2025

AP® Precalculus

Sample Student Responses and Scoring Commentary

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Free-Response Question 4

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Question 4: Symbolic Manipulations
Part B: Graphing calculator not allowed — 6 points

Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

	Model Solution	Scoring	
A	The functions g and h are given by $g(x) = 2\log_3 x$ $h(x) = 4\cos^2 x$ **(i)** Solve $g(x) = 4$ for values of x in the domain of g. **(ii)** Solve $h(x) = 3$ for values of x in the interval $\left[0, \frac{\pi}{2}\right)$.		
	(i) $g(x) = 4$ $2\log_3 x = 4$ $\log_3 x = 2$ $3^2 = x$ $x = 9$	Solution to $g(x) = 4$	**Point A1**
	(ii) $h(x) = 3$ $4\cos^2 x = 3$ $\cos^2 x = \frac{3}{4}$ $\cos x = \pm\frac{\sqrt{3}}{2}$ Because x is in $\left[0, \frac{\pi}{2}\right)$, $x = \frac{\pi}{6}$	Solution to $h(x) = 3$	**Point A2**

Scoring Notes for Part A

- **Point A1** and **Point A2** both require supporting work. "Scratchwork" can be ignored; the use of a variable other than x is acceptable. Arithmetic errors following a complete and correct solution may be considered scratchwork. The use of "$x =$" is not required.
- A logarithmic expression that adds one or both parentheses around the full argument of the logarithm is eligible to earn **Point A1**.
- A response that includes correct values of x outside of the interval $\left[0, \frac{\pi}{2}\right)$ is eligible to earn **Point A2** (e.g., $x = \frac{5\pi}{6}$, $x = \frac{7\pi}{6}$, or $x = \frac{11\pi}{6}$).
- The use of $\pm$ is not required in supporting work for **Point A2**.
- Where applicable, answers that have not been evaluated according to bullets two and three in the Directions do not earn the point. Rationalizing denominators is not required.

Partial Credit for Part A

A response that **does not** earn either **Point A1** or **Point A2** is eligible for **partial credit** in part A if the response has one criteria from the first column AND one criteria from the second column.

Partial credit response is scored **1** for **Point A1** and **0** for **Point A2**.

First Column	**Second Column**
Correct answer in part A (i) without supporting work.	Correct answer in part A (ii) without supporting work.
Correct answer in part A (i) with supporting work, but the answer has not been evaluated according to bullets two and three in the Directions (e.g., $x = 3^2$). No incorrect work.	Correct answer in part A (ii) with supporting work, but the answer has not been evaluated according to bullets two and three in the Directions. This includes an answer of $x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$. No incorrect work.
Answer in part A (i) is reported as $x^2 = 3^4$ OR $x^2 = 81$. No incorrect work follows.	Answer in part A (ii) is reported as $\cos x = \pm\frac{\sqrt{3}}{2}$ OR $\cos x = \frac{\sqrt{3}}{2}$. No incorrect work follows.

B The functions j and k are given by

$$j(x) = \log_2 x + 3\log_2 2$$

$$k(x) = \frac{6}{\tan x\left(\csc^2 x - 1\right)}$$

(i) Rewrite $j(x)$ as a single logarithm base 2 without negative exponents in any part of the expression. Your result should be of the form $\log_2(\text{expression})$.

(ii) Rewrite $k(x)$ as an expression in which $\tan x$ appears exactly once and no other trigonometric functions are involved.

(i) $j(x) = \log_2 x + 3\log_2 2$ $j(x) = \log_2 x + \log_2 2^3$ $j(x) = \log_2(8x),\ x > 0$	Expression for $j(x)$	**Point B1**
(ii) $k(x) = \frac{6}{\tan x\left(\csc^2 x - 1\right)}$ $k(x) = \frac{6}{\tan x\left(\cot^2 x\right)}$ $k(x) = \frac{6}{\cot x}$ $k(x) = 6\tan x,\ \tan x \neq 0,\ \cot x \neq 0$	Expression for $k(x)$	**Point B2**

Scoring Notes for Part B

- **Point B1** is earned with a correct expression for $j(x)$ without supporting work, provided no incorrect work is included. "Scratchwork" can be ignored; the use of a variable other than x is acceptable. The use of "$j(x) =$" is not required.
- **Point B2** requires supporting work. Scratchwork can be ignored; the use of a variable other than x is acceptable. The use of "$k(x) =$" is not required.
- Domain restrictions are not required to be included and are not scored regardless if correct or incorrect.
- Where applicable, answers that have not been evaluated according to bullets two and three in the Directions do not earn the point.
- A logarithmic expression that is missing one or both parentheses around the full argument of the logarithm is still eligible to earn **Point B1**.
- If a response is presented as a complex fraction, the complex fraction must be unambiguous in structure. Parentheses must be used correctly, and/or the fraction bars must be clearly and correctly proportioned.

