2025

# AP® Calculus AB

## Sample Student Responses and Scoring Commentary

**Inside:**

Free-Response Question 4

- ☑ Scoring Guidelines
- ☑ Student Samples
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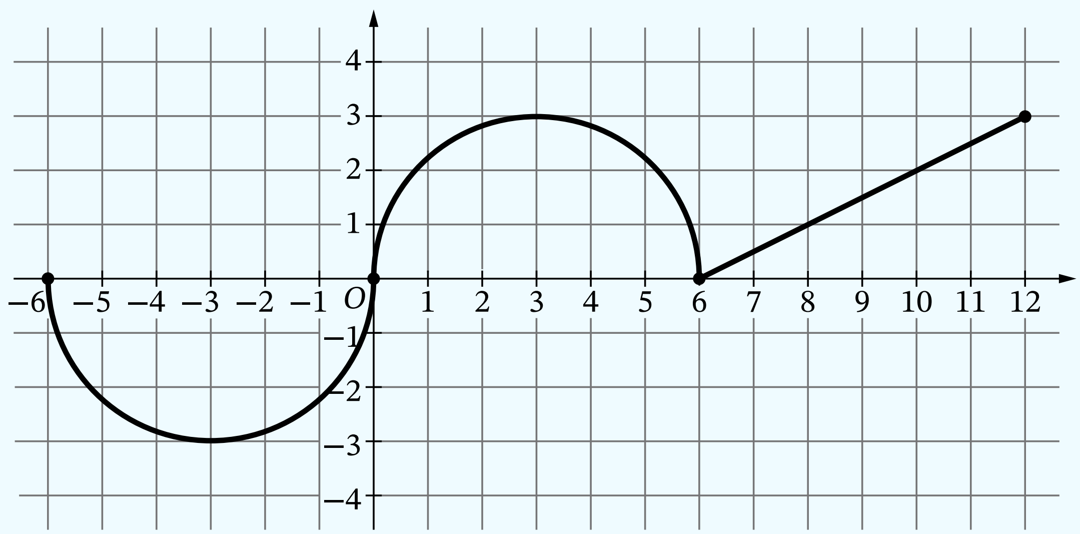
## Part A (AB or BC): Graphing calculator not allowed
## Question 4 — 9 points

### General Scoring Notes

- The model solution is presented using standard mathematical notation.
- Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be accurate to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

The continuous function $f$ is defined on the closed interval $-6 \le x \le 12$. The graph of $f$, consisting of two semicircles and one line segment, is shown in the figure.




Graph of $f$

Let $g$ be the function defined by $g(x) = \int_6^x f(t)\,dt$.

| | Model Solution | Scoring | |
|---|---|---|---|
| **A** | Find $g'(8)$. Give a reason for your answer. | | |
| | $g'(x) = f(x)$ | Considers $g'(x) = f(x)$ | **Point 1 (P1)** |
| | $g'(8) = f(8) = 1$ | Answer | **Point 2 (P2)** |

**Scoring Notes for Part A**

- **P1** is earned for $g' = f$, $g'(x) = f(x)$, or $g'(8) = f(8)$ in part A.
- A response of $g'(8) = f(8) = 1$ earns both **P1** and **P2**.
- A response that does not earn **P1** can earn **P2** with an implied application of the Fundamental Theorem of Calculus (e.g., $g'(8) = 1$ or $f(8) = 1$).
- A response of $g'(8) = f(8) - f(6) = 1$ earns **P2** but not **P1**.

**B** Find all values of $x$ in the open interval $-6 < x < 12$ at which the graph of $g$ has a point of inflection. Give a reason for your answer.

| | | |
|---|---|---|
| The graph of $g$ has a point of inflection where $g'' = f'$ changes sign, which is where $g' = f$ changes from decreasing to increasing or vice versa.<br><br>The graph of $g$ has points of inflection at $x = -3$ and $x = 6$ because $f$ changes from decreasing to increasing there.<br><br>The graph of $g$ also has a point of inflection at $x = 3$ because $f$ changes from increasing to decreasing there. | Answer<br>Reason | **Point 3 (P3)**<br>**Point 4 (P4)** |

