2025

AP® Calculus AB

Sample Student Responses and Scoring Commentary

Inside:

Free-Response Question 5

- ☑ **Scoring Guidelines**
- ☑ **Student Samples**
- ☑ **Scoring Commentary**

Part B (AB): Graphing calculator not allowed
Question 5 — 9 points

General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Two particles, H and J, are moving along the x-axis. For $0 \le t \le 5$, the position of particle H at time t is given by $x_H(t) = e^{t^2-4t}$ and the velocity of particle J at time t is given by $v_J(t) = 2t\left(t^2-1\right)^3$.

	Model Solution	Scoring	
A	Find the velocity of particle H at time $t = 1$. Show the work that leads to your answer.		
	$x_H'(t) = v_H(t) = (2t-4)e^{t^2-4t}$	Considers x_H'	**Point 1 (P1)**
	$x_H'(1) = v_H(1) = -2e^{-3}$	Answer	**Point 2 (P2)**

Scoring Notes for Part A

- **P1** can be earned by presenting $x_H'(t)$, $x_H'(1)$, $x'(t)$, $x'(1)$, $(2t-4)e^{t^2-4t}$, or $(2\cdot 1-4)e^{1^2-4\cdot 1}$.
- An unsupported answer of $-2e^{-3}$ earns **P2** but not **P1**.

B During what open intervals of time t, for $0 < t < 5$, are particles H and J moving in opposite directions? Give a reason for your answer.

From part A, $x_H'(t) = v_H(t) = (2t - 4)e^{t^2-4t}$.

$x_H'(t) = (2t - 4)e^{t^2-4t} = 0 \Rightarrow t = 2$

$x_H'(t) < 0$ for $0 < t < 2$, and $x_H'(t) > 0$ for $2 < t < 5$.

Thus, particle H is moving to the left for $0 < t < 2$ and moving to the right for $2 < t < 5$.

$v_J(t) = 2t\left(t^2 - 1\right)^3 = 0$ for $0 < t < 5 \Rightarrow t = 1$

$v_J(t) < 0$ for $0 < t < 1$, and $v_J(t) > 0$ for $1 < t < 5$.

Thus, particle J is moving to the left for $0 < t < 1$ and moving to the right for $1 < t < 5$.

Therefore, particles H and J are moving in opposite directions for $1 < t < 2$.

Considers sign of $x_H'(t)$ or $v_J(t)$	**Point 3 (P3)**
Analysis for one particle	**Point 4 (P4)**
Answer with reason	**Point 5 (P5)**

Scoring Notes for Part B

- To earn **P3**, a response can do one of the following:
  - Set $x_H'(t) = 0$, $v_H(t) = 0$, or $(2t - 4)e^{t^2-4t} = 0$
  - Set $v_J(t) = 0$ or $2t\left(t^2 - 1\right)^3 = 0$
  - Identify $t = 2$ for particle H and no other values in the interval $0 < t < 5$
  - Identify $t = 1$ for particle J and no other values in the interval $0 < t < 5$
  - Identify the interval $1 < t < 2$
- To earn **P4**, a response can provide an analysis of signs of velocity or direction of motion on the interval $0 < t < 5$ for either particle H or particle J.
- To be eligible for **P5**, a response must provide correct analyses of signs of velocity or direction of motion on the interval $0 < t < 5$ for both particles.
- Only analysis within the interval $0 < t < 5$ will be considered in scoring.

C It can be shown that $v_J'(2) > 0$. Is the speed of particle J increasing, decreasing, or neither at time $t = 2$? Give a reason for your answer.

$v_J(2) > 0$ and $v_J'(2) > 0$. Because $v_J(2)$ and $v_J'(2)$ have the same sign, the speed of particle J is increasing at $t = 2$.	Answer with reason **Point 6 (P6)**

Scoring Notes for Part C

- An evaluation of $v_J(2)$ is not necessary, but if a value is presented, it must be correct. The correct value is $v_J(2) = 108$.
- An evaluation of $v_J'(2)$ is not necessary, but if a value is presented, it must be correct. The correct value is $v_J'(2) = 486$.
- A response can either import the analysis for the sign of $v_J(2)$ from part B or restart.
- A response that stated "$v_J(t) > 0$ for $1 < t < 5$" in part B does not need to restate $v_J(2) > 0$ and earns **P6** for "$v_J(2)$ and $v_J'(2)$ have the same sign, so the speed is increasing."

