

2024

AP[®]



AP[®] Precalculus

Sample Student Responses and Scoring Commentary

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Free-Response Question 4

- ✓ **Scoring Guidelines**
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Question 4: Symbolic Manipulations**Part B: Graphing calculator not allowed****6 points****Directions:**

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

Model Solution**Scoring****(A)** The functions g and h are given by

$$g(x) = e^{(x+3)}$$

$$h(x) = \arcsin\left(\frac{x}{2}\right).$$

(i) Solve $g(x) = 10$ for values of x in the domain of g .(ii) Solve $h(x) = \frac{\pi}{4}$ for values of x in the domain of h .

(i) $g(x) = 10$ $e^{(x+3)} = 10$ $\ln e^{(x+3)} = \ln 10$ $x + 3 = \ln 10$ $x = -3 + \ln 10$	Solution to $g(x) = 10$	1 point
(ii) $h(x) = \frac{\pi}{4}$ $\arcsin\left(\frac{x}{2}\right) = \frac{\pi}{4}$ $\frac{x}{2} = \sin\left(\frac{\pi}{4}\right)$ $\frac{x}{2} = \frac{\sqrt{2}}{2}$ (OR $\frac{1}{\sqrt{2}}$) $x = \sqrt{2}$	Solution to $h(x) = \frac{\pi}{4}$	1 point

Scoring notes:

- Supporting work is required in (i) and (ii). “Scratchwork” can be ignored; the use of a variable other than x is acceptable. Arithmetic errors following a correct solution may be considered scratchwork.
- Supporting work in (ii) must include $\frac{x}{2} = \sin\left(\frac{\pi}{4}\right)$ OR $\frac{x}{2} = \frac{\sqrt{2}}{2}$ OR $\frac{x}{2} = \frac{1}{\sqrt{2}}$.
- An alternate solution for (i) is $x = \frac{\log_b 10}{\log_b e} - 3$, where $b > 0$, $b \neq 1$, and the result is evaluated according to bullets 2 and 3 in the directions.
- Where applicable, answers that have not been evaluated according to bullets 2 and 3 in the directions do not earn the point. Rationalizing denominators is not required.
- A response that does not earn either point in Part (A) is eligible for **partial credit** in Part (A) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **1-0** in Part (A).

First Column	Second Column
Correct answer in (i) without supporting work	Correct answer in (ii) without supporting work
Correct answer in (i) with supporting work, but the answer has not been evaluated according to bullets 2 and 3 in the directions. No incorrect work.	Correct answer in (ii) with supporting work, but the answer has not been evaluated according to bullets 2 and 3 in the directions. No incorrect work.
Answer in (i) is reported as $x + 3 = \ln 10$. No incorrect work follows.	Answer in (ii) is reported as $\frac{x}{2} = \sin\left(\frac{\pi}{4}\right)$. No incorrect work follows.
	Answer in (ii) is reported as $\frac{x}{2} = \frac{\sqrt{2}}{2}$. No incorrect work follows.
	Answer in (ii) is reported as $\frac{x}{2} = \frac{1}{\sqrt{2}}$. No incorrect work follows.

(B) The functions j and k are given by

$$j(x) = \log_{10}(8x^5) + \log_{10}(2x^2) - 9\log_{10}x$$

$$k(x) = \left(\frac{1 - \sin^2 x}{\sin x}\right)\sec x.$$

- (i) Rewrite $j(x)$ as a single logarithm base 10 without negative exponents in any part of the expression. Your result should be of the form $\log_{10}(\text{expression})$.
- (ii) Rewrite $k(x)$ as a single term involving $\tan x$.

<p>(i) $j(x) = \log_{10}(8x^5) + \log_{10}(2x^2) - 9\log_{10}x$</p> $j(x) = \log_{10}(8x^5 \cdot 2x^2) - \log_{10}x^9$ $j(x) = \log_{10}\left(\frac{8x^5 \cdot 2x^2}{x^9}\right)$ $j(x) = \log_{10}\left(\frac{16x^7}{x^9}\right)$ $j(x) = \log_{10}\left(\frac{16}{x^2}\right), x > 0$	<p>Expression for $j(x)$ 1 point</p>
<p>(ii) $k(x) = \left(\frac{1 - \sin^2 x}{\sin x}\right)\sec x$</p> $k(x) = \left(\frac{\cos^2 x}{\sin x}\right)\left(\frac{1}{\cos x}\right)$ $k(x) = \left(\frac{\cos x}{\sin x}\right)$ $k(x) = \frac{1}{\tan x}, \sin x \neq 0, \cos x \neq 0$	<p>Expression for $k(x)$ 1 point</p>

Scoring notes:

- Supporting work is required in (i) and (ii). “Scratchwork” can be ignored; the use of a variable other than x is acceptable.
- Domain restrictions are not required to be included and are not scored.
- Where applicable, answers that have not been evaluated according to bullets 2 and 3 in the directions do not earn the point.
- To earn the first point, use of “log” rather than “ \log_{10} ” is acceptable.
- The expression $j(x) = \log_{10}\left(\frac{4}{x}\right)^2$ earns the point in (i) with supporting work.
- A logarithmic expression that is missing one or both parentheses around the full argument of the logarithm is still eligible to earn the point.
- If a response is presented as a complex fraction, the complex fraction must be unambiguous in structure. Parentheses must be used correctly, and/or the fraction bars must be clearly and correctly proportioned.

