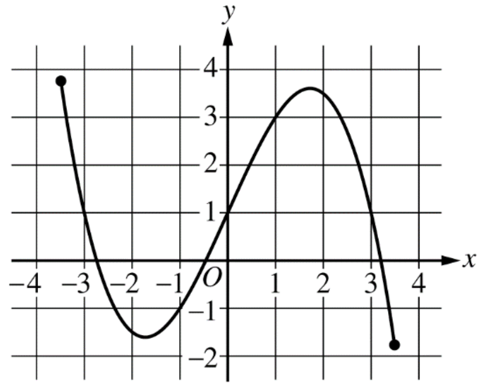
2024

AP® Precalculus

Scoring Guidelines

Question 1: Function Concepts
Part A: Graphing calculator required — 6 points



Graph of f

The figure shows the graph of the function f on its domain of $-3.5 \le x \le 3.5$. The points $(-3, 1)$, $(0, 1)$, and $(3, 1)$ are on the graph of f. The function g is given by $g(x) = 2.916 \cdot (0.7)^x$.

	Model Solution	Scoring	
(A)	(i) The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(3)$ as a decimal approximation, or indicate that it is not defined. (ii) Find all values of x for which $f(x) = 1$, or indicate that there are no such values.		
	(i) $h(3) = g(f(3)) = g(1) = 2.041$	Value	**1 point**
	(ii) From the graph, $f(x) = 1$ when $x = -3$, $x = 0$, and $x = 3$.	Values	**1 point**

General Scoring Notes for Question 1 Parts (A), (B), and (C):

- Decimal approximations must be correct to three places after the decimal point by rounding or truncating. Decimal values of 0 in final digits need not be reported ($2.000 = 2.00 = 2.0 = 2$).
- A **decimal presentation error** occurs when a response is complete and correct, but the answer is reported to fewer digits than required.
- The first decimal presentation error in Question 1 does not earn the point. For each additional part of Question 1 that requires a decimal approximation and contains a decimal presentation error, the response is eligible to earn the point.

Scoring notes:

- The first point is earned for a correct decimal approximation of 2.041.
- The second point does not require supporting work.

- A response that does not earn either point in Part (A) is eligible for **partial credit** in Part (A) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **0-1** in Part (A).

First Column	**Second Column**
Correct value in (i) that is not expressed as a decimal approximation	Only one correct value in (ii) with no incorrect values included
Correct value in (i) with a decimal presentation error	Only two correct values in (ii) with no incorrect values included

---

(B) (i) Find all values of x, as decimal approximations, for which $g(x) = 2$, or indicate that there are no such values.

(ii) Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

(i) $g(x) = 2 \Rightarrow 2.916(0.7)^x = 2$ $x = 1.057$	Value	**1 point**
(ii) As x increases without bound, the output values of g get arbitrarily close to 0. Therefore, $\lim\limits_{x\to\infty} g(x) = 0$.	End behavior with limit notation	**1 point**

Scoring notes:

- The first point is earned for a correct decimal approximation of 1.057. No incorrect values may be included.
- The second point requires a correct limit statement with four components: “lim,” “$x \to \infty$,” the function g, and 0. Examples that earn the point include:
  - $\lim\limits_{x\to\infty} g(x) = 0$ OR $\lim\limits_{x\to\infty} g = 0$
  - $\lim\limits_{x\to\infty} g(x) \to 0$ OR $\lim\limits_{x\to\infty} g \to 0$
  - $\lim\limits_{x\to\infty} g(x) \quad 0$ OR $\lim\limits_{x\to\infty} g \quad 0$

  If the response includes an additional, complete limit statement (e.g., $\lim\limits_{x\to-\infty} g(x) = \infty$), the value of the limit must be correct.

- A response that does not earn either point in Part (B) is eligible for **partial credit** in Part (B) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **1-0** in Part (B).

First Column	**Second Column**
Correct answer in (i) that is not expressed as a decimal approximation	Correct end behavior statement in (ii) without use of limit notation
Correct value in (i) with a decimal presentation error	Correct end behavior statement in (ii) with incorrect limit notation
	Correct limit statement in (ii) that is missing "$x \to \infty$"

(C) (i) Determine if f has an inverse function.