D Particle J is at position $x = 7$ at time $t = 0$. Find the position of particle J at time $t = 2$. Show the work that leads to your answer.

$x_J(2) = x_J(0) + \int_0^2 v_J(t)\,dt = 7 + \int_0^2 2t\left(t^2 - 1\right)^3 dt$ $= 7 + \left[\frac{1}{4}\left(t^2 - 1\right)^4\right]_0^2$ $= 7 + \frac{1}{4}\left((3)^4 - (-1)^4\right) = 7 + \frac{1}{4}(80) = 27$	Integrand **Point 7 (P7)** Antiderivative **Point 8 (P8)** Answer **Point 9 (P9)**

Scoring Notes for Part D

- To earn **P7**, a response must present an indefinite or definite integral with an integrand of $v_J(t)$ or $2t\left(t^2 - 1\right)^3$. (See below for notes on how to handle a missing differential dt.)
- **P8** is earned for an antiderivative of the form $k\left(t^2 - 1\right)^4$ or equivalent, for $k > 0$. If $k \neq \frac{1}{4}$, then the response is not eligible to earn **P9**.
- A response of $7 + \frac{1}{4}\left((3)^4 - (-1)^4\right)$ or equivalent banks **P9** (i.e., subsequent errors in simplification will not be considered in scoring for **P9**).

  Note: An ambiguous response, such as $7 + \frac{1}{4}\left((3)^4 - (-1)^4$, does not bank **P9** and therefore must go on to resolve the ambiguity with a correct final answer (e.g., $7 + \frac{1}{4}(80)$ or 27) to earn **P9**.

- If the differential dt is missing:
  - Writing $\int_0^2 v_J(t)$ earns **P7** and is eligible to earn **P8** and **P9**.
  - Writing $7 + \int_0^2 v_J(t)$ earns **P7** and is eligible to earn **P8** and **P9**.
  - Writing $\int_0^2 v_J(t) + 7$ introduces an ambiguity for the intended integrand.
    - $\int_0^2 v_J(t) + 7 = \left[\frac{1}{4}\left(t^2 - 1\right)^4\right]_0^2 + 7$ resolves the ambiguity.

      Therefore, this earns **P7** and **P8** and is eligible for **P9**.
    - $\int_0^2 v_J(t) + 7 = \left[\frac{1}{4}\left(t^2 - 1\right)^4 + 7t\right]_0^2$ confirms that an incorrect integrand was used.

      Therefore, this does not earn **P7**, earns **P8**, and is not eligible for **P9**.
    - If the ambiguity is not resolved, this does not earn **P7**, **P8**, or **P9**.
- Alternate solution using u-substitution:

  Let $u = t^2 - 1$, then $du = 2t\,dt$.

  $t = 0 \Rightarrow u = -1$

  $t = 2 \Rightarrow u = 3$

  $$x_J(2) = x_J(0) + \int_0^2 v_J(t)\,dt = 7 + \int_0^2 2t\left(t^2 - 1\right)^3 dt$$

  $$= 7 + \int_{-1}^3 u^3\,du = 7 + \left[\frac{1}{4}u^4\right]_{-1}^3$$

  $$= 7 + \frac{1}{4}\left((3)^4 - (-1)^4\right) = 7 + \frac{1}{4}(80) = 27$$

- Alternate solution using indefinite integral:

  $$\int 2t\left(t^2 - 1\right)^3 dt = \frac{1}{4}\left(t^2 - 1\right)^4 + C$$

  $$x_J(0) = 7 = \frac{1}{4}\left(0^2 - 1\right)^4 + C \Rightarrow C = \frac{27}{4}$$

  $$x_J(t) = \frac{1}{4}\left(t^2 - 1\right)^4 + \frac{27}{4}$$

  $$x_J(2) = \frac{1}{4}\left(2^2 - 1\right)^4 + \frac{27}{4} = \frac{108}{4} = 27$$

Q5 NO CALCULATOR ALLOWED Q5

Answer QUESTION 5 PARTS A and B on this page.

