AP® Calculus AB

Sample Student Responses and Scoring Commentary

Inside:

Free-Response Question 6

☑ **Scoring Guidelines**

☑ **Student Samples**

☑ **Scoring Commentary**

Part B (AB): Graphing calculator not allowed
Question 6   9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.

- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Consider the curve G defined by the equation $y^3 - y^2 - y + \frac{1}{4}x^2 = 0$.

Model Solution	Scoring

A Show that $\dfrac{dy}{dx} = \dfrac{-x}{2(3y^2 - 2y - 1)}$.

$\dfrac{d}{dx}\left(y^3 - y^2 - y + \dfrac{1}{4}x^2\right) = \dfrac{d}{dx}(0)$ $\Rightarrow\ 3y^2 \cdot \dfrac{dy}{dx} - 2y \cdot \dfrac{dy}{dx} - \dfrac{dy}{dx} + \dfrac{x}{2} = 0$	Implicit differentiation **Point 1 (P1)**
$\Rightarrow\ (3y^2 - 2y - 1)\dfrac{dy}{dx} = -\dfrac{x}{2}$ $\Rightarrow\ \dfrac{dy}{dx} = \dfrac{-x}{2(3y^2 - 2y - 1)}$	Verification **Point 2 (P2)**

Scoring Notes for Part A

- **P1** is earned only for the correct implicit differentiation of $y^3 - y^2 - y + \frac{1}{4}x^2 = 0$. Responses may use alternative notations for $\dfrac{dy}{dx}$, such as y'.

- To be eligible for **P2**, a response must have earned **P1**.

- It is sufficient to present $(3y^2 - 2y - 1)\dfrac{dy}{dx} = -\dfrac{x}{2}$ to earn **P2**, provided there are no subsequent errors.

B   There is a point P on the curve G near $(2, -1)$ with x-coordinate 1.6. Use the line tangent to the curve at $(2, -1)$ to approximate the y-coordinate of point P.

$\left.\dfrac{dy}{dx}\right|_{(x,\,y)=(2,\,-1)} = \dfrac{-2}{2(3+2-1)} = -\dfrac{1}{4}$	Slope of tangent line	**Point 3 (P3)**
$y \approx -1 - \dfrac{1}{4}(1.6 - 2) = -0.9$	Tangent line approximation	**Point 4 (P4)**

Scoring Notes for Part B

- A response can earn **P3** with $\left.\dfrac{dy}{dx}\right|_{(x,\,y)=(2,\,-1)} = -\dfrac{1}{4}$, $\dfrac{dy}{dx} = -\dfrac{1}{4}$, "slope is $-\dfrac{1}{4}$," or equivalent.

- A response that presents a linear approximation with a slope of $-\dfrac{1}{4}$ also earns **P3**.

- A response that declares $\left.\dfrac{dy}{dx}\right|_{(x,\,y)=(2,\,-1)}$ (or the slope) equal to any nonzero value $k \neq -\dfrac{1}{4}$ does not earn **P3**. Such a response earns **P4** for a presented approximation mathematically equivalent to $-1 + k(-0.4)$.

- **P4** cannot be earned with a linear approximation using a slope other than $-\dfrac{1}{4}$ if that slope has not been declared to be the value of $\left.\dfrac{dy}{dx}\right|_{(x,\,y)=(2,\,-1)}$.

- A response does not have to present the tangent line equation but must clearly demonstrate its use at $x = 1.6$ in finding the requested approximation to be eligible for **P4**.

- A response of $-1 - \dfrac{1}{4}(-0.4)$ earns both **P3** and **P4**.

- A response of $-1 - \dfrac{1}{4}(1.6 - 2)$ or equivalent banks **P4** (i.e., subsequent errors in simplification will not be considered in scoring for **P4**).

  Note: An ambiguous response, such as $-1 - \dfrac{1}{4}(1.6 - 2$, does not bank **P4** and therefore must go on to resolve the ambiguity with a correct final answer (e.g., -0.9) to earn **P4**.

