2025

AP® Precalculus

Sample Student Responses and Scoring Commentary

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Free-Response Question 1

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Question 1: Function Concepts
Part A: Graphing calculator required — 6 points

The function f is decreasing and is defined for all real numbers. The table gives values for $f(x)$ at selected values of x.

x	–2	–1	0	1	2
$f(x)$	14	7	3.5	1.75	0.875

The function g is given by $g(x) = -0.167x^3 + x^2 - 1.834$.

A

(i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(1)$ as a decimal approximation or indicate that it is not defined. Show the work that leads to your answer.

(ii) Find the value of $f^{-1}(3.5)$, or indicate that it is not defined.

Model Solution	Scoring	
(i) $h(1) = g(f(1)) = g(1.75) = 0.333$	Value	**Point A1**
(ii) From the table, $f^{-1}(3.5) = 0$.	Value	**Point A2**

General Scoring Notes for Question 1 Parts A, B, and C

- Decimal approximations must be accurate to three places after the decimal point by rounding or truncating. Decimal values of 0 in final digits need not be reported ($2.000 = 2.00 = 2.0 = 2$).
- A **decimal presentation error** occurs when a response is complete and correct, but the answer is reported to fewer digits than required.
- The first decimal presentation error in Question 1 does not earn the point. For each additional part of Question 1 that requires a decimal approximation and contains a decimal presentation error, the response is eligible to earn the point.

Scoring Notes for Part A

- **Point A1** is earned for a correct decimal approximation of 0.333 with supporting work of "$f(1)$" OR "$g(1.75)$" OR "1.75."
- **Point A2** does not require supporting work. **Point A2** is earned with a response of "0."

Partial Credit for Part A

A response that **does not** earn either **Point A1** or **Point A2** is eligible for **partial credit** in part A if the response has one criteria from the first column AND one criteria from the second column.

Partial credit response is scored **0** for **Point A1** and **1** for **Point A2**.

First Column	Second Column
Correct value in part A (i) without supporting work.	$f^{-1}(3.5)$ exists in part A (ii) without giving specific value.
Correct value in part A (i) that is not expressed as a decimal approximation.	
Correct value in part A (i) with a decimal presentation error.	

B

(i) Find all values of x, as decimal approximations, for which $g(x) = 0$, or indicate that there are no such values.

(ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

(i) $g(x) = 0 \Rightarrow -0.167x^3 + x^2 - 1.834 = 0$ $x = -1.233$, $x = 1.578$, $x = 5.643$	Values	**Point B1**
(ii) As x increases without bound, the output values of g decrease without bound. Therefore, $\lim_{x\to\infty} g(x) = -\infty$.	End behavior with limit notation	**Point B2**

Scoring Notes for Part B

- **Point B1** does not require supporting work. **Point B1** is earned with the three correct decimal approximations -1.233, 1.578, and 5.643. The use of "$x =$" is not required.
- **Point B2** requires a correct limit statement with four components: "lim", "$x \to \infty$", the function g, and $-\infty$. Examples that earn **Point B2** include:
  - $\lim_{x\to\infty} g(x) = -\infty$ OR $\lim_{x\to\infty} g = -\infty$
  - $\lim_{x\to\infty} g(x) \to -\infty$ OR $\lim_{x\to\infty} g \to -\infty$
  - $\lim_{x\to\infty} g(x)\ \ -\infty$ OR $\lim_{x\to\infty} g\ \ -\infty$
- If the response includes an additional, complete limit statement (e.g., $\lim_{x\to-\infty} g(x) = \infty$), the limit statement must be correct.

Partial Credit for Part B

A response that **does not** earn either **Point B1** or **Point B2** is eligible for **partial credit** in part B if the response has one criteria from the first column AND one criteria from the second column.

Partial credit response is scored **1** for **Point B1** and **0** for **Point B2**.

