2025

AP® Calculus BC

Sample Student Responses and Scoring Commentary

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Free-Response Question 5

☑ Scoring Guidelines

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Part B (BC): Graphing calculator not allowed
Question 5 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = (3 - x)y^2$ with initial condition $f(1) = -1$.

	Model Solution	Scoring	
A	Find $f''(1)$, the value of $\frac{d^2y}{dx^2}$ at the point $(1, -1)$. Show the work that leads to your answer.		
	$\frac{d^2y}{dx^2} = -y^2 + (3 - x)2y\frac{dy}{dx}$	Product rule	**Point 1 (P1)**
		Chain rule	**Point 2 (P2)**
	$f'(1) = \left.\frac{dy}{dx}\right\rvert_{(x,y)=(1,\,-1)} = (3 - 1)(-1)^2 = 2$ $f''(1) = \left.\frac{d^2y}{dx^2}\right\rvert_{(x,y)=(1,\,-1)} = -(-1)^2 + (3 - 1)(2)(-1)(2) = -9$	$f''(1)$	**Point 3 (P3)**

Scoring Notes for Part A

- The expression $\frac{d^2y}{dx^2} = -y^2 + (3-x)2y$ or $\frac{d^2y}{dx^2} = 6y - y^2 - 2xy$ earns **P1** but not **P2**. Such a response is not eligible for **P3**.
- The expression $\frac{d^2y}{dx^2} = -2y\frac{dy}{dx}$ or $\frac{d^2y}{dx^2} = 6y\left(\frac{dy}{dx}\right) - 2y\left(\frac{dy}{dx}\right)$ earns **P2** but not **P1**. Such a response is eligible for **P3** for a consistent answer of $f''(1) = 4$ or $f''(1) = -8$, respectively, which is found by correctly substituting correct values for x, y, and $\frac{dy}{dx}$.
- A response of $-(-1)^2 + (3-1)(2)(-1)(2)$ earns **P1**, **P2**, and **P3** regardless of any subsequent errors in simplification.
- Alternate approach (using separation of variables):

  The particular solution for the differential equation that passes through the point $(1,-1)$ is $y = \frac{2}{x^2 - 6x + 3}$. Therefore, $\frac{dy}{dx} = \frac{-2(2x-6)}{\left(x^2 - 6x + 3\right)^2}$.

  This response has not yet earned **P1**, **P2**, or **P3**.
  - A response that correctly applies the quotient rule (or product rule) and the chain rule to find that $\frac{d^2y}{dx^2} = \frac{-4\left(x^2-6x+3\right)^2 + 4(2x-6)\left(x^2-6x+3\right)(2x-6)}{\left(x^2-6x+3\right)^4}$ earns **P1** and **P2** and is eligible to earn **P3** for the correct answer of $f''(1) = -9$.
  - A response that correctly applies the quotient rule (or product rule) but does not correctly apply the chain rule (e.g., $\frac{d^2y}{dx^2} = \frac{-4\left(x^2-6x+3\right)^2 + 4(2x-6)\left(x^2-6x+3\right)}{\left(x^2-6x+3\right)^4}$) earns **P1**, does not earn **P2**, and is not eligible to earn **P3**.
  - A response that does not correctly apply the quotient rule (or product rule) but does correctly apply the chain rule (e.g., $\frac{d^2y}{dx^2} = \frac{-4}{2\left(x^2-6x+3\right)(2x-6)}$) does not earn **P1**, earns **P2**, and is eligible to earn **P3** for a consistent answer.

B	Write the second-degree Taylor polynomial for f about $x = 1$.		
	$f'(1) = 2$ and $f''(1) = -9$ $P_2(x) = -1 + 2(x-1) - \frac{9}{2}(x-1)^2$	Two terms	**Point 4 (P4)**
		Remaining term	**Point 5 (P5)**

Scoring Notes for Part B

- **P4** and **P5** can be earned with an answer consistent with incorrect values of $f'(1)$ and $f''(1)$ imported from part A.
- Any terms of degree greater than two or "$+\cdots$" does not earn **P5**.
- A response of $-1 + 2(x-1) - \frac{9}{2}(x-1)^2$ earns **P4** and **P5**, regardless of any subsequent algebraic simplification.
- A response that does not present the polynomial as powers of $(x-1)$ but instead presents a correct expanded/simplified form of the polynomial (e.g., $-\frac{9}{2}x^2 + 11x - \frac{15}{2}$) earns **P4** but not **P5**.