Partial Credit for Part B

A response that **does not** earn either **Point B1** or **Point B2** is eligible for **partial credit** in part B if the response has one criteria from the first column AND one criteria from the second column.

Partial credit response is scored **1** for **Point B1** and **0** for **Point B2**.

First Column	Second Column
Expression in part B (i) is reported as $\log_2 x + 3$. No incorrect work follows.	Correct expression in part B (ii) without supporting work.
Expression in part B (i) is reported as $\log_2\left(2^3 \cdot x\right)$. No incorrect work follows.	Expression in part B (ii) is reported as $\frac{6}{\cot x}$ OR $\frac{6\sin x}{\cos x}$. No incorrect work follows.
Expression in part B (i) is reported using logarithm base b, $b > 0$, and $b \neq 2$, and has the correct argument.	Expression in part B (ii) includes a correct application of a Pythagorean identity with no incorrect work.

C The function m is given by

$$m(x) = e^{2x} - e^x - 12.$$

Find all input values in the domain of m that yield an output value of 0.

$m(x) = 0 \Rightarrow e^{2x} - e^x - 12 = 0$ $\left(e^x\right)^2 - e^x - 12 = 0$ Let $y = e^x$. $y^2 - y - 12 = 0$ $(y-4)(y+3) = 0$ $y - 4 = 0$ or $y + 3 = 0$ $e^x = 4$ or $e^x = -3 \Rightarrow x = \ln 4$	Quadratic form with e^x	**Point C1**
	Value of x	**Point C2**

Scoring Notes for Part C

- **Point C1** and **Point C2** both require supporting work. "Scratchwork" can be ignored; the use of a variable other than x is acceptable. The use of "$x =$" is not required.
- **Point C1** is earned for a substitution of $y = e^x$ and factored form of $(y \pm 4)(y \pm 3)$ [the use of a variable other than y is acceptable] OR for presenting $m(x)$ in factored form as $\left(e^x \pm 4\right)\left(e^x \pm 3\right)$.
- To earn **Point C2**, no incorrect values for x are included.

Answer QUESTION 4 PART A on this page.

PART A

i $4 = 2\log_3 x$

$3^2 = x$

$\boxed{x = 9}$

ii $3 = 4\cos^2 x \quad [0, \frac{\pi}{2})$

$\pm \frac{\sqrt{3}}{2} = \cos x$

$\boxed{\frac{\pi}{6}}$

Answer QUESTION 4 PARTS B and C on this page.

PART B

i $\log_2 x + \log_2 2^3$ → 8

$\log_2 8x$

ii $6 \div \left(\frac{\sin x}{\cos x}\left(\frac{1-\sin^2 x}{\sin^2 x}\right)\right)$

$6 \div \left(\tan x\left(\frac{\cos^2 x}{\sin^2 x}\right)\right)$

$\cot^2 x$

$\tan x \cdot \frac{1}{\tan^2 x} = \cot x$

$\frac{6}{\cot x} = 6\tan x$

PART C

$e^{2x} - e^x - 12 = 0$

$e^{2x} - e^x = 12$

$(e^2)^x (e)^x = 12$

$y = e^x$

$y^2 - y - 12 = 0$

$(y-4)(y+3) = 0$

$e^x - 4 = 0, e^x + 3 = 0$

$e^x = 4$ ~~$e^x = -3$~~

$\ln 4 = x$

Answer QUESTION 4 PART A on this page.

PART A

i $g(x) = 2\log_3 x$

$4 = 2\log_3 x$

$2 = \log_3 x$

3^2

$x = 9$

ii $3 = 4\cos^2 x$

$\frac{3}{4} = \cos^2 x$

$\sqrt{\frac{3}{4}}$

$\frac{\sqrt{3}}{2} = \cos x$

$x = \frac{\pi}{6}$

Q4 NO CALCULATOR ALLOWED Q4

Answer QUESTION 4 PARTS B and C on this page.

