AP

AP® Precalculus

Sample Student Responses and Scoring Commentary

Inside:

Free-Response Question 3

☑ **Scoring Guidelines**

☑ **Student Samples**

☑ **Scoring Commentary**

Question 3: Modeling a Periodic Context
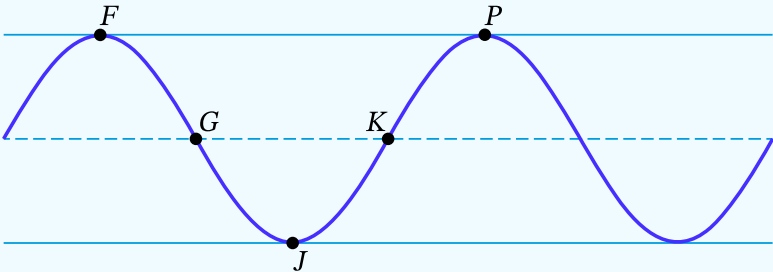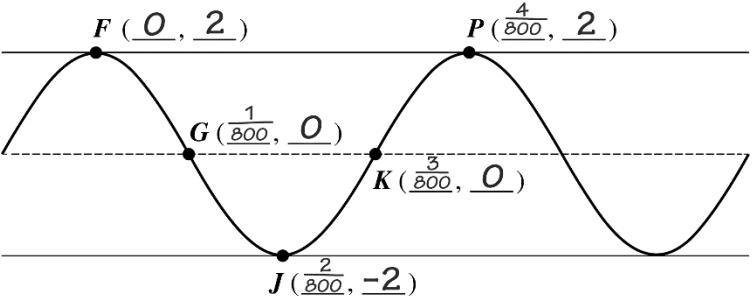## Part B: Graphing calculator not allowed   6 points

For a guitar to make a sound, the strings need to vibrate, or move up and down or back and forth, in a motion that can be modeled by a periodic function.

At time $t = 0$ seconds, point X on one vibrating guitar string starts at its highest position, 2 millimeters above its resting position. Then it passes through its resting position and moves to its lowest position, 2 millimeters below the resting position. Point X then passes through its resting position and returns to 2 millimeters above the resting position. This motion occurs 200 times in 1 second.

The sinusoidal function h models how far point X is from its resting position, in millimeters, as a function of time t, in seconds. A positive value of $h(t)$ indicates the point is above the resting position; a negative value of $h(t)$ indicates the point is below the resting position.

Model Solution	Scoring

A The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented.
Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.

$F\left(\underline{0}, \underline{2}\right)$ $P\left(\frac{4}{800}, \underline{2}\right)$ $G\left(\frac{1}{800}, \underline{0}\right)$ $K\left(\frac{3}{800}, \underline{0}\right)$ $J\left(\frac{2}{800}, \underline{-2}\right)$ Note: t-coordinates will vary. A correct set of coordinates for one full cycle of h as pictured is acceptable.	$h(t)$-coordinates **Point A1**
	t-coordinates **Point A2**

Scoring Notes for Part A

- No supporting work is required.

- $h(t)$-coordinates and/or t-coordinates may appear in a list.

- Negative t-coordinates are acceptable. Fractions do not need to be reduced; equivalent fractions and exact decimal values are acceptable.

- t-coordinates must be $0 + \frac{4}{800}k$, $\frac{1}{800} + \frac{4}{800}k$, $\frac{2}{800} + \frac{4}{800}k$, $\frac{3}{800} + \frac{4}{800}k$, $\frac{4}{800} + \frac{4}{800}k$ for a specific integer k.

- If the graph is used to record coordinates, that work is scored. In this case, other work is considered scratchwork and is not scored. Use of the graph is not required.

Partial Credit for Part A

A response that **does not** earn either **Point A1** or **Point A2** is eligible for **partial credit** in part A if the response meets one of the following criteria:

- All 5 points are in the form $(h(t), t)$ with correct input values and correct output values swapped.

- 3 out of the 5 points are correct.

- All 5 points $(t, h(t))$ meet these requirements:

  - t-coordinates are in arithmetic sequence with $\Delta t = \dfrac{1}{800}$.

  - $h(t)$-coordinates are such that

    1. F and P have **same** $h(t)$-coordinate.

    2. G and K have **same** $h(t)$-coordinate, which is **less than** $h(t)$-coordinate of F and P.

    3. Difference in $h(t)$-coordinates for F and G **equals** Difference in $h(t)$-coordinates for G and J.

Partial credit response is scored **0** for **Point A1** and **1** for **Point A2**.

B	The function h can be written in the form $h(t) = a\sin(b(t + c)) + d$. Find values of constants a, b, c, and d.