C	The second-degree Taylor polynomial for f about $x = 1$ is used to approximate $f(1.1)$. Given that $\lvert f'''(x)\rvert \le 60$ for all x in the interval $1 \le x \le 1.1$, use the Lagrange error bound to show that this approximation differs from $f(1.1)$ by at most 0.01.		
	$\lvert f(1.1) - P_2(1.1)\rvert \le \dfrac{\max\limits_{1\le x\le 1.1}\lvert f'''(x)\rvert}{3!}\lvert 1.1-1\rvert^3 \le \dfrac{60}{6}(0.1)^3 = 0.01$	Form of error bound	**Point 6 (P6)**
		Analysis	**Point 7 (P7)**

Scoring Notes for Part C

- **P6** is earned for presenting either $\dfrac{\max\limits_{1\le x\le 1.1}\lvert f'''(x)\rvert}{3!}\lvert 1.1-1\rvert^3$ or $\dfrac{60}{6}(0.1)^3$. Subsequent errors in simplification will not earn **P7**.
- To earn **P7**, a response must have earned **P6** and must explicitly connect the error bound with 0.01; for example by communicating $\text{Error} \le 0.01$, $\text{Error Bound} = 0.01$, or equivalent.
- A response that declares the error is equal to 0.01 (or any equivalent form of this value) does not earn **P7**.

D	Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the work that leads to your answer.		
	$f(1.2) \approx f(1) + (1.2 - 1) \cdot \left.\frac{dy}{dx}\right\|_{(1,\,-1)}$ $= -1 + 0.2(2) = -0.6$	First step of Euler's method	**Point 8 (P8)**
	$f(1.4) \approx f(1.2) + (1.4 - 1.2) \cdot \left.\frac{dy}{dx}\right\|_{(1.2,\,-0.6)}$ $\approx -0.6 + 0.2(3 - 1.2)(-0.6)^2 = -0.4704$ An approximation of $f(1.4)$ is -0.47.	Answer with supporting work	**Point 9 (P9)**

Scoring Notes for Part D

- To earn **P8**, a response must demonstrate the first step of Euler's method, with the correct initial condition, correct step size, and correct (or imported) expression for the derivative.

  Note: Any subsequent error in simplification or rounding will not affect the scoring for **P8**.
- The two steps of Euler's method may be explicit expressions or may be presented in a table. For example:

x	y	$\frac{dy}{dx} \cdot \Delta x$ (or $\frac{dy}{dx} \cdot 0.2$)
1	-1	0.4
1.2	-0.6	0.1296
1.4	-0.4704	

  Note: In the presence of a correct answer, a table does not need to be labeled to earn both **P8** and **P9**. In the presence of no answer or an incorrect answer, such a table must be correctly labeled to earn **P8**.
- A response of $-0.6 + 0.2(3 - 1.2)(-0.6)^2$ earns **P9**, regardless of any subsequent errors in simplification or rounding.
- A response that imports an incorrect value for $f'(1)$ from part A or part B is eligible to earn **P9** with a consistent answer.
- A response may report the final answer as -0.47, $(1.4, -0.47)$, $-\frac{294}{625}$, or equivalent.

Q5 NO CALCULATOR ALLOWED Q5

Answer QUESTION 5 PARTS A and B on this page.

PART A

$f''(x) = \frac{d^2y}{dx^2} = \frac{d}{dx}[(3-x)y^2] = -y^2 + 2(3-x)y\frac{dy}{dx} = 2(3-x)^2y^3 - y^2$

$f''(1) = 2(2)^2(-1)^3 - (-1)^2$

PART B

$T_2(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2}(x-1)^2 = -1 + 2(x-1) - \frac{9}{2}(x-1)^2$

$f'(1) = (3-1)(-1)^2 = 2$

Q5 NO CALCULATOR ALLOWED Q5

Answer QUESTION 5 PARTS C and D on this page.