C For $x > 0$ and $y > 0,$ there is a point S on the curve G at which the line tangent to the curve at that point is vertical. Find the y-coordinate of point $S.$ Show the work that leads to your answer.

For $x > 0,$ the curve G has a vertical tangent line when $2(3y^2 - 2y - 1) = 0$.	Sets denominator equal to 0	**Point 5 (P5)**
$2(3y^2 - 2y - 1) = 0 \implies 2(3y + 1)(y - 1) = 0$ Because $y > 0$, it follows that $y = 1$. The line tangent to the curve is vertical at the point on the curve where $y = 1$.	Answer	**Point 6 (P6)**

Scoring Notes for Part C

- **P5** is earned with any of $2(3y^2 - 2y - 1) = 0$, $3y^2 - 2y - 1 = 0$, $2(3y \pm 1)(y \pm 1) = 0$, or $(3y \pm 1)(y \pm 1) = 0$.

- To be eligible for **P6**, a response must have earned **P5**.

- A response does not need to consider the numerator of $\dfrac{dy}{dx} = \dfrac{-x}{2(3y^2 - 2y - 1)}$ to earn **P5** or **P6**; considering the denominator is sufficient.

- A response that states solutions of $y = -\dfrac{1}{3}$ and $y = 1$, but does not clearly identify that $y = 1$ is the only solution to the prompt, does not earn **P6**.

- In the presence of algebraic work to find the value of y, **P6** is earned only if the algebraic work is correct.

- A response of "$2(3y^2 - 2y - 1) = 0$, $y = 1$" earns both **P5** and **P6**.

D A particle moves along the curve H defined by the equation $2xy + \ln y = 8$. At the instant when the particle is at the point $(4, 1)$, $\dfrac{dx}{dt} = 3$. Find $\dfrac{dy}{dt}$ at that instant. Show the work that leads to your answer.

$\dfrac{d}{dt}(2xy + \ln y) = \dfrac{d}{dt}(8)$ $2\dfrac{dx}{dt}y + 2x\dfrac{dy}{dt} + \dfrac{1}{y}\dfrac{dy}{dt} = 0$	Attempts implicit differentiation with respect to t	**Point 7 (P7)**
	$2\dfrac{dx}{dt}y + 2x\dfrac{dy}{dt} + \dfrac{1}{y}\dfrac{dy}{dt} = 0$	**Point 8 (P8)**
$2(3)(1) + 2(4)\dfrac{dy}{dt} + \dfrac{1}{1}\dfrac{dy}{dt} = 0$ $\Rightarrow \; 6 + 9\dfrac{dy}{dt} = 0 \;\Rightarrow\; \dfrac{dy}{dt} = -\dfrac{2}{3}$	Answer	**Point 9 (P9)**

Scoring Notes for Part D

- **P7** is earned for implicitly differentiating $2xy + \ln y = 8$ with respect to t with at most one error.

- **P8** is earned for an equation equivalent to $2\dfrac{dx}{dt}y + 2x\dfrac{dy}{dt} + \dfrac{1}{y}\dfrac{dy}{dt} = 0$.

- To be eligible for **P9**, a response must have earned **P7** and **P8**, with no errors in implicit differentiation.

- **P9** is earned only for the value of $-\dfrac{2}{3}$.

- Alternate solution:

  $$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

  $$\frac{d}{dx}(2xy + \ln y) = \frac{d}{dx}(8) \;\Rightarrow\; 2y + 2x\frac{dy}{dx} + \frac{1}{y}\frac{dy}{dx} = 0 \;\Rightarrow\; \frac{dy}{dx} = \frac{-2y}{2x + \dfrac{1}{y}}$$

  $$\left.\frac{dy}{dx}\right|_{(x,\,y)=(4,\,1)} = \frac{-2(1)}{2(4) + \dfrac{1}{1}} = -\frac{2}{9}$$

  $$\left.\frac{dy}{dt}\right|_{(x,\,y)=(4,\,1)} = \left.\frac{dy}{dx} \cdot \frac{dx}{dt}\right|_{(x,\,y)=(4,\,1)} = -\frac{2}{9} \cdot 3 = -\frac{2}{3}$$

  - **P7** is earned for an implicit differentiation with respect to x with at most one error, as long as $\dfrac{dy}{dx}$ is eventually correctly linked to $\dfrac{dx}{dt}$.