(ii) Give a reason for your answer based on the definition of a function and the graph of $y = f(x)$.

(i) f does not have an inverse function on its domain of $-3.5 \le x \le 3.5$.	Answer	**1 point**
(ii) There are output values of f that are not mapped from unique input values; for example, $f(-3) = f(0) = f(3) = 1$.	Reason	**1 point**

Scoring notes:

- The first point is earned for a correct answer.
- Both points may be earned in (ii) provided there is no incorrect response in (i).
- The second point requires an implicit or explicit reference to the definition of a function AND support for the reason by referencing specific function values.
- A response such as "f does not have an inverse function because $f(-3) = f(0) = 1$" OR "f does not have an inverse function because there are two input values mapped to 1" earns both points.
- A response such as "f is not one-to-one" OR "f fails the horizontal line test" OR "There are output values that are not mapped from unique input values" is not sufficient to earn the second point.
- The second point cannot be earned if there are any errors in Part (C) (ii).
- A response that indicates that f **has** an inverse function in Part (C) (i) without a reason in Part (C) (i) <u>combined</u> with a response in Part (C) (ii) that provides both the correct answer <u>and</u> a correct reason is scored **0-1**.

Total for question 1 — 6 points

Question 2: Modeling a Non-Periodic Context
Part A: Graphing calculator required — 6 points

On the initial day of sales ($t = 0$) for a new video game, there were 40 thousand units of the game sold that day. Ninety-one days later ($t = 91$), there were 76 thousand units of the game sold that day.

The number of units of the video game sold on a given day can be modeled by the function G given by $G(t) = a + b\ln(t+1)$, where $G(t)$ is the number of units sold, in thousands, on day t since the initial day of sales.

Model Solution	Scoring

(A) (i) Use the given data to write two equations that can be used to find the values for constants a and b in the expression for $G(t)$.

(ii) Find the values for a and b as decimal approximations.

(i) Because $G(0) = 40$ and $G(91) = 76$, two equations to find a and b are $a + b\ln(0+1) = 40$ $a + b\ln(91+1) = 76$.	Two equations	**1 point**
(ii) $a = 40 - b\ln 1 = 40$ $b = \dfrac{(76-40)}{\ln 92} = 7.961451$ $G(t) = 40 + 7.961\ln(t+1)$	Values of a and b	**1 point**

General Scoring Notes for Question 2 Parts (A), (B), and (C):

- Decimal approximations must be correct to three places after the decimal point by rounding or truncating. Decimal values of 0 in final digits need not be reported ($2.000 = 2.00 = 2.0 = 2$).
- A **decimal presentation error** occurs when a response is complete and correct, but the answer is reported to fewer digits than required.
- The first decimal presentation error in Question 2 does not earn the point. For each additional part of Question 2 that requires a decimal approximation and contains a decimal presentation error, the response is eligible to earn the point.

Scoring notes:

- The first point is earned for presenting two equations involving a and b that use the given input-output pairs.
- The second point is earned for correct values of a and b with or without supporting work. If correct values are identified, work should be ignored.
- The second point is earned for correct values of a and b presented as either stand-alone values OR in an expression for $G(t)$.
- A response is eligible to earn both points with a correct translation to "thousands." Use of $40{,}000$ and $76{,}000$ results in values of $a = 40{,}000$ and $b = 7961$.
- A response that does not earn either point in Part (A) is eligible for **partial credit** in Part (A) if the response has one correct equation in the presence of two equations involving a and b AND one correct value. Partial credit response is scored **1-0** in Part (A).

(B) (i) Use the given data to find the average rate of change of the number of units of the video game sold, in thousands per day, from $t = 0$ to $t = 91$ days. Express your answer as a decimal approximation. Show the computations that lead to your answer.

(ii) Use the average rate of change found in (i) to estimate the number of units of the video game sold, in thousands, on day $t = 50$. Show the work that leads to your answer.

(iii) Let A_t represent the estimate of the number of units of the video game sold, in thousands, using the average rate of change found in (i). For A_{50}, found in (ii), it can be shown that $A_{50} < G(50)$. Explain why, in general, $A_t < G(t)$ for all t, where $0 < t < 91$.