First Column	Second Column
Three correct values in part B (i) with values that are not expressed as decimal approximations	Correct end behavior statement in part B (ii) without use of limit notation
Three correct values in part B (i) with a decimal presentation error	Correct end behavior statement in part B (ii) with incorrect limit notation
Only one correct value in part B (i) with no incorrect values included (may have a decimal presentation error)	Correct limit statement in part B (ii) that is missing "$x \to \infty$"
Only two correct values in part B (i) with no incorrect values included (may have a decimal presentation error)	

C **(i)** Based on the table, which of the following function types best models function f: linear, quadratic, exponential, or logarithmic?

(ii) Give a reason for your answer in part C (i) based on the relationship between the change in the output values of f and the change in the input values of f. Refer to the values in the table in your reasoning.

(i) An exponential function best models f.	Answer	**Point C1**
(ii) In this case, the input-value intervals all have length 1. Examining the ratios of successive output values gives $\frac{f(-1)}{f(-2)} = \frac{f(0)}{f(-1)} = \frac{f(1)}{f(0)} = \frac{f(2)}{f(1)} = 0.5$. Because the successive output values over equal-length input-value intervals are proportional, an exponential model is best.	Reasoning	**Point C2**

Scoring Notes for Part C

- **Point C1** is earned for a correct function model with no incorrect function models listed. A response of "exponential" earns **Point C1**.
- Both **Point C1** and **Point C2** may be earned in part C (ii) provided there is no incorrect response in part C (i).
- **Point C2** requires an implicit or explicit reference to output values and input values from the table as support for the reason. For example, $\frac{f(n+1)}{f(n)} = 0.5$ OR "successive output values are decreasing proportionately by a factor of 0.5." The reasoning must demonstrate that the ratio of 0.5 applies to more than one pair of successive output values.
- A reason that references "exponential regression," "r values," OR "r^2 values" is not sufficient to earn **Point C2**.
- Special case: A response that indicates that f is best modeled by a **linear**, **quadratic**, or **logarithmic** function in part C (i) without a reason in part C (i) <u>combined</u> with a response in part C (ii) that provides both the correct answer <u>and</u> a correct reason is scored **0** for **Point C1** and **1** for **Point C2**.

Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1

Answer QUESTION 1 PART A on this page.

PART A

i

$f(1) = 1.75$

$h(1) = g(f(1))$

$g(1.75)$

$-0.167(1.75)^3 + (1.75)^2 - 1.834$

$= 0.3334944$

$= 0.333$

ii

$f^{-1}(3.5) = 0$

Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1

Answer QUESTION 1 PARTS B and C on this page.

PART B

i $-0.167x^3 + x^3 - 1.834 = 0$

$x = -1.233207$

$x = 1.5780618$

$x = 5.6431687$

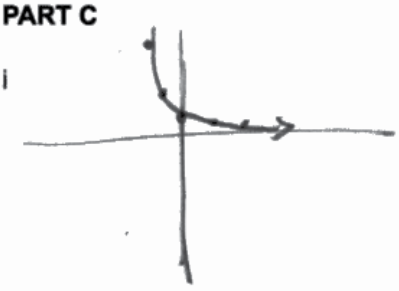
Zeroes are at $(-1.233, 0)$ $(1.578, 0)$, $(5.643, 0)$

ii

$\lim_{x \to \infty} g(x) = -\infty$

PART C

i



Exponential.

ii The function f is exponential because over equal intervals of input values, the output values change proportionally. For example, over an input of (0, 1), where the input changes by 1, the output value is proportional & over equal input intervals halving is constantly done

Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1

Answer QUESTION 1 PART A on this page.

PART A

i

$h(x) = g(f(x))$ ~~$h(1)$~~

$f(1) = 1.75$

$g(1.75) = -0.167(1.75)^3 + (1.75)^2 - 1.834$

$h(1) = 0.333$

ii

$f^{-1}(3.5) = 0$

Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1

Answer QUESTION 1 PARTS B and C on this page.