  - **P8** is earned for finding $\dfrac{dy}{dx}$ and multiplying the result by $\dfrac{dx}{dt} = 3$.

  - **P8** can be earned for a stated incorrect $\dfrac{dy}{dx}$, as long as it is multiplied by 3.

  - **P9** is only earned for a correct value of $-\dfrac{2}{3}$ or equivalent.

  - Stating $\dfrac{dy}{dt} = \dfrac{dy}{dx} \cdot \dfrac{dx}{dt}$ alone does not earn any points.

Answer QUESTION 6 PARTS A and B on this page.

PART A

$$y^3 - y^2 - y + \frac{1}{4}x^2 = 0$$

$$y^3 - y^2 - y = -\frac{1}{4}x^2$$

$$3y^2\frac{dy}{dx} - 2y\frac{dy}{dx} - \frac{dy}{dx} = -\frac{1}{2}x$$

$$\frac{dy}{dx}\left(3y^2 - 2y - 1\right) = -\frac{1}{2}x$$

$$\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)} \checkmark$$

PART B

$$\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)} = \frac{-2}{2(3(-1)^2 - 2(-1) - 1)} = \frac{-2}{2(3+2-1)} = \frac{-2}{8} = \frac{-1}{4}$$

$$y - -1 = -\frac{1}{4}(x - 2)$$

$$y + 1 = -\frac{1}{4}x + \frac{1}{2}$$

$$y = -\frac{1}{4}x - \frac{1}{2}$$

$$y \approx \frac{-1}{4}\left(\frac{16}{10}\right) - \frac{1}{2}$$

$$y \approx \frac{-4}{10} - \frac{1}{2}$$

$$\approx \frac{-4}{10} - \frac{5}{10} = \boxed{\frac{-9}{10}} \text{ or } \boxed{-0.9}$$

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Q5522/14

Q6 ⊠ **NO CALCULATOR ALLOWED** **Q6**

Answer QUESTION 6 PARTS C and D on this page.

PART C

$2(3y^2 - 2y - 1) = 0$

$3y^2 - 2y - 1 = 0$

$3y^2 - 3y + y - 1 = 0$

$3y(y-1) + 1(y-1) = 0$

$(3y+1)(y-1) = 0$

$3y+1 = 0 \qquad y-1 = 0$

$y = \dfrac{-1}{3} \qquad\quad y = 1$

The y-coordinate is $1 \rightarrow \boxed{y = 1}$

PART D

$(2x)\left(\dfrac{dy}{dt}\right) + \left(2\dfrac{dx}{dt}\right)(y) + \dfrac{1}{y}\dfrac{dy}{dt} = 0$

$2(4)\dfrac{dy}{dt} + 2(3)(1) + \dfrac{1}{1}\dfrac{dy}{dt} = 0$

$8\dfrac{dy}{dt} + 6 + \dfrac{dy}{dt} = 0$

$8\dfrac{dy}{dt} + \dfrac{dy}{dt} = -6$

$\dfrac{dy}{dt}(8+1) = -6$

$\dfrac{dy}{dt} = \dfrac{-6}{9} = \boxed{\dfrac{-2}{3}}$

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Q5522/15

Answer QUESTION 6 PARTS A and B on this page.