(i) $\dfrac{G(91) - G(0)}{91 - 0} = \dfrac{(76 - 40)}{91} = 0.395604$ The average rate of change is 0.396 (or 0.395) thousand units per day.	Average rate of change	**1 point**
(ii) The average rate of change is $r = \dfrac{G(91) - G(0)}{91 - 0} = 0.395604$. The secant line between point $(0, G(0))$ and point $(91, G(91))$ is given by $y = y_1 + r(x - x_1)$, where (x_1, y_1) can be either one of the points. Estimates using the average rate of change are given by $y = G(0) + r(t - 0)$ OR $y = G(91) + r(t - 91)$. Both of these produce the same estimate. For $t = 50$, $y = 40 + r(50 - 0) = 59.780$ OR $y = 76 + r(50 - 91) = 59.780$. The number of units sold on day $t = 50$ was approximately 59.780 thousand.	Estimate using average rate of change	**1 point**
(iii) The estimate A_t is the y-coordinate of a point on the secant line that passes through $(0, G(0))$ and $(91, G(91))$. Because the graph of G is concave down on the interval $(0, 91)$, this secant line is below the graph of G on the interval $(0, 91)$. Therefore, the estimate A_t is less than the value of $G(t)$ for all t on the interval $(0, 91)$.	Answer with explanation	**1 point**

Scoring notes:

- Supporting work is required in (i) and (ii).
- The first point is earned for a correct decimal approximation in the presence of a quotient that uses the given data values. Units are not needed and are ignored if presented.
- Eligibility for the second point:
    - If a response earned the point in (i) without a decimal presentation error, then an estimate in the range $[59.750, 59.805]$ earns the second point in the presence of supporting work.
    - If a response in (i) has a decimal presentation error, the reported value in (i) as the average rate of change can be used to arrive at an estimate in (ii). To earn the second point, the estimate in (ii) must be consistent with both the reported value in (i) and the endpoint used in the supporting work in (ii).
    - If a response in (i) is incorrect, the reported value in (i) as the average rate of change can be used to arrive at an estimate in (ii). To earn the second point, the estimate in (ii) must be consistent with both the reported value in (i) and the endpoint used in the supporting work in (ii).
- The final number in (ii) may be reported as 59 thousand or 60 thousand provided the supporting work has a correct decimal approximation for the estimate.
- A response is eligible to earn both points with a correct translation to "thousands."
    - Use of $40,000$ and $76,000$ results in an answer of 395.604 in (i).
    - If a response earned the point in (i) without a decimal presentation error, then an estimate in the range $[59,750, 59,805]$ earns the second point in the presence of supporting work.
    - If a response in (i) has a decimal presentation error, the reported value in (i) as the average rate of change can be used to arrive at an estimate in (ii). To earn the second point, the estimate in (ii) must be consistent with both the reported value in (i) and the endpoint used in the supporting work in (ii).
    - If a response in (i) is incorrect, the reported value in (i) as the average rate of change can be used to arrive at an estimate in (ii). To earn the second point, the estimate in (ii) must be consistent with both the reported value in (i) and the endpoint used in the supporting work in (ii).
- A response that does not earn either point in Part (B) (i) and Part (B) (ii) is eligible for **partial credit** in Part (B) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **1-0** in Part (B)(i)/(ii).

First Column	**Second Column**
A correct quotient that uses the given data values that is not expressed as a decimal approximation	A correct estimate in (ii) that does not include supporting work
A correct quotient that uses the given data values and has a decimal presentation error	Correct supporting work in (ii) that does not provide an estimate
A correct average rate of change in (i) that does not include supporting work	

- To earn the third point, the reasoning must include:
  - The graph of G is concave down OR the rate of change of G is decreasing
  - A reference to the use of a secant line on $0 < t < 91$ OR the use of a linear function with reference to endpoints 0 and 91 that provide the placement of the line

(C) The makers of the video game reported that daily sales of the video game decreased each day after $t = 91$. Explain why the error in the model G increases after $t = 91$.