PART C

$$|f(1.1)-T_2(1.1)| \le \left|\frac{60}{3!}(1.1-1)^3\right| \le 0.01$$

PART D

$$\Delta x = \frac{1.4-1}{2} = 0.2$$

$$f(1.2) \approx -1 + 0.2\left((3-1)(-1)^2\right) = -0.6$$

$$f(1.4) \approx -0.6 + 0.2\left((3-1.2)(-0.6)^2\right)$$

Q5 NO CALCULATOR ALLOWED Q5

Answer QUESTION 5 PARTS A and B on this page.

PART A

$f''(x) = \frac{d^2y}{dx^2}$ $f'(x) = (3-x)y^2$

$f''(x) = (3-x)2y + y^2(-1)$

$\cancel{f''(1) = d^2y}$

$f''(1) = \left.\frac{d^2y}{dx^2}\right|_{(1,-1)} = (3-1)2(-1) + (-1)^2(-1)$

$= (2)(-2) - 1 = -5$

PART B

$\cancel{f'(x)}$ $\cancel{f'(1) = (3-1)}$

$\left.\frac{dy}{dx}\right|_{(1,-1)} = (3-1)(-1)^2 = 2.$

$f(x) + f'(x)x + \frac{f''(x)x^2}{2!}$ (ex.)

$-1 + 2(x-1) \cancel{+} - \frac{5(x-1)^2}{2!}$

Q5 NO CALCULATOR ALLOWED Q5

Answer QUESTION 5 PARTS C and D on this page.

PART C

Lagrange error bound *

$$|\text{Error}| \le \frac{60}{3!}|1.1-1|^3$$

PART D

$(1,-1)$	$y+1=2(x-1)$	$(1.2,-.6)$
$(1.2,-.6)$	$y+.6=\frac{81}{125}(x-1.2)$	$(1.4,$

$$\frac{dy}{dx}=(3-x)(y^2)$$

$$(3-1.2)(.6)^2$$

$$\left(\frac{\cancel{180}^{\,9}}{\cancel{100}_{\,25}}\right)\left(\frac{\cancel{36}^{\,9}}{\cancel{100}_{\,5}}\right)=\frac{81}{125}$$

1.8

$$2\left(\frac{2}{10}\right)+-1$$

$$\cancel{f(1.4)\approx\frac{81}{125}(1.4-1.2)-.6\;\frac{4}{10}-\frac{10}{10}}$$

$$\left(\frac{2}{10}\right)(2)=\frac{4}{10}-\frac{10}{10}=\frac{-6}{10}$$

$$f(1.4)\approx(3-1.2)(-.6)^2(1.4-1.2)-.6$$

$$\approx(1.8)(-.6)^2(.2)-.6$$

Q5 NO CALCULATOR ALLOWED Q5

Answer QUESTION 5 PARTS A and B on this page.

PART A

$\frac{dy}{dx} = (3-x)(y^2)$

$\int y^{-2}\,dy = \int (3-x)\,dx$

$-\frac{1}{3}y^{-3} = -1 + C$

$-\frac{1}{3y^3} = -1 + C$

$\frac{-1}{3(-1)^3} = -1 + C$

$\frac{-1}{-3} = -1 + C$

$\frac{1}{3} + 1 = C$

$C = \frac{4}{3}$

$-\frac{1}{3y^3} = -1 + \frac{4}{3}$

$\frac{1}{3y^3} = 1 - \frac{4}{3}$

$\frac{1}{y^3} = 3 - 4$

$\frac{1}{y^3} = -1$

$1 = -y^3$

$-1 = y^3 \quad y = -1$

$f''(1) = -1$

?

PART B

$f(1) = -1$

$f'(1) =$

$f''(1) = -1$

$\frac{f'(x)(x-c)^n}{n!}$

Q5 NO CALCULATOR ALLOWED Q5

Answer QUESTION 5 PARTS C and D on this page.