PART A

$$y^3 - y^2 - y + \frac{1}{4}x^2 = 0$$

$$3y^2 \frac{dy}{dx} - 2y \frac{dy}{dx} - \frac{dy}{dx} + \frac{1}{2}x = 0$$

$$(3y^2 - 2y - 1)\frac{dy}{dx} = -\frac{x}{2}$$

$$\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}$$

PART B

$$y + 1 = \frac{-1}{4}(x - 2)$$

$$\frac{dy}{dx} = \frac{-2}{2(3(-1)^2 - 2(-1) - 1)} = \frac{-2}{2(3 + 2 - 1)} \quad -\frac{2}{8} = -\frac{1}{4}$$

$$-\frac{1}{4}x + \frac{1}{2} - \frac{2}{2} = \left(-\frac{1}{4}x - \frac{1}{2}\right)4$$

$$-1.6 - 2 = \boxed{-3.6}$$

$$-x - 2 =$$

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Answer QUESTION 6 PARTS C and D on this page.

PART C

$$\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)} \times \frac{1}{0}$$

$$2(3y^2 - 2y - 1) = 0$$

$$6y^2 - 4y - 2 = 0$$

$$2(3y^2 - 2y - 1)$$

$$3y^2 - 3y + y - 1$$

$$3y(y-1) + 1(y-1)$$

$$3y + 1 \qquad y - 1$$

$$\bcancel{y = \frac{-1}{3}} \qquad \boxed{y = 1}$$

PART D

$$2xy + \ln y = 8 \qquad\qquad \begin{array}{l} y = 1 \\ x = 4 \end{array} \quad \frac{dx}{dt} = 3$$

$$2x\frac{dy}{dt} \cdot y\, 2\frac{dx}{dt} + \frac{1}{y}\frac{dy}{dt} = 0$$

$$\frac{dy}{dt}\left(2x + \frac{1}{y}\right) \cdot 2(1)(3)$$

$$\frac{dy}{dt}\left(2(4)+1\right) \cdot 2(1)(3)$$

$$\frac{dy}{dx} = \boxed{\frac{1}{54}}$$

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Q6   ⊠ NO CALCULATOR ALLOWED   Q6

Answer QUESTION 6 PARTS A and B on this page.

PART A

$$3y^2\frac{dy}{dx} - 2y\frac{dy}{dx} - \frac{dy}{dx} + \frac{1}{2}x = 0$$

$$\frac{dy}{dx}\frac{(3y^2 - 2y - 1)}{3y^2 - 2y - 1} = \frac{-\frac{1}{2}x}{3y^2 - 2y - 1}$$

$$\frac{dy}{dx} = \frac{-x}{2(3y^2 - 2y - 1)}$$

PART B

$y - y_1 = m(x - x_1)$

(x, y)

$(2, -1)$ $y + 1 = m(x - 2)$

$y + 1 = 8(x - 2)$

$y = 8(x - 2) + 1$

$8(1.6 - 2) + 1$

$8(-.4) + 1$

$-3.2 + 1$

$= -2.2$

$-(3+1)^2 - (-1) + \frac{1}{4}(2)^2$

$-1 - 1 + 1 + \frac{1}{4}(4)$

$-1 + \frac{1}{4}(4)$

$-1 +$

$\frac{-2}{2(3+1)^2 - 2(-1) - 1}$

$(3 + 2 - 1)$

$2(4) = 8$

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Answer QUESTION 6 PARTS C and D on this page.

PART C

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$$2(3y^2 - 2y - 1) = 0$$

$$6y^2 - 4y - 2 = 0$$

$$6y^2 - 4y = 2 \qquad \text{point } s = \frac{2}{3}$$

$$2y(3y - 2) = 0$$

PART D

$$2y + 2x\frac{dy}{dx} + e^y\frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{2x + e^4}{2x + e^4} \qquad = \frac{-2y}{2x + e^4}$$

$$\frac{dy}{dt}\bigg|_{(4,1)} \quad \frac{-2(1)}{2(4) + e^0} = \frac{-2}{8} = -\frac{1}{4}$$

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Question 6

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

NEW for 2025: The question overviews can be found in the *Chief Reader Report on Student Responses* on [AP Central](#).