On day $t = 91$, the output for daily sales and $G(91)$ are the same. For $t > 91$, daily sales are decreasing and G is increasing. Therefore, the absolute value of the difference between the actual daily sales and the daily sales predicted by G is increasing each day for $t > 91$.	Answer with reason	**1 point**

Scoring notes:

- To earn the point, the reasoning must include an implicit or explicit connection between the "function model is increasing" and "daily sales are decreasing."

Total for question 2 **6 points**

Question 3: Modeling a Periodic Context
Part B: Graphing calculator not allowed — 6 points

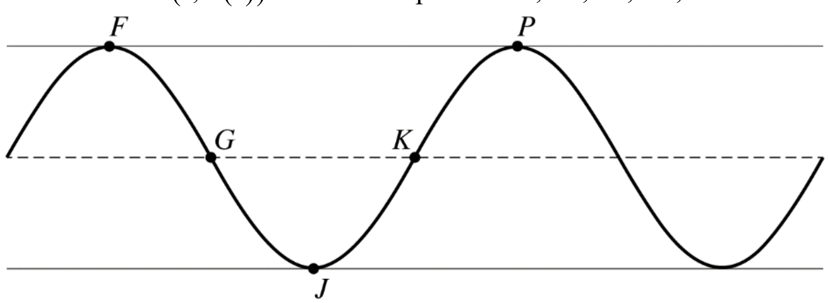
Note: Figure not drawn to scale.

The tire of a car has a radius of 9 inches, and a person rolls the tire forward at a constant rate on level ground, as shown in the figure. Point W on the edge of the tire touches the ground at time $t = \frac{1}{2}$ second. The tire completes a full rotation, and the next time W touches the ground is at time $t = \frac{5}{2}$ seconds. The maximum height of W above the ground is 18 inches. As the tire rolls, the height of W above the ground periodically increases and decreases.

The sinusoidal function h models the height of point W above the ground, in inches, as a function of time t, in seconds.

Model Solution	Scoring

(A) The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented.
Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.



Model Solution	Scoring	
F has coordinates $\left(\frac{3}{2}, 18\right)$. G has coordinates $(2, 9)$. J has coordinates $\left(\frac{5}{2}, 0\right)$. K has coordinates $(3, 9)$. P has coordinates $\left(\frac{7}{2}, 18\right)$. OR	$h(t)$-coordinates	**1 point**
	t-coordinates	**1 point**

F has coordinates $\left(-\frac{1}{2}, 18\right)$.

G has coordinates $(0, 9)$.

J has coordinates $\left(\frac{1}{2}, 0\right)$.

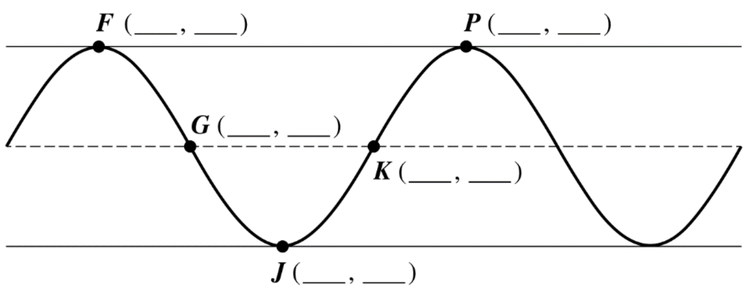
K has coordinates $(1, 9)$.

P has coordinates $\left(\frac{3}{2}, 18\right)$.

Note: t-coordinates will vary. A correct set of coordinates for one full cycle of h as pictured is acceptable.

Scoring notes:

- No supporting work is required.
- $h(t)$-coordinates and/or t-coordinates may appear in a list. Negative t-coordinates are acceptable.
- t-coordinates must be $-\frac{1}{2} + 2k, 0 + 2k, \frac{1}{2} + 2k, 1 + 2k, \frac{3}{2} + 2k$, for a specific integer k.
- If the graph is used to record coordinates, that work is scored. In this case, other work is considered scratchwork and is not scored. Use of the graph is not required.



