that each rock song has probability $\frac{100}{1,000} = 0.10$ of being selected each hour and each song is independent from every other song. Therefore, X has a binomial distribution with $n = 20$ independent trials and probability of success $p = 0.10$ for each trial. ii. The expected value for the number of rock songs played in one hour is $np = 20(0.10) = 2$ songs.	**Essentially correct (E)** if the response satisfies at least three of the following four components: 1. In part B (i) the response defines the random variable as the number of rock songs played in one hour. 2. In part B (i) the response describes the distribution as binomial. 3. In part B (i) or B (ii) the response states that $n = 20$ and $p = 0.10$. 4. In part B (ii) the response correctly calculates the expected value AND provides supporting work for the calculation of the correct expected value. **Partially correct (P)** if the response satisfies only two of the four components required for E. **Incorrect (I)** if the response does not meet the criteria for E or P.

Scoring Notes:

- When defining the random variable, the response must include both "the number of rock songs" and "in one hour" or "out of 20 songs."
- A response that states $X \sim B(20,\ 0.1)$ satisfies components 2 and 3.
- If a response states the random variable has a distribution other than binomial (e.g., normal, left skewed, or uniform), part B cannot be scored E.
- Stating that songs are distributed randomly is not a distribution and should be considered extraneous.
- Examples that satisfy components 3 and 4 include:
  - $np = 20(0.10) = 2$
  - $np = 20\left(\frac{100}{1,000}\right) = 2$
  - $n = 20,\ p = 0.10,\ np = 2$
- An example that satisfies component 4 only:
  - $20(0.10) = 2$
- An arithmetic or transcription error in a response can be ignored if correct work is shown.

Model Solution	Scoring
C i. The probability that in a particular hour 4 or more rock songs will be played is $P(X \geq 4) = 1 - P(X \leq 3)$ $P(X \geq 4) = 1 - \left[\binom{20}{0}(0.10)^0(0.90)^{20} + \right.$ $\binom{20}{1}(0.10)^1(0.90)^{19} + \binom{20}{2}(0.10)^2(0.90)^{18} +$ $\left. \binom{20}{3}(0.10)^3(0.90)^{17} \right]$ $P(X \geq 4) = 1 - 0.867 = 0.133$. ii. No, the probability that 4 or more rock songs would be played in an hour is 0.133, which is high enough to be reasonably attributed to chance alone. This probability is not small enough to provide evidence that the selection process was not truly random.	**Essentially correct (E)** if the response satisfies the following four components: 1. In part C (i) the response provides a correct probability. 2. In part C (i) the response shows work that supports the correct probability. 3. In part C (ii) the response indicates that there is not a strong reason to believe that the selection process was not truly random. 4. In part C (ii) the response provides an explanation that correctly links the probability to the decision. **Partially correct (P)** if the response satisfies only two or three of the four components required for E. **Incorrect (I)** if the response does not meet the criteria for E or P.

Scoring Notes:

- A response may satisfy component 2 by any of the following:
  - <u>Graphical display</u>: Displaying a bar graph of binomial probabilities including axes with scale with appropriate bars shaded.
  - <u>Probability formula</u>: For example,
    $$1 - \binom{20}{0}(0.10)^0(0.90)^{20} - \binom{20}{1}(0.10)^1(0.90)^{19} - \binom{20}{2}(0.10)^2(0.90)^{18} - \binom{20}{3}(0.10)^3(0.90)^{17}.$$
  - <u>Calculator function notation</u>: Using calculator function notation with clearly defined arguments. For example:
    - $1 - \text{binomcdf}(n = 20, p = 0.10, \text{upper bound} = 3)$ satisfies component 2 because the boundary value is clearly labeled.
    - $1 - \text{binomcdf}(n = 20, p = 0.10, 3)$ does not satisfy component 2 because the boundary value is not labeled.
    - $\text{Binomcdf}(n = 20, p = 0.10, \text{lower bound} = 4, \text{upper bound} = 20)$ satisfies component 2 because the boundary value is clearly labeled.
  - <u>Random Variable</u>: $P(X \geq 4)$ or $1 - P(X \leq 3)$ with identification of the binomial distribution with correct parameters ($n = 20$ and $p = 0.10$) included in part C satisfies component 2.
- An arithmetic or transcription error in a response can be ignored if correct work is shown.
- A response that indicates that the manager does have a strong argument that the selection process was not truly random (or responds "yes") that is adequately supported by an explanation based on an incorrectly calculated probability in part C (i) may satisfy components 3 and 4.
- A response that indicates that the manager does have a strong argument that the selection process was not truly random (or responds "yes") that is supported by a statement claiming the probability is low may satisfy components 3 and 4.