**Scoring Notes for Part B**

- **P3** is earned only for an answer of $x = -3$, $x = 3$, and $x = 6$. If any other/additional values of $x$ in $-6 < x < 12$ are declared to be points of inflection, the response does not earn either **P3** or **P4**. Consideration of $x = -6$ or of $x = 12$ does not impact scoring.
- To earn **P4**, a response must tie the reason to the given graph of $f$.
  - A response of "$g$ has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because $f$ changes from increasing to decreasing or decreasing to increasing there" earns both **P3** and **P4**.
  - A response of "$g$ has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because the slope of $f$ changes sign there" earns both **P3** and **P4**.
  - A response of "$g$ has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because $f$ attains relative extrema there" earns both **P3** and **P4**.
  - A response of "$g$ has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because $g$ changes concavity there" earns **P3** but not **P4**.
  - A response of "$g$ has a point of inflection at $x = -3$, $x = 3$, and $x = 6$ because $g'' = f'$ changes sign there" earns **P3** but not **P4**.
  - A response that relies upon an ambiguous term such as "the function" or "the graph" does not earn **P4**.
- **Special case:** A response with two of the three correct $x$-values with correct reasoning and no other/additional values of $x$ declared to be points of inflection earns **P4** but not **P3**.

| C | Find $g(12)$ and $g(0)$. Label your answers. | | |
|---|---|---|---|
| | $g(12) = \int_6^{12} f(t)\,dt = \frac{1}{2}\cdot 6\cdot 3 = 9$ | $g(12)$ | **Point 5 (P5)** |
| | $g(0) = \int_6^0 f(x)\,dx = -\int_0^6 f(x)\,dx = -\frac{\pi}{2}3^2 = -\frac{9\pi}{2}$ | $g(0)$ | **Point 6 (P6)** |

**Scoring Notes for Part C**

- Unlabeled values do not earn either **P5** or **P6**.
- **P5** is earned for a response of $g(12) = 9$, with or without supporting work.
- **P6** is earned for a response of $g(0) = -\frac{9\pi}{2}$, with or without supporting work.

  Note: Incorrect communication between the label "$g(0)$" and the answer will be treated as scratch work and will not impact scoring. For example, $g(0) = \int_0^6 f(x)\,dx = -\frac{9\pi}{2}$ earns **P6**.

| | | | |
|---|---|---|---|
| **D** | Find the value of $x$ at which $g$ attains an absolute minimum on the closed interval $-6 \le x \le 12$. Justify your answer. | | |
| | For $-6 \le x \le 12$, $g$ attains a minimum either when $g'(x) = f(x) = 0$ or at an endpoint.<br><br>$g'(x) = f(x) = 0$ | Considers $g'(x) = 0$ | **Point 7 (P7)** |
| | $\Rightarrow \ x = 0,\ x = 6$<br><br>$x$ \| $g(x)$<br>$-6$ \| $0$<br>$0$ \| $-\frac{9\pi}{2}$<br>$6$ \| $0$<br>$12$ \| $9$ | Justification | **Point 8 (P8)** |
| | Therefore, on the closed interval $-6 \le x \le 12$, $g$ attains an absolute minimum value at $x = 0$. | Answer | **Point 9 (P9)** |

**Scoring Notes for Part D**

- **P7** is earned for considering $g'(x) = 0$ or $f(x) = 0$. **P7** is not earned by just presenting $x = 0$ and $x = 6$.

  A response that discusses the sign of $g'(x)$ or $f(x)$ changing OR uses the phrase "critical points of $g$" also earns **P7**.
- To earn **P8** using a candidates test, a response must make a global argument by providing evaluations or reasoning for each of $g(-6)$, $g(0)$, $g(6)$, and $g(12)$ (and no other $x$-values).
- Alternate justification and answer:

  Because $g'(x) \le 0$ (or $f(x) \le 0$) for $-6 \le x < 0$ and $g'(x) \ge 0$ (or $f(x) \ge 0$) for $0 < x \le 12$, the absolute minimum of $g$ occurs at $x = 0$.
- A response that presents a local argument (such as a First Derivative Test or a Second Derivative Test) or an incorrect global argument does not earn **P8** but is eligible for **P9** with the correct answer of $x = 0$.
- For **P8**, values of $g(0)$ and $g(12)$ can be imported from part C. A response can earn **P9** with an answer that is consistent with the imported values.