- A response that does not earn either point in Part (A) is eligible for **partial credit** in Part (A) if the response meets one of the following criteria. Partial credit response is scored **0-1** in Part (A).
  - All 5 points in the form $(h(t), t)$ with correct input values and correct output values swapped
  - 3 correct points out of the 5 points
  - All 5 points $(t, h(t))$ meet these requirements:
    - t-coordinates in arithmetic sequence with $\Delta t = \frac{1}{2}$
    - $h(t)$-coordinates are such that
      (1) F and P have same $h(t)$-coordinate
      (2) G and K have same $h(t)$-coordinate, which is less than $h(t)$-coordinate of F and P
      (3) Difference in $h(t)$-coordinates for F and G equals difference in $h(t)$-coordinates for G and J

(B) The function h can be written in the form $h(t) = a\sin(b(t+c)) + d$. Find values of constants a, b, c, and d.

$h(t) = a\sin(b(t+c)) + d$ $a = 9$ $\frac{2\pi}{b} = 2$, so $b = \frac{2\pi}{2} = \pi$ $c = -1$ $d = 9$ $h(t) = 9\sin(\pi(t-1)) + 9$ OR $a = -9$ $\frac{2\pi}{b} = 2$, so $b = \frac{2\pi}{2} = \pi$ $c = 0$ $d = 9$ $h(t) = -9\sin(\pi t) + 9$ Note: Based on horizontal shifts and reflections, there are other correct forms for $h(t)$.	Vertical transformations: Values of a and d Horizontal transformations: Values of b and c	**1 point** **1 point**

Scoring notes:

- No supporting work is required.
- Points are earned for correct values in a list OR for correct values in an expression for $h(t)$. Only one of these answer presentations is required.
- If the answer box is used to record values, that work is scored. In this case, other work is considered scratchwork and is not scored. Use of the answer box is not required.

$a =$ ______________________

$b =$ ______________________

$c =$ ______________________

$d =$ ______________________

- Other correct values of c:
  - $h(t) = 9\sin(\pi(t+c)) + 9 \Rightarrow c = -1 + 2k$, for any integer k
  - $h(t) = -9\sin(\pi(t+c)) + 9 \Rightarrow c = 0 + 2k$, for any integer k
- Full credit for Part (B) is possible based on the correct use of an imported response from Part (A) that meets these criteria:
  - $a \neq 1$, $b \neq 1$, $d \neq 0$, and if $a > 0$, then $c \neq 0$
  - All 5 points $(t, h(t))$ meet these requirements:
    - t-coordinates in arithmetic sequence with $\Delta t = \frac{1}{2}$
    - $h(t)$-coordinates are such that

      (1) F and P have same $h(t)$-coordinate

      (2) G and K have same $h(t)$-coordinate, which is less than $h(t)$-coordinate of F and P

      (3) Difference in $h(t)$-coordinates for F and G equals difference in $h(t)$-coordinates for G and J

- A response that does not earn either point in Part (B) is eligible for **partial credit** in Part (B) if the response meets one of the following criteria. Partial credit response is scored **1-0** in Part (B).
  - Values of a and b [Values of a and b could be $\pm$]
  - Values of b and d [Value of b could be $\pm$]
  - Response uses $h(t) = a\mathbf{cos}(b(t + c)) + d$ with values as follows:
    - $a = 9$; $b = \pi$; $c = -\frac{3}{2} + 2k$, for a specific integer k ; $d = 9$
    - $a = -9$; $b = \pi$; $c = -\frac{1}{2} + 2k$, for a specific integer k ; $d = 9$

(C) Refer to the graph of h in part (A). The t-coordinate of K is t_1, and the t-coordinate of P is t_2.

(i) On the interval (t_1, t_2), which of the following is true about h ?

a. h is positive and increasing.
b. h is positive and decreasing.
c. h is negative and increasing.
d. h is negative and decreasing.

(ii) Describe how the rate of change of h is changing on the interval (t_1, t_2).