PART B

i $g(x) = -0.167x^3 + x^2 - 1.834 = 0$

$x = -1.212$ and $x = 1.666$ and $x = 5.681$

ii $\lim_{x \to -\infty} g(x) = \infty$

$\lim_{x \to \infty} g(x) = -\infty$

PART C

i exponential

ii The function f(x) is an exponential regression because the input values increase at a constant while the output values decrease proportionally by a factor of .50.

Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1

Answer QUESTION 1 PART A on this page.

h(11

PART A

i $g(f(x)) = -0.167(1.75)^3 + (1.75)^2 - 1.834)$

The answer is approximately

0.3335

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ii

According to the graph the function equation for f(x) would be $y = 3.5(\frac{1}{2})^x$.

The inverse function would be $\log_{\frac{1}{2}} 3.5x - y$

$f^{-1}(3.5) = -3.6147$

$x = \frac{3.5(\frac{1}{2})^y}{3.5}$

$3.5x = (\frac{1}{2})^y$

$\log_{\frac{1}{2}} 3.5x = y$

Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1 Q1

Answer QUESTION 1 PARTS B and C on this page.

PART B

i $g(x)$ equals zero at approximately

$x = -1.23, 1.57, 5.64$

ii

$\lim_{x \to -\infty} g(x) = \infty$, $\lim_{x \to \infty} g(x) = -\infty$

PART C

i This function best models an exponential function,

ii This best models an exponential function because as the input values increase at a constant rate, the output values decrease exponentially at a factor of 2.

Question 1: Function Concepts

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

NEW for 2025: The question overviews can be found in the *Chief Reader Report on Student Responses* on AP Central.

Sample: 1A
Score: 6

Point A1: 1

The response earned the point with 0.333 in line 5 with supporting work provided. Answers must be accurate, rounded or truncated, to the third place after the decimal point. The presence of $g(f(1))$ in line 1 is sufficient supporting work.

Point A2: 1

The response earned the point with the statement "$f^{-1}(3.5) = 0$."

Point B1: 1

The response earned the point with the three correct decimal approximations for x.

Point B2: 1

The response earned the point by presenting the correct limit notation that expresses the correct end behavior.

Point C1: 1

The response earned the point with the correct answer: "exponential."

Point C2: 1

The response earned the point with "where the input changes by 1, the output value is proportional & haves."

Question 1: Function Concepts (continued)

Sample: 1B
Score: 5

Point A1: 1

The response earned the point with 0.333 in line 4 with supporting work provided. Answers must be accurate, rounded or truncated, to the third place after the decimal point. The presence of $f(1)$ in line 2 is sufficient supporting work.

Point A2: 1

The response earned the point with statement "$f^{-1}(3.5) = 0$."

Point B1: 0

The response did not earn the point because the correct values are not presented.

Point B2: 1

The response earned the point by presenting the correct limit notation that expresses the correct end behavior. The additional presented limit statement is correct.

Point C1: 1

The response earned the point with the correct answer: "exponential."

Point C2: 1

The response earned the point with "the input values increase at a constant while the output values decrease proportionally by a factor of .50."

Question 1: Function Concepts (continued)

Sample: 1C
Score: 3

Point A1: 1

The response earned the point with 0.3335 in line 4 with supporting work provided. Answers must be accurate, rounded or truncated, to the third place after the decimal point. Only the first three decimal places after the decimal point are read for credit. The presence of 1.75 substituted in $g(x)$ for x in line 2 is sufficient supporting work.

Point A2: 0

The response did not earn the point. The value -3.6147 is incorrect.

Point B1: 0

The response did not earn the point because although the values presented are accurate, they are given only to two decimal places. This is a decimal presentation error.

Point B2: 1

The response earned the point by presenting the correct limit notation that expresses the correct end behavior. The additional presented limit statement is correct.

Point C1: 1

The response earned the point with the correct answer: "exponential."

Point C2: 0

The response did not earn the point. The declaration of "decrease exponentially at a factor of 2" in lines 4–5 is incorrect.