Q4 NO CALCULATOR ALLOWED Q4

**Answer QUESTION 4 PARTS A and B on this page.**

**PART A**

$\frac{d}{dx} g(x) = \frac{d}{dx} \int_{6}^{x} f(t)\,dt$

$g'(x) = f(x)$

$g'(8) = f(8) = \boxed{1}$

fundamental theorem of calculus states that $\frac{d}{dx}\left[\int f(x)dx\right] = f(x)$

**PART B**

g has a pt of inflection at $x=-3$ because f (or g') changes from decreasing to increasing

g has a pt of inflection at $x=3$ because f (or g') changes from increasing to decreasing

g has a pt of inflection $x=6$ because f (or g') changes from decreasing to increasing



Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q4 NO CALCULATOR ALLOWED Q4

**Answer QUESTION 4 PARTS C and D on this page.**

**PART C**

$g(12) = \int_6^{12} f(t)\,dt$

$g(12) = \frac{(6)(3)}{2}$

$g(12) = 9$

$g(0) = \int_6^{0} f(t)\,dt$

$g(0) = -\int_0^{6} f(t)\,dt$

$g(0) = -\left(\frac{1}{2}\pi(3)^2\right)$

$g(0) = -\left(\frac{9}{2}\pi\right)$

$g(0) = -\frac{9}{2}\pi$

**PART D**

$g$ is continuous on $[-6, 12]$

$\therefore$ EVT applies

$g'(x) = f(x)$

$0 = f(x)$

$x = 0 \quad x = 6$

| $x$ | $g(x)$ |
|---|---|
| $-6$ | $0$ |
| $0$ | $-\frac{9}{2}\pi$ |
| $6$ | $0$ |
| $12$ | $9$ |

$g(0) < g(-6) = g(6) < g(12)$

$\therefore$ $g$ attains an absolute minimum value at $x = 0$

Q4 NO CALCULATOR ALLOWED Q4

Answer QUESTION 4 PARTS A and B on this page.

PART A

$g'(x) = f(x)$

$g'(8) = f(8)$

$g'(8) = 1$

f(8) appears to be 1 when looking at the graph

PART B

when x=0 there appears to be a point of inflection as f goes from concave up to concave down.

Q4 NO CALCULATOR ALLOWED Q4

**Answer QUESTION 4 PARTS C and D on this page.**

**PART C**

$g(12) = 9$

$g(0) = -\frac{9}{2}\pi$

**PART D**

$g'(x) = f(x) = 0$

$x = 0, 6$

minimum when $x = 0$ knowing that $g'(x) = f(x)$ using the FTC, 0 is a minimum as $f(0) = 0$ meaning the slope of g is 0 and f goes from negative to ~~positive~~ ~~at~~ that point making it a minimum,

Q4 NO CALCULATOR ALLOWED Q4

Answer QUESTION 4 PARTS A and B on this page.

PART A

$g(x) = \int_0^x f(t)\,dt$

$g'(x) = f(x)$

$g'(8) = f(8)$

$g'(8) = 1$

$g'(8) = 1$ because $g'(x) = f(x)$. when $x = 8$, $f(8) = 1$ and because the derivative of $g(x)$ equals $f(x)$, $g'(8)$ will also equal 1.

PART B $g''(x) = f'(x)$

POI: $x = 0, 6$

Because $g''(x) = f'(x)$, when the slope of $f(x)$ changes signs, $g(x)$ will have a point of inflation.

Q4 NO CALCULATOR ALLOWED Q4

**Answer QUESTION 4 PARTS C and D on this page.**

**PART C**

$g(x) = \int_6^x f(t)\,dt$

$g(12) = \int_6^{12} f(t)\,dt$

$g(12) = 18$

$g(0) = \int_6^0 f(t)\,dt = -\int_0^6 f(t)\,dt$

$g(0) = -\frac{1}{2}\pi(3)^2$

$g(0) = -\frac{9\pi}{2}$

**PART D**

$g'(x) = f(x)$

$g(6) = 0$

$g(-6) = 0$

$g(12) = 18$

g attains an absolute maximum at $x = 12$ because $g(x) = \int_6^x f(t)\,dt$, and at $x = 6$, the integral equals 12, so $g(6) = 12$.



Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5522/11

## Question 4

**Note:** Student samples are quoted verbatim and may contain spelling and grammatical errors.

### Overview

**NEW for 2025:** The question overviews can be found in the *Chief Reader Report on Student Responses* on AP Central.

### Sample: 4A
### Score: 9 (1-1-1-1-1-1-1-1-1)

The response earned 9 points: 2 points in part A, 2 points in part B, 2 points in part C, and 3 points in part D.

In part A the response earned **P1** with the equation $g'(x) = f(x)$ in line 2. The response earned **P2** with the statement $g'(8) = f(8) = 1$ in line 3. This statement alone would have earned both **P1** and **P2**. Note: The comment presented on the right regarding the Fundamental Theorem of Calculus was not considered in scoring **P1** or **P2**.

In part B the response earned **P3** with the presentation of the correct answer $x = -3$, $x = 3$, and $x = 6$, with no additional values presented. The response earned **P4** with the correct reasoning that "$f$ (or $g'$) changes from decreasing to increasing" in the first and third sentences and "$f$ (or $g'$) changes from increasing to decreasing" in the second sentence.

In part C the response earned **P5** with the boxed statement $g(12) = 9$ in line 3 on the left. The numerical expression $g(12) = \frac{(6)(3)}{2}$ in line 2 on the left would have earned the point with no simplification. The response earned **P6** with the boxed statement $g(0) = -\frac{9}{2}\pi$ in line 5 on the right. The numerical expression $g(0) = -\left(\frac{1}{2}\pi(3)^2\right)$ in line 3 on the right would have earned the point with no simplification.

In part D the response earned **P7** with the statement $0 = f(x)$ in line 4 on the left. The response earned **P8** with the presentation of the correct table on the right. The response earned **P9** for the boxed statement that "$g$ attains an absolute minimum value at $x = 0$" on the right.

### Sample: 4B
### Score: 6 (1-1-0-0-1-1-1-0-1)

The response earned 6 points: 2 points in part A, 0 points in part B, 2 points in part C, and 2 points in part D.

In part A the response earned **P1** with the equation $g'(x) = f(x)$ in line 1. The response also would have earned this point with the equation $g'(8) = f(8)$ in line 2. The response earned **P2** with the equation $g'(8) = 1$ in line 3.

In part B the response did not earn **P3** because the answer $x = 0$ in line 1 is incorrect. The response did not earn **P4** because the response does not have at least two of the three correct $x$-values and presents incorrect reasoning.

In part C the response earned **P5** with the correct labeled value of 9 for $g(12)$ in line 1. The response earned **P6** with the correct labeled value of $-\frac{9}{2}\pi$ for $g(0)$ in line 2.

## Question 4 (continued)

In part D the response earned **P7** with the equation $g'(x) = f(x) = 0$ in line 1. The response did not earn **P8** because a correct justification of an absolute minimum is not presented. The justification presented is not sufficient to establish that $x = 0$ is an absolute minimum on the given interval. The response earned **P9** for presenting the correct answer of $x = 0$ and a local argument that this is the location of a minimum.

**Sample: 4C**
**Score: 3 (1-1-0-0-0-1-0-0-0)**

The response earned 3 points: 2 points in part A, 0 points in part B, 1 point in part C, and 0 points in part D.

In part A the response earned **P1** with the equation $g'(x) = f(x)$ in line 2. The equation $g'(8) = f(8)$ in line 3 would also have earned the point. The response earned **P2** with the boxed statement $g'(8) = 1$ in line 4.

In part B the response did not earn **P3** because $x = 0$ is presented as an answer in line 2. The response did not earn **P4**. An incorrect value of $x = 0$ is presented, making the response ineligible for **P3** or **P4**.

In part C the response did not earn **P5**. An incorrect value for $g(12)$ is presented. The response earned **P6** with the correct labeled value of $-\frac{9\pi}{2}$ for $g(0)$ in the boxed response given in the last line. The numeric expression $g(0) = -\frac{1}{2}\pi(3)^2$ would have earned the point in line 5 with no further simplification.

In part D the response did not earn **P7**. No evidence is presented that shows consideration of $g'(x) = 0$ or $f(x) = 0$. The response did not earn **P8**. A complete candidates test is not presented, as $g(0)$ is not considered. The response is eligible for **P9**. The response did not earn **P9** because the correct location of the minimum is not presented.