$\dfrac{2\pi}{b} = \dfrac{1}{200}$, so $b = 400\pi$

$d = 0$

Method 1:

Using $a = 2$,

$c = \dfrac{1}{800}$ OR $c = \dfrac{1}{800} + \dfrac{4}{800}k$, for any integer k

$$
\begin{aligned}
a &= \underline{\qquad 2 \qquad} \\
b &= \underline{\qquad 400\pi \qquad} \\
c &= \underline{\qquad \tfrac{1}{800} \qquad} \\
d &= \underline{\qquad 0 \qquad}
\end{aligned}
$$

For example, $h(t) = 2\sin\left(400\pi\left(t + \dfrac{1}{800}\right)\right)$. Based on horizontal shifts, there are other correct forms for $h(t)$.

Vertical transformations: Values of a and d — **Point B1**

Horizontal transformations: Values of b and c — **Point B2**

Method 2:

Using $a = -2$,

$c = -\dfrac{1}{800}$ OR $c = -\dfrac{1}{800} + \dfrac{4}{800}k$, for any integer k

$a =$ _____ -2 _____

$b =$ _____ 400π _____

$c =$ _____ $-\dfrac{1}{800}$ _____

$d =$ _____ 0 _____

For example, $h(t) = -2\sin\left(400\pi\left(t - \dfrac{1}{800}\right)\right)$. Based on

horizontal shifts, there are other correct forms for $h(t)$.

Scoring Notes for Part B

- No supporting work is required.

- Points are earned for correct values in a list OR for correct values in an expression for $h(t)$. Only one of these answer presentations is required.

- Fractions do not need to be reduced; equivalent fractions and exact decimal values are acceptable.

- If the answer box is used to record values, that work is scored. In this case, other work is considered scratchwork and is not scored. Use of the answer box is not required.

- **Point B1** and **Point B2** may be earned based on the correct use of an imported response from part A that meets these criteria:

  - $a \neq 1$, $b \neq 1$, and $c \neq 0$.

  - All 5 points $(t, h(t))$ from part A meet these requirements:

    o t-coordinates are in arithmetic sequence with $\Delta t = \dfrac{1}{800}$.

    o $h(t)$-coordinates are such that

      1. F and P have **same** $h(t)$-coordinate.

      2. G and K have **same** $h(t)$-coordinate, which is **less than** $h(t)$-coordinate of F and P.

      3. Difference in $h(t)$-coordinates for F and G **equals** Difference in $h(t)$-coordinates for G and J.

Partial Credit for Part B

A response that **does not** earn either **Point B1** or **Point B2** is eligible for **partial credit** in part B if the response meets one of the following criteria:

- Correct values of a and b [Values of a and b could be $\pm$]

- Correct values of b and d [Value of b could be $\pm$]

- Response uses $h(t) = a\cos(b(t + c)) + d$ with values as follows:

  ○ $a = 2$; $b = 400\pi$; $c = 0 + \dfrac{4}{800}k$, for a specific integer k; $d = 0$

  ○ $a = -2$; $b = 400\pi$; $c = -\dfrac{2}{800} + \dfrac{4}{800}k$, for a specific integer k; $d = 0$

Partial credit response is scored **1** for **Point B1** and **0** for **Point B2**.

C   Refer to the graph of h in part A. The t-coordinate of G is t_1, and the t-coordinate of J is t_2.

(i) On the interval (t_1, t_2), which of the following is true about h?

    a. h is positive and increasing.

    b. h is positive and decreasing.

    c. h is negative and increasing.

    d. h is negative and decreasing.

(ii) On the interval (t_1, t_2), describe the concavity of the graph of h and determine whether the rate of change of h is increasing or decreasing.

(i) Choice d.	Function behavior	**Point C1**
(ii) The graph of h is concave up on the interval (t_1, t_2), and the rate of change of h is increasing on the interval (t_1, t_2).	Concavity of graph and behavior of rate of change	**Point C2**

Scoring Notes for Part C

- No supporting work is required.

- **Point C1** is earned only for a correct answer of "d" OR "negative and decreasing." If both the letter choice and written description are included, the written description is scored.

- To earn **Point C2**, both descriptions must be correct. **Point C2** is not earned for a response that only includes "the graph of h is concave up" OR only includes "the rate of change of h is increasing."

- To earn **Point C2**, "concave up" AND "increasing" is acceptable.

- To earn **Point C2**, "concave up" AND "function h is decreasing at an increasing rate" is acceptable.

- A response with an isolated statement "decreasing at an increasing rate" does not earn **Point C2**. The implied subject is "the rate of change of h."

- A response with a statement that "the rate of change of h is increasing at an increasing (or decreasing) rate" does not earn **Point C2**. Analysis to make such a conclusion requires calculus.