PART A

$X_H(t) = e^{t^2-4t}$

$V_H(t) = (2t-4)\cdot e^{t^2-4t}$

$V_H(1) = (2(1)-4)\cdot e^{1-4}$

$\boxed{V_H(1) = -2e^{-3}}$

$X_H(4) = e^{t^2-4t}$

$V_H(4) = (2t-4)\cdot e^{t^2-4t}$

$V_H(1) = (2-4)\cdot e^{1-4}$

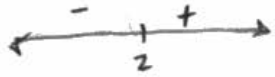
$= -2e$

PART B

$V_H(t) = (2t-4)\cdot e^{t^2-4t}$; $V_H(t) = 0$

$0 = (2t-4)\cdot e^{t^2-4t}$

$t = 2$



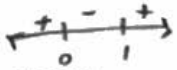
H moves left from 0 to 2
right from 2 to 5

J moves left from 0 to 1
right from 1 to 5

$V_J(t) = 2t + (t^2-1)^3$; $V_J(t) = 0$

$0 = 2t + (t^2-1)^3$

$t = 0, 1, -1$



H and J move in opposite directions from $1 < t < 2$ they have opposite sign velocity at this interval

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

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Q5 NO CALCULATOR ALLOWED Q5

Answer QUESTION 5 PARTS C and D on this page.

PART C

$V_J(2) = 2(2)((2)^2-1)^3$

$V_J(2) = 4(3)^3$

$V_J(2)$ is positive $(V_J(2) > 0)$

$V_J'(2)$ is positive $(V_J'(2) > 0)$

Since velocity and acceleration have the same sign, the particle is speeding up at $t=2$.

PART D

$X_J(t) = \int V_J(t)\,dt$

$X_J(t) = \int 2t(t^2-1)^3\,dt$

$(t^2-1)(t^2-1)$

$(t^4-2t^2+1)(t^2-1)$

$t^6-t^4-2t^4+2t^2+t^2-1$

$X_J(t) = \int 2t(t^6-3t^4+3t^2-1)$

$X_J(t) = \int 2t^7-6t^5+6t^3-2t$

$X_J(t) = \frac{1}{4}t^8-t^6+\frac{3}{2}t^4-t^2+C$

$7 = 0+C$

$C = 7$

$X_J(2) = \frac{1}{4}(2)^8-(2)^6+\frac{3}{2}(2)^4-(2^2)+7$

Q5 NO CALCULATOR ALLOWED Q5

Answer QUESTION 5 PARTS A and B on this page.

PART A

H

$X_H(t) = e^{t^2-4t}$

J

$V_J(t) = 2t(t^2-1)^3$

Find $V_H(1)$

$X_H(t) = e^{t^2-4t}$

$V_H(t) = e^{t^2-4t} \cdot (2t-4)$

$V_H(1) = e^{1-4} \cdot (2-4)$

$V_H(1) = e^{-3} \cdot -2$

$V_H(1) = -2e^{-3}$

PART B

During what open intervals for $0<t<5$ are H and J moving in opposite directions

$-V_H(t) = V_J(t)$

Answer QUESTION 5 PARTS C and D on this page.

PART C

The speed of particle J is increasing at time $t=2$. $V_J'(2)$ describes the particles acceleration, and it is said to be >0. If acceleration is positive, speed is increasing.

PART D

$X_J(0) = 7$ Find $x_J(2)$

$V_J(t) = 2t(t^2-1)^3$

$\int 2t(t^2-1)^3$

$X_J(t) = \frac{1}{4}(t^2-1)^4$

$x_J(2) = \frac{1}{4}(4-1)^4$

$x_J(2) = \frac{1}{4}(3)^4$

$x_J(2) = \frac{81}{4}$

Q5 NO CALCULATOR ALLOWED Q5

Answer QUESTION 5 PARTS A and B on this page.

PART A

$X'_H(t) = e^{t^2-4t} \cdot 2t-4$

$X'_H(1) = e^{1^2-4(1)} \cdot 2(1)-4$

PART B

$v_J(t) = 2t(t^2-1)^3 = 0$ $X'_H(t) =$

$2t=0$ $t^2=1$

$t=0$ $t=1,-1$

on the interval $(0,1)$ the particles are moving in opposite directions !!

Answer QUESTION 5 PARTS C and D on this page.