- A response that indicates that the manager does not have a strong argument that the selection process was not random supported by the calculation of the standard deviation of the binomial distribution, 1.342, and an explanation based on 4 being within two standard deviations of the expected value (mean) may earn credit for components 3 and 4.
- If a response gives two arguments, treat them as parallel solutions and score the weaker solution.
- A response that finds the probability of exactly 4 songs playing (e.g., $\text{binompdf}(20, 0.1, 4) = 0.089$) and explains that this is not strong evidence that the selection process was not truly random may still satisfy components 3 and 4.

Scoring for Question 3	Score
Complete Response Three parts essentially correct	4
Substantial Response Two parts essentially correct and one part partially correct	3

Developing Response
Two parts essentially correct and no part partially correct
    OR
One part essentially correct and one or two parts partially correct
    OR
Three parts partially correct
	2

Minimal Response One part essentially correct and no parts partially correct *OR* No part essentially correct and two parts partially correct	1

Question 3

Begin your response to **QUESTION 3** on this page.

A) i) $P(rock) = \dfrac{100 \text{ rock songs}}{1000 \text{ total songs}} = \dfrac{1}{10} = 0.1$

ii) $P(rock \text{ twice}) = P(rock) \cdot P(rock) = (0.1)(0.1) = 0.01$

B) i) the random variable is the number of rock songs plays out of 20 songs, and it is a binomial distribution

ii) $X = $ number of rock songs
$n = $ number of total songs $= 20$
$P = $ proportion of songs that are rock $= 0.1$

$\mu_x = nP = 20(0.1) = 2$

expected value is 2 rock songs in an hour.

C) i) $P(x \geq 4) = 1 - (P(x < 4)) = 1 - (P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$P(X = x) = \binom{n}{x} P^x (1-P)^{n-x} = \binom{20}{x}(0.1)^x (0.9)^{20-x}$

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Q5518/06

Question 3

Continue your response to **QUESTION 3** on this page.

$$= 1 - \left(\binom{20}{0}(0.9)^{20} + \binom{20}{1}(0.1)(0.9)^{19} + \binom{20}{2}(0.1)^2(0.9)^{18} + \binom{20}{3}(0.1)^3(0.9)^{17} \right)$$

$$= 1 - (0.867) = \underline{0.133}$$

ii) this does not provide strong evidence that the selection process isn't random. there is a 13.3% chance of 4 or more rock songs playing in an hour, which is a high enough probability that it has a good chance of happening naturally given a random selection process

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Question 3

Begin your response to **QUESTION 3** on this page.

A-i) $100 / 1,000 = 10\%$ chance a rock song is played.
 ⌞rock songs⌟ ⌞total songs⌟

ii) $10\% \times 10\% = 1\%$ chance both songs are rock songs.

B-i) the random variable of interest is rock songs played at random, which has a 10% probability for every 1 song.

ii) $.1 \times 20 = 2$ rock songs per hour.
  p n

C-i) binom cdf $(20, .1, 3) = .867$ $1-.867 = .133$
     n p x
  $\sim 13\%$ chance 4 or more rock songs will be selected in 1 hour.

ii) No, because an hour is short term and will not follow exact probabilities. the random generator would be closer to the distribution if played for a long period of time.

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Q5518/08

Question 3

Begin your response to **QUESTION 3** on this page.