Sample: 6A
Score: 9 (1-1-1-1-1-1-1-1-1)

The response earned 9 points: 2 points in part A, 2 points in part B, 2 points in part C, and 3 points in part D.

In part A the response earned **P1** in the third line with a correct implicit differentiation equation. The response earned **P2** with appropriate work in line 4 leading to the correct equation for $\frac{dy}{dx}$ in line 5.

In part B the response earned **P3** with correct simplification of $\frac{-1}{4}$ at the end of line 1. The response would have earned the point in line 1 with $\dfrac{-2}{2\left(3(-1)^2 - 2(-1) - 1\right)}$ without simplification. The response earned **P4** in line 2 on the right with the expression $y \approx \frac{-1}{4}\left(\frac{16}{10}\right) - \frac{1}{2}$. All further simplification presented is correct but unnecessary.

In part C the response earned **P5** with the equation $2\left(3y^2 - 2y - 1\right) = 0$ in line 1 on the left. The response earned **P6** with algebra in lines 2 through 7 on the left and the correct value of y in line 1 on the right.

In part D the response earned **P7** in line 1 with the implicit differentiation with respect to t with at most one error. The response earned **P8** in line 1 because the implicit differentiation contains no errors. The response earned **P9** with the correct value of $\frac{-2}{3}$. The response would have earned the point with the unsimplified form $\frac{dy}{dt} = \frac{-6}{9}$.

Sample: 6B
Score: 6 (1-1-1-0-1-1-1-0-0)

The response earned 6 points: 2 points in part A, 1 point in part B, 2 points in part C, and 1 point in part D.

In part A the response earned **P1** with the correct implicit differentiation in line 2. The response earned **P2** with appropriate work in line 3 leading to the correct equation in line 4.

In part B the response earned **P3** in line 1 with $y + 1 = -\frac{1}{4}(x - 2)$, which is a linear approximation with a slope of $-\frac{1}{4}$. The response could also have earned the point with the correct declaration of $\frac{dy}{dx}$ in line 2. The response did not earn **P4** because the correct answer is not given.

In part C the response earned **P5** with the equation $2\left(3y^2 - 2y - 1\right) = 0$ in line 1 on the right. The response earned **P6** with subsequent algebra in lines 2 through 6 and with the circled $y = 1$ in line 7 on the right.

Question 6 (continued)

In part D the response earned **P7** with $2x\dfrac{dy}{dt} \cdot y2\dfrac{dx}{dt} + \dfrac{1}{y}\dfrac{dy}{dt} = 0$, which is an attempt at implicit differentiation with respect to t with at most one error. The response did not earn **P8** because the implicit differentiation has one error. The multiplication indicated with the " · " should have been addition. The response did not earn **P9** because it did not earn both **P7** and **P8** and is therefore ineligible for **P9**.

Sample: 6C
Score: 3 (1-1-0-0-1-0-0-0-0)

The response earned 3 points: 2 points in part A, 0 points in part B, 1 point in part C, and 0 points in part D.

In part A the response earned **P1** in the first line with correct implicit differentiation. The response earned **P2** with appropriate work in line 2 leading to the correct equation in line 3.

In part B the response did not earn **P3** because the slope used in the linear approximation is not $\dfrac{-1}{4}$. The response did not earn **P4** because the linear approximation in line 5 on the left, $8(1.6 - 2) + 1$, is not of the form $-1 + k(-0.4)$ where k is a slope other than $-\dfrac{1}{4}$ and connected to a declared value of $\dfrac{dy}{dx}$ at the given point.

In part C the response earned **P5** in the first algebraic line with the equation $2(3y^2 - 2y - 1) = 0$. The response did not earn **P6** because the correct answer is not given.

In part D the response did not earn **P7** because the presented implicit differentiation expression in line 1 has more than one error. The response did not earn **P8** because the presented implicit differentiation expression in line 1 has more than one error. The response did not earn **P9** because it did not earn **P7** and **P8** and is therefore ineligible for **P9**.