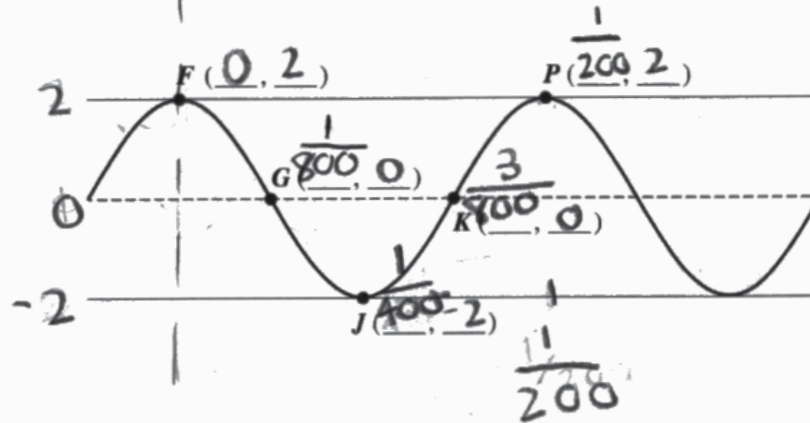- **Point C2** cannot be earned if there are any errors in part C (ii).

Answer QUESTION 3 PART A on this page.

PART A

$$F(0, 2)$$

$$G\left(\frac{1}{800}, 0\right)$$

$$J\left(\frac{1}{400}, -2\right)$$

$$K\left(\frac{3}{800}, 0\right)$$

$$P\left(\frac{1}{200}, 2\right)$$

Page 8

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

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Q5527/08

Q3   ☒ **NO CALCULATOR ALLOWED**   **Q3**

Answer QUESTION 3 PARTS B and C on this page.

PART B

amp. = 2

reflect across x-axis

period = $\frac{1}{200}$

phase shift = $\frac{1}{800}$

$$\frac{2\pi}{\frac{1}{200}} = \frac{2\pi}{1} \cdot \frac{200}{1} = 400\pi$$

$$\frac{2\pi}{b} = \frac{1}{200}$$

$$h(t) = -2\sin\left(400\pi\left(t - \frac{1}{800}\right)\right)$$

b = 400π

a =	−2
b =	400π
c =	−1/800
d =	0

PART C

i

    d. h is negative and decreasing

ii

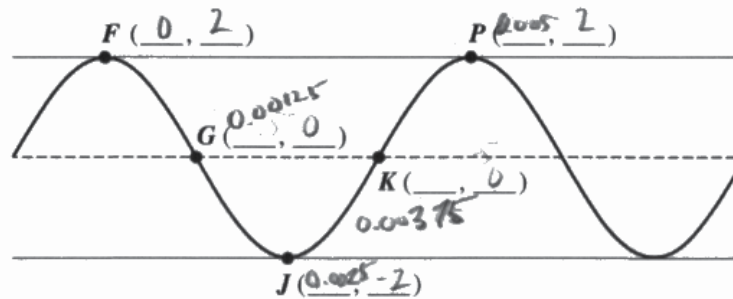On the interval (t_1, t_2), h is concave up and the rate of change of h is increasing

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5527/09

Answer QUESTION 3 PART A on this page.

PART A

F (_0_ , _2_)   P (_0.005_ , _2_)

G (_0.00125_ , _0_)

K (___ , _0_)

0.00375

J (_0.0025_ , _-2_)

$$\frac{1}{200} = \frac{1}{100} \div 2 = 0.01 \div 2 = 0.005$$

period = 0.005 seconds

$$\frac{0.005}{2} = 0.0025$$

$$\frac{0.0025}{2} = 0.00125$$

F : (0,2)
G : (0.00125, 0)
J : (0.0025, -2)
K : (0.00375, 0)
P : (0.005, 2)

Page 8

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

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Q5527/08

Answer QUESTION 3 PARTS B and C on this page.

PART B

$$a \sin(b(t+c)) + d$$

midline at 0, $d = 0$

min $= -2$, max $= 2$, $a = 2$

$a =$	2
$b =$	400π
$c =$	0.00375
$d =$	0

$$\frac{2\pi}{b} = 0.005$$

$$\frac{2\pi}{0.005} = b \qquad b = 400\pi$$

$$2\pi \cdot 200 = 400\pi$$

PART C

i Choice D

ii On the interval (t_1, t_2) the graph of h is concave up so the rate of change is increasing.