(i) Choice a.	Function behavior	**1 point**
(ii) Because the graph of h is concave down on the interval (t_1, t_2), the rate of change of h is decreasing on the interval (t_1, t_2).	Change in rate of change	**1 point**

Scoring notes:

- No supporting work is required.
- The first point is earned for a correct answer of "a" OR "positive and increasing." If both the letter choice and written description are included, the written description is scored.
- To earn the second point, "decreasing" OR "function h is increasing at a decreasing rate" is acceptable. If concavity of the graph of h is referenced, it must be correct.
- The second point is not earned for a response that only includes "the graph of h is concave down."
- A response with a statement that the rate of change of h is decreasing at an increasing (or decreasing) rate does not earn the second point. Analysis to make such a conclusion requires calculus.
- The second point is not earned for a response that states "increasing at a decreasing rate" without a subject. The implied subject is "the rate of change of h."
- The second point cannot be earned if there are any errors in Part (C) (ii).

Total for question 3 **6 points**

Question 4: Symbolic Manipulations
Part B: Graphing calculator not allowed — 6 points

Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

Model Solution	Scoring

(A) The functions g and h are given by

$$g(x) = e^{(x+3)}$$
$$h(x) = \arcsin\left(\frac{x}{2}\right).$$

(i) Solve $g(x) = 10$ for values of x in the domain of g.

(ii) Solve $h(x) = \frac{\pi}{4}$ for values of x in the domain of h.

Model Solution	Scoring	
(i) $g(x) = 10$ $e^{(x+3)} = 10$ $\ln e^{(x+3)} = \ln 10$ $x + 3 = \ln 10$ $x = -3 + \ln 10$	Solution to $g(x) = 10$	**1 point**
(ii) $h(x) = \frac{\pi}{4}$ $\arcsin\left(\frac{x}{2}\right) = \frac{\pi}{4}$ $\frac{x}{2} = \sin\left(\frac{\pi}{4}\right)$ $\frac{x}{2} = \frac{\sqrt{2}}{2}$ (OR $\frac{1}{\sqrt{2}}$) $x = \sqrt{2}$	Solution to $h(x) = \frac{\pi}{4}$	**1 point**

Scoring notes:

- Supporting work is required in (i) and (ii). "Scratchwork" can be ignored; the use of a variable other than x is acceptable. Arithmetic errors following a correct solution may be considered scratchwork.
- Supporting work in (ii) must include $\frac{x}{2} = \sin\left(\frac{\pi}{4}\right)$ OR $\frac{x}{2} = \frac{\sqrt{2}}{2}$ OR $\frac{x}{2} = \frac{1}{\sqrt{2}}$.
- An alternate solution for (i) is $x = \frac{\log_b 10}{\log_b e} - 3$, where $b > 0$, $b \neq 1$, and the result is evaluated according to bullets 2 and 3 in the directions.
- Where applicable, answers that have not been evaluated according to bullets 2 and 3 in the directions do not earn the point. Rationalizing denominators is not required.
- A response that does not earn either point in Part (A) is eligible for **partial credit** in Part (A) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **1-0** in Part (A).

First Column	**Second Column**
Correct answer in (i) without supporting work	Correct answer in (ii) without supporting work
Correct answer in (i) with supporting work, but the answer has not been evaluated according to bullets 2 and 3 in the directions. No incorrect work.	Correct answer in (ii) with supporting work, but the answer has not been evaluated according to bullets 2 and 3 in the directions. No incorrect work.
Answer in (i) is reported as $x + 3 = \ln 10$. No incorrect work follows.	Answer in (ii) is reported as $\frac{x}{2} = \sin\left(\frac{\pi}{4}\right)$. No incorrect work follows.
	Answer in (ii) is reported as $\frac{x}{2} = \frac{\sqrt{2}}{2}$. No incorrect work follows.
	Answer in (ii) is reported as $\frac{x}{2} = \frac{1}{\sqrt{2}}$. No incorrect work follows.

(B) The functions j and k are given by

$$j(x) = \log_{10}\left(8x^5\right) + \log_{10}\left(2x^2\right) - 9\log_{10} x$$

$$k(x) = \left(\frac{1 - \sin^2 x}{\sin x}\right)\sec x .$$

(i) Rewrite $j(x)$ as a single logarithm base 10 without negative exponents in any part of the expression. Your result should be of the form $\log_{10}(\text{expression})$.

(ii) Rewrite $k(x)$ as a single term involving $\tan x$.