PART C

$v_J'(2) > 0$ ∴ the Speed of the particle is increasing at $t=2$

PART D

$$\int_0^2 2t(t^2-1)^3\,dt \qquad u = t^2-1, \quad dt = \frac{du}{2t}$$

$$\int_0^2 2t(u)^3 \frac{du}{2t}$$

$$\frac{1}{4}(t^2-1)^3\Big]_0^2 = \frac{1}{4}(2^2-1)^3 - \frac{1}{4}(0^2-1)^3$$

Question 5

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

NEW for 2025: The question overviews can be found in the *Chief Reader Report on Student Responses* on AP Central.

Sample: 5A
Score: 9 (1-1-1-1-1-1-1-1-1)

The response earned 9 points: 2 points in part A, 3 points in part B, 1 point in part C, and 3 points in part D.

In part A the response earned **P1** with the presentation of $(2t-4)\cdot e^{t^2-4t}$ in line 2. The response earned **P2** with the numerical expression $(2(1)-4)\cdot e^{1-4}$ in line 3.

In part B the response earned **P3** with the presentation of $v_H(t)=0$ in line 1 on the right. The response earned **P4** with a correct analysis of motion of H on the interval $0<t<5$ in lines 2 and 3 on the right. The response earned **P5** with a correct analysis of motion of J on the interval $0<t<5$ in lines 4 and 5 on the right and the correct boxed conclusion in lines 6–10 on the right.

In part C the response earned **P6** with a correct answer of "$v_J(2)$ is positive" in line 3, "$v_J'(2)$ is positive" in line 4, and the correct conclusion in lines 5 and 6.

In part D the response earned **P7** on the left with $\int v_J(t)\,dt$ in line 1. The response earned **P8** with $\frac{1}{4}t^8-t^6+\frac{3}{2}t^4-t^2$ in line 5. The response earned **P9** with the boxed answer $\frac{1}{4}(2)^8-(2)^6+\frac{3}{2}(2)^4-\left(2^2\right)+7$.

Sample: 5B
Score: 4 (1-1-0-0-0-0-1-1-0)

The response earned 4 points: 2 points in part A, 0 points in part B, 0 points in part C, and 2 points in part D.

In part A the response earned **P1** with the presentation of $e^{t^2-4t}\cdot(2t-4)$ in line 4. The response earned **P2** with the numerical expression $e^{1-4}\cdot(2-4)$ in line 5.

In part B the response did not earn **P3**. The response did not consider the sign of $x_H'(t)$ or $v_J(t)$. The response did not earn **P4** because there was no analysis of signs of velocity or direction of motion for either particle on the interval $0<t<5$. The response did not earn **P5** because it was not eligible without earning **P4**.

In part C the response did not earn **P6** because only acceleration is considered.

In part D the response earned **P7** with $\int 2t\left(t^2-1\right)^3$ in line 3. The response earned **P8** with $\frac{1}{4}\left(t^2-1\right)^4$ in line 4. The response did not earn **P9** because the stated answer of $\frac{1}{4}(4-1)^4$ in line 5 is incorrect.

Question 5 (continued)

Sample: 5C
Score: 3 (1-0-1-0-0-0-1-0-0)

The response earned 3 points: 1 point in part A, 1 point in part B, 0 points in part C, and 1 point in part D.

In part A the response earned **P1** with the presentation of $x_H'(t)$ in line 1. The response did not earn **P2** because the numerical expression $e^{1^2-4(1)} \cdot 2(1) - 4$ in line 2 is missing parentheses and the error is not resolved.

In part B the response earned **P3** with the equation $2t\left(t^2-1\right)^3 = 0$. The response did not earn **P4** because there was no analysis of signs of velocity or direction of motion for either particle on the interval $0 < t < 5$. The response did not earn **P5** because it was not eligible without earning **P4**.

In part C the response did not earn **P6** because only acceleration is considered.

In part D the response earned **P7** with $\int_0^2 2t\left(t^2-1\right)^3\,dt$ in line 1. The response did not earn **P8**. The exponent in the expression $\frac{1}{4}\left(t^2-1\right)^3$ in line 3 on the right is incorrect. The response did not earn **P9** because the stated answer of $\frac{1}{4}\left(2^2-1\right)^3 - \frac{1}{4}\left(0^2-1\right)^3$ in line 3 on the right is incorrect.