PART B

i

$\log_2 x + 3\log_2 2$

$j(x) = \log_2(8x)$

ii

$\frac{6}{\tan x(\csc^2 x - 1)}$ $\frac{6}{\tan^3 x}$

$\frac{6}{\tan x(\tan^2 x)}$

PART C

$0 = e^{2x} - e^x - 12$ $u = e^x$

$0 = u^2 - u - 12$

$0 = (u-4)(u+3)$

$u = 4$ $u = -3$

$e^x = 4$ $e^x = -3$

$x = \ln 4$ ~~$x = \ln -3$~~

Answer QUESTION 4 PART A on this page.

PART A

i

$$\frac{4}{2} = \frac{2\log_3 x}{2}$$

$$2 = \log_3 x$$

$$x = 3^2$$

$$x = 9$$

ii

$$\frac{3}{4} = \frac{4\cos^2 x}{4}$$

$$\sqrt{\frac{3}{4}} = \sqrt{\cos^2 x}$$

$$\frac{\sqrt{3}}{\sqrt{4}} = \cos x$$

$$\frac{\sqrt{3}}{2} = \cos x$$

$$x = \frac{\pi}{6}$$

Answer QUESTION 4 PARTS B and C on this page.

PART B

i

$$j(x) = \log_2 x + 2^3$$

$$j(x) = \log_2 x + 8$$

ii

$$K(x) = \frac{6}{\tan x \ (\tan x)}$$

$$K(x) = \frac{6}{\tan^2 x}$$

PART C

$$0 = e^{2x} - e^x - 12$$

$$(e^x + 3)(e^x - 4)$$

$$e^x = -3 \qquad e^x = 4$$

$$\ln \qquad \ln$$

$$x = \ln(-3) \qquad x = \ln(4)$$

Question 4: Symbolic Manipulations

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

NEW for 2025: The question overviews can be found in the *Chief Reader Report on Student Responses* on AP Central.

Sample: 4A
Score: 6

Point A1: 1

The response earned the point for presenting the correct solution with supporting work.

Point A2: 1

The response earned the point for presenting the correct solution with supporting work. The supporting work presented is sufficient. To be eligible for Point A2, the supporting work must include either $\cos^2 x = \frac{3}{4}$ or $\cos x = \frac{\sqrt{3}}{2}$.

Point B1: 1

The response earned the point with a correct answer that is presented according to the instructions given in the second and third bullets in the Directions, together with correct supporting work. Although the use of parentheses around the logarithm argument is encouraged, the response is eligible for Point B1 if no parentheses are provided according to the fifth bullet point of the Scoring Notes for part B.

Point B2: 1

The response earned the point for correctly rewriting the function as an expression in which $\tan x$ appears exactly once together with supporting work.

Point C1: 1

The response earned the point for providing the quadratic form $y^2 - y - 12$ after defining the substitution as $y = e^x$.

Point C2: 1

The response earned the point for the correct solution. The response provides the extraneous solution and eliminates it. The extraneous solution is not necessary in the supporting work to be eligible for Point C2.

Question 4: Symbolic Manipulations (continued)

Sample: 4B
Score: 5

Point A1: 1

The response earned the point by presenting the correct solution with supporting work.

Point A2: 1

The response earned the point for presenting the correct solution with supporting work. The use of $\pm$ is not required before $\frac{\sqrt{3}}{2}$. The positive value is correct for the given domain.

Point B1: 1

The response earned the point with a correct answer that is presented according to the instructions given in the second and third bullets in the Directions. Supporting work is not required to earn Point B1.

Point B2: 0

The response did not earn the point because both the supporting work and the answer are incorrect.

Point C1: 1

The response earned the point for providing the quadratic form $0 = u^2 - u - 12$ and defining the substitution as $u = e^x$.

Point C2: 1

The response earned the point for the correct answer. The response provides the extraneous solution and eliminates it. The extraneous solution is not necessary to be eligible for Point C2.

Question 4: Symbolic Manipulations (continued)

Sample: 4C
Score: 3

Point A1: 1

The response earned the point for presenting the correct solution with supporting work.

Point A2: 1

The response earned the point for presenting the correct solution with supporting work. The use of $\pm$ is not required before $\frac{\sqrt{3}}{2}$. The positive value is correct for the given domain.

Point B1: 0

The response did not earn the point because the answer presented is not in the form $\log_2(\text{expression})$.

Point B2: 0

The response did not earn the point because both the supporting work and the answer are incorrect.

Point C1: 1

The response earned the point by presenting $m(x)$ as $\left(e^x + 3\right)\left(e^x - 4\right)$.

Point C2: 0

The response did not earn the point because the extraneous solution is not eliminated.