(i) $j(x) = \log_{10}\left(8x^5\right) + \log_{10}\left(2x^2\right) - 9\log_{10} x$ $j(x) = \log_{10}\left(8x^5 \cdot 2x^2\right) - \log_{10} x^9$ $j(x) = \log_{10}\left(\dfrac{8x^5 \cdot 2x^2}{x^9}\right)$ $j(x) = \log_{10}\left(\dfrac{16x^7}{x^9}\right)$ $j(x) = \log_{10}\left(\dfrac{16}{x^2}\right),\ x > 0$	Expression for $j(x)$	**1 point**
(ii) $k(x) = \left(\dfrac{1 - \sin^2 x}{\sin x}\right)\sec x$ $k(x) = \left(\dfrac{\cos^2 x}{\sin x}\right)\left(\dfrac{1}{\cos x}\right)$ $k(x) = \left(\dfrac{\cos x}{\sin x}\right)$ $k(x) = \dfrac{1}{\tan x},\ \sin x \neq 0,\ \cos x \neq 0$	Expression for $k(x)$	**1 point**

Scoring notes:

- Supporting work is required in (i) and (ii). "Scratchwork" can be ignored; the use of a variable other than x is acceptable.
- Domain restrictions are not required to be included and are not scored.
- Where applicable, answers that have not been evaluated according to bullets 2 and 3 in the directions do not earn the point.
- To earn the first point, use of "$\log$" rather than "$\log_{10}$" is acceptable.
- The expression $j(x) = \log_{10}\left(\frac{4}{x}\right)^2$ earns the point in (i) with supporting work.
- A logarithmic expression that is missing one or both parentheses around the full argument of the logarithm is still eligible to earn the point.
- If a response is presented as a complex fraction, the complex fraction must be unambiguous in structure. Parentheses must be used correctly, and/or the fraction bars must be clearly and correctly proportioned.

- A response that does not earn either point in Part (B) is eligible for **partial credit** in Part (B) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **1-0** in Part (B).

First Column	**Second Column**
Correct expression in (i) without supporting work	Correct expression in (ii) without supporting work
Expression in (i) is reported as $\log_{10}\left(\frac{8x^5 \cdot 2x^2}{x^9}\right)$. No incorrect work follows.	Expression in (ii) is reported as $\frac{\cos x}{\sin x}$ OR $\cot x$. No incorrect work follows.
Expression in (i) is reported as $\log_{10}\left(\frac{16x^7}{x^9}\right)$. No incorrect work follows.	Expression in (ii) includes a correct application of a Pythagorean identity with no incorrect work.
Expression in (i) is reported as $2\log_{10}\left(\frac{4}{x}\right)$. No incorrect work follows.	
Expression in (i) is reported as $-\log_{10}\left(\frac{x^2}{16}\right)$. No incorrect work follows.	
Expression in (i) is reported as $-2\log_{10}\left(\frac{x}{4}\right)$. No incorrect work follows.	
The expression in (i) is reported using natural logarithm and has the correct argument OR any of the expressions in partial credit rows two through six above are presented with natural logarithm.	

(C) The function m is given by

$$m(x) = \cos^{-1}(\tan(2x)).$$

Find all values in the domain of m that yield an output value of 0.

$m(x) = 0 \Rightarrow \cos^{-1}(\tan(2x)) = 0$ $\tan(2x) = \cos(0)$	One value of x	**1 point**
$\tan(2x) = 1$ $2x = \frac{\pi}{4} + \pi n$ $x = \frac{\pi}{8} + \frac{\pi}{2}n$, where n is any integer	All values of x	**1 point**

Scoring notes:

- Supporting work is required. "Scratchwork" can be ignored; the use of a variable other than x is acceptable.
- A response with supporting work that gives all correct values for x, such as $x = \frac{\pi}{8} + \pi n$ and $x = \frac{5\pi}{8} + \pi n$, earns both points.
- When expressing a general solution for all values for x (e.g., $x = \frac{\pi}{8} + \frac{\pi}{2}n$), the response can use i, k, n, or any letter except x, which is the variable used in the function.
- To earn the second point, "where n is any integer" is not required to be included.
- To earn the second point, no incorrect values for x are included.

Total for question 4 **6 points**