PART C

$$\frac{f(z)(x-c)^{n+1}}{(n+1)!} \to \frac{(60)(0.1)^3}{3!} \le 0.01$$

$$\frac{\cancel{6}0(0.001)}{\cancel{6}}$$

$$10(0.001) \le 0.01$$

$$0.01 \le 0.01$$

0.1
0.1
01
000
001
0.1
001
0000
00.001

PART D $\Delta x = 0.2$ $\frac{dy}{dx} = (3-x)(y^2)(dx)$

$(1,-1)$ $(3-1)((-1)^2)(0.2) = (2)(0.2) = 0.4$

$(1.2, -0.6)$ $(3-1.2)((-0.6)^2)(0.2) = (1.8)(0.36)(0.2)$

$(1.4, -0.470)$

$f(1.4) \approx -0.470$

3
0.6
0.6
.36
000

1.8
0.2
.36
08

0.36
0.36
216
1080
.1296

0.6000
.1296
4704

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5525/13

Question 5

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

NEW for 2025: The question overviews can be found in the *Chief Reader Report on Student Responses* on AP Central.

Sample: 5A
Score: 9 (1-1-1-1-1-1-1-1-1)

The response earned 9 points: 3 points in part A, 2 points in part B, 2 points in part C, and 2 points in part D.

In part A the response earned **P1** with the correct application of the product rule in the expression $-y^2 + 2(3-x)y\frac{dy}{dx}$ in line 1. The response earned **P2** with the correct application of the chain rule in the expression $-y^2 + 2(3-x)y\frac{dy}{dx}$ in line 1. The response earned **P3** with the equation $f''(1) = 2(2)^2(-1)^3 - (-1)^2$ in line 2.

In part B the response earned **P4** with the two correct terms, -1 and $2(x-1)$, of the Taylor polynomial in line 1. The response earned **P5** with the third correct term, $-\frac{9}{2}(x-1)^2$, in line 1.

In part C the response earned **P6** with the expression $\left|\frac{60}{3!}(1.1-1)^3\right|$ in line 1. The response earned **P7** by connecting the expression for error, $|f(1.1) - T_2(1.1)|$, and 0.01 to the value of the error bound with a correct inequality.

In part D the response earned **P8**. The first step of Euler's method is shown on line 2 with the expression $f(1.2) \approx -1 + 0.2\left((3-1)(-1)^2\right)$. The response earned **P9** with the expression $f(1.4) \approx -0.6 + 0.2\left((3-1.2)(-0.6)^2\right)$ on line 3.

Sample: 5B
Score: 6 (1-0-0-1-1-1-0-1-1)

The response earned 6 points: 1 point in part A, 2 points in part B, 1 point in part C, and 2 points in part D.

In part A the response earned **P1** with the equation $f''(x) = (3-x)2y + y^2(-1)$ in line 2. The response did not earn **P2**. The response does not present the chain rule factor of $\frac{dy}{dx}$ with the first term in the product rule. The response did not earn **P3**. The response is not eligible for this point because **P2** is not earned.

In part B the response earned **P4** with the two correct terms, -1 and $2(x-1)$, of the Taylor polynomial in line 4. The response earned **P5** with the term $\frac{-5(x-1)^2}{2!}$, which is consistent with the value of $f''(1)$ imported from part A.

Question 5 (continued)

In part C the response earned **P6** with the expression $\frac{60}{3!}|1.1-1|^3$. The response did not earn **P7** because there is no explicit connection to 0.01.

In part D the response earned **P8**. The first step of Euler's method is shown on line 1 of the table with the presentation of the correct local linearization $y+1=2(x-1)$ and its evaluation to produce the point $(1.2, -.6)$. The response earned **P9** with the expression $f(1.4) \approx (3-1.2)(-.6)^2(1.4-1.2)-.6$ in the second to last line.

Sample: 5C
Score: 3 (0-0-0-0-0-1-0-1-1)

The response earned 3 points: 0 points in part A, 0 points in part B, 1 point in part C, and 2 points in part D.

In part A the response did not earn **P1**, **P2**, or **P3**. In an attempt at the alternate solution using separation of variables, the response produces an incorrect solution for the function y and is not eligible for any points in part A.

In part B the response did not earn **P4** or **P5** because no terms of the Taylor polynomial are presented.

In part C the response earned **P6** with the expression $\frac{(60)(0.1)^3}{3!}$ in the middle of line 1. The response did not earn **P7** because there is no explicit connection to error or error bound.

In part D the response earned **P8**. The first step of Euler's method is started on line 2 with the presentation of the correct initial value, value of the derivative, and correct step size. It is completed with the presentation of the point $(1.2, -0.6)$ on line 3. The response earned **P9** with the correct work on line 3 and the presentation of the answer $f(1.4) \approx -0.470$ on line 5.