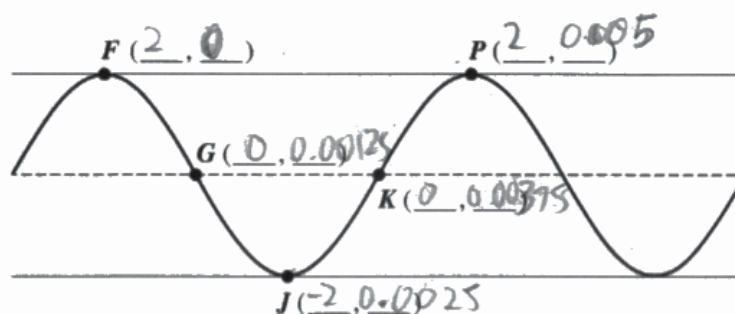**Answer QUESTION 3 PART A on this page.**

PART A

F (2 , 0)   P (2 , 0.005)

G (0 , 0.00125)

K (0 , 0.00375)

J (-2 , 0.0025)

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

0011966

Q5527/08

Q3   ⬛ **NO CALCULATOR ALLOWED**   **Q3**

Answer QUESTION 3 PARTS B and C on this page.

PART B

$$h(t) = a \sin(b(t+c)) + d$$

$$a \, ! \, 2$$

$$\frac{1}{200} \quad b = \frac{2\pi}{0.005} = 400\pi$$

$$= 0.005$$

$$C = 0.00125$$

$$d = 0$$

$a =$	2
$b =$	400π
$c =$	0.00125
$d =$	0

PART C

i   C.

- -

ii

The graph is concave up because the rate of change of h is increasing.

Q5527/09

Question 3: Modeling a Periodic Context

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

NEW for 2025: The question overviews can be found in the *Chief Reader Report on Student Responses* on [AP Central](#).

Sample: 3A
Score: 6

Point A1: 1

The response earned the point. The $h(t)$ -coordinates for the five points on the graph are correct. Use of the graph is not required. If the graph is used to record coordinates, that work is scored.

Point A2: 1

The response earned the point. The *t*-coordinates for the five points on the graph are correct.

Point B1: 1

The response earned the point. The values of *a* and *d* are correct. The response also presents a complete, correct expression for $h(t)$. Because the answer box is used to record values, the other work is considered scratchwork and is not scored.

Point B2: 1

The response earned the point for correct values of *b* and *c*. The response also presents a complete, correct expression for $h(t)$. Because the answer box is used to record values, the other work is considered scratchwork and is not scored.

Point C1: 1

The response "*h* is negative and decreasing" earned the point. The correct choice "d" is also presented. Had these responses not matched, the written statement is scored to determine whether the point is earned.

Point C2: 1

The response "On the interval (t_1, t_2), *h* is concave up and the rate of change of *h* is increasing" earned the point.

Question 3: Modeling a Periodic Context (continued)

Sample: 3B
Score: 5

Point A1: 1

The response earned the point. The $h(t)$-coordinates for the five points on the graph are correct.

Point A2: 1

The response earned the point. The *t*-coordinates for the five points on the graph are correct. The values 0, 0.00125, 0.0025, 0.00375, and 0.005 are exact decimal values corresponding to 0, $\frac{1}{800}$, $\frac{2}{800}$, $\frac{3}{800}$, and $\frac{4}{800}$.

Point B1: 1

The response earned the point for the correct values of *a* and *d*. Because the answer box is used to record values, the other work is considered scratchwork and is not scored.

Point B2: 0

The response did not earn the point. Although the value of *b* is correct, the value $c = 0.00375$ is not correct. For $a = 2$, $c = -0.00375$ is a possible value for *c*.

Point C1: 1

The response earned the point with the correct selection of choice "D." A written description is not required.

Point C2: 1

The response earned the point with the statement, "On the interval (t_1, t_2) the graph of *h* is concave up so the rate of change is increasing."

Question 3: Modeling a Periodic Context (continued)

Sample: 3C
Score: 4

Point A1: 0

The response did not earn the point. Although the correct $h(t)$ -values are on the graph, the (input, output) pairs are written in the form (output, input).

Point A2: 1 (from Partial Credit)

The response did not initially earn the point. Although the correct *t*-values are on the graph, the (input, output) pairs are written in the form (output, input). A response that does not earn either Point A1 or Point A2 in part A is eligible for Partial Credit. This Partial Credit is earned because of the Partial Credit bullet in the Scoring Guidelines that states, "All 5 points are in the form $(h(t), t)$ with correct input values and correct output values swapped." The values 0, 0.00125, 0.0025, 0.00375, and 0.005 are exact decimal values corresponding to 0, $\frac{1}{800}$, $\frac{2}{800}$, $\frac{3}{800}$, and $\frac{4}{800}$. The response meets the criteria for Partial Credit and receives a score of 0 for Point A1 and 1 for Point A2.

Point B1: 1

The response earned the point for correct values of *a* and *d*.

Point B2: 1

The response earned the point for correct values of *b* and *c*. The value $c = 0.00125$ is the exact decimal equivalent of $\frac{1}{800}$.

Point C1: 0

The response "c" is incorrect and did not earn the point.

Point C2: 1

The response "The graph is concave up because the rate of change of *h* is increasing" earned the point.