A.

i. $P(x \text{ is rock})$

$\frac{100}{1000} = .10$

There is a 10% chance the a randomly selected song is rock.

ii. $P(\text{rock}) = .10$

The probability that both randomly selected songs are rock is still 10% because the events are independent and songs can be replayed.

B.

i. The random variable of interest is the amount of rock songs played in 1 hour.

ii. $P(\text{rock}) = .10$

$.10(20) = 2$

Out of 20 songs played in a 1 hour period we can expect 2 of these songs to be rock songs.

C.

i. $P(x \geq 4)$ binomialCDF

rock songs played

The probability that 4 or more rock songs are played is .0432

ii. Because the event that 4 rock songs are played in a 1 hour period has a roughly 4% chance of happening there is no strong evidence that the selection process isn't random because there is a chance it will happen.

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Q5518/08

Question 3

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

NEW for 2025: The question overviews can be found in the *Chief Reader Report on Student Responses* on [AP Central](#).

Sample: 3A
Score: 4

The response earned the following: part A – E, part B – E, and part C – E.

In part A (i) the response calculates the correct probability with supporting work, satisfying components 1 and 2. In part A (ii) the response calculates the correct probability with supporting work, satisfying components 3 and 4. This part of the response was scored essentially correct (E).

In part B (i) the response correctly defines the random variable, satisfying component 1, and describes the distribution as binomial, satisfying component 2. In part B (ii) the response labels "n" and "p" and correctly calculates the expected value with supporting work, satisfying components 3 and 4. This part of the response was scored essentially correct (E).

In part C (i) the response provides a correct probability with supporting work, satisfying components 1 and 2. In part C (ii) the response indicates that "this does not provide strong evidence that the selection process isn't random," satisfying component 3. The response provides an explanation that links the probability to the decision with the statement, "which is a high enough probability that it has a good chance of happening naturally," satisfying component 4. This part of the response was scored essentially correct (E).

Sample: 3B
Score: 2

The response earned the following: part A – E, part B – P, and part C – P.

In part A (i) the response calculates the correct probability with supporting work, satisfying components 1 and 2. In part A (ii) the response calculates the correct probability with supporting work, satisfying components 3 and 4. This part of the response was scored essentially correct (E).

In part B (i) the response does not define the random variable as the number of rock songs played in one hour, so component 1 is not satisfied. The response does not satisfy component 2 as the distribution is not described as binomial. In part B (ii) the response labels "n" and "p" and correctly calculates the expected value with supporting work, satisfying components 3 and 4. For satisfying only two of the four components, this part of the response was scored partially correct (P).

In part C (i) the response provides a correct probability, satisfying component 1, and parameters are identified in the supporting calculator notation, satisfying component 2. In part C (ii) the response indicates "No," satisfying component 3. The response does not link the decision to the probability, so component 4 is not satisfied. For satisfying only three of the four components, this part of the response was scored partially correct (P).

Question 3 (continued)

Sample: 3C
Score: 1

The response earned the following: part A – P, part B – P, and part C – I.

In part A (i) the response calculates the correct probability with supporting work, satisfying components 1 and 2. In part A (ii) the response does not calculate the correct probability with supporting work, so it does not satisfy components 3 and 4. For satisfying two of the four components, this part of the response was scored partially correct (P).

In part B (i) the response correctly defines the random variable, satisfying component 1. The response does not satisfy component 2 as the distribution is not described as binomial, and in part B (ii) it does not satisfy component 3 as it does not label "n." The response correctly calculates the expected value with supporting work, satisfying component 4. For satisfying only two of the four components, this part of the response was scored partially correct (P).

In part C (i), the response does not provide a correct probability with supporting work, so components 1 and 2 are not satisfied. In part C (ii), the response indicates that "there is no strong evidence that the selection process isn't random," satisfying component 3. The response does not provide an explanation that links the probability to the decision, so component 4 is not satisfied. Because only one of four components is satisfied, this part of the response was scored incorrect (I).