- A response that does not earn either point in Part (B) is eligible for **partial credit** in Part (B) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **1-0** in Part (B).

First Column	Second Column
Correct expression in (i) without supporting work	Correct expression in (ii) without supporting work
Expression in (i) is reported as $\log_{10}\left(\frac{8x^5 \cdot 2x^2}{x^9}\right)$. No incorrect work follows.	Expression in (ii) is reported as $\frac{\cos x}{\sin x}$ OR $\cot x$. No incorrect work follows.
Expression in (i) is reported as $\log_{10}\left(\frac{16x^7}{x^9}\right)$. No incorrect work follows.	Expression in (ii) includes a correct application of a Pythagorean identity with no incorrect work.
Expression in (i) is reported as $2\log_{10}\left(\frac{4}{x}\right)$. No incorrect work follows.	
Expression in (i) is reported as $-\log_{10}\left(\frac{x^2}{16}\right)$. No incorrect work follows.	
Expression in (i) is reported as $-2\log_{10}\left(\frac{x}{4}\right)$. No incorrect work follows.	
The expression in (i) is reported using natural logarithm and has the correct argument OR any of the expressions in partial credit rows two through six above are presented with natural logarithm.	

(C) The function m is given by

$$m(x) = \cos^{-1}(\tan(2x)).$$

Find all values in the domain of m that yield an output value of 0.

$m(x) = 0 \Rightarrow \cos^{-1}(\tan(2x)) = 0$ $\tan(2x) = \cos(0)$ $\tan(2x) = 1$ $2x = \frac{\pi}{4} + \pi n$ $x = \frac{\pi}{8} + \frac{\pi}{2}n$, where n is any integer	One value of x	1 point
	All values of x	1 point

Scoring notes:

- Supporting work is required. “Scratchwork” can be ignored; the use of a variable other than x is acceptable.
- A response with supporting work that gives all correct values for x , such as $x = \frac{\pi}{8} + \pi n$ and $x = \frac{5\pi}{8} + \pi n$, earns both points.
- When expressing a general solution for all values for x (e.g., $x = \frac{\pi}{8} + \frac{\pi}{2}n$), the response can use i , k , n , or any letter except x , which is the variable used in the function.
- To earn the second point, “where n is any integer” is not required to be included.
- To earn the second point, no incorrect values for x are included.

Total for question 4

6 points

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NO CALCULATOR ALLOWED

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Answer QUESTION 4 part (A) on this page.

Response for question 4(A)

(i)

$$g(x) = 10$$

$$e^{(x+3)} = 10$$

$$\ln 10 = x + 3$$

$$x = \ln 10 - 3$$

(ii)

$$h(x) = \sin^{-1}\left(\frac{x}{2}\right)$$

Domain: $[-1, 1]$

$$h(x) = \frac{\pi}{4}$$

$$\sin^{-1}\left(\frac{x}{2}\right) = \frac{\pi}{4}$$



$$\frac{x}{2} = \frac{\sqrt{2}}{2}$$

$$x = \sqrt{2}$$

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4 4 4 4 4 NO CALCULATOR ALLOWED 4 4 4 4 4

Answer QUESTION 4 parts (B) and (C) on this page.

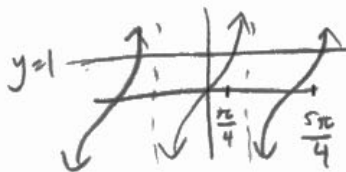
Response for question 4(B)

$$\begin{aligned}
 \text{(i)} \quad j(x) &= \log_{10}(8x^5) + \log_{10}(2x^2) - 9 \log_{10} x \\
 &= \log_{10} \frac{(\cancel{8x^5})(\cancel{2x^2})}{\cancel{x^9}} \\
 &= \log_{10} \frac{16}{x^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad k(x) &= \left(\frac{1 - \sin^2 x}{\sin x} \right) \sec x \\
 &= \frac{\cancel{\cos^2 x}}{\sin x} \cdot \frac{1}{\cancel{\cos x}} \\
 &= \frac{1}{\tan x}
 \end{aligned}$$

Response for question 4(C)

$$\begin{aligned}
 m(x) &= 0 \\
 \cos^{-1}(\tan(2x)) &= 0 \\
 \tan(2x) &= 1 \\
 2x &= \frac{\pi}{4} \\
 x &= \frac{\pi}{8}
 \end{aligned}$$



$$x = \frac{\pi}{8} + \frac{\pi}{2}k \quad \forall k \in \mathbb{Z}$$

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NO CALCULATOR ALLOWED

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Answer QUESTION 4 part (A) on this page.

Response for question 4(A)

$$(i) \quad g(x) = e^{x+3}$$

$$10 = e^{x+3}$$

$$\ln 10 = x + 3$$

$$x = \ln 10 - 3$$

(ii)

$$\frac{\pi}{4} = \arcsin\left(\frac{x}{2}\right)$$

$$\sin\left(\frac{\pi}{4}\right) = \frac{x}{2}$$

$$2 \cdot \frac{\sqrt{2}}{2} = \frac{x}{2} \cdot 2$$

$$x = \sqrt{2}$$

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Answer QUESTION 4 parts (B) and (C) on this page.

Response for question 4(B)

$$\begin{aligned}
 \text{(i)} \quad & \log_{10}(8x^5) + \log_{10}(2x^2) - 9\log_{10}x \\
 & \log_{10}(8x^5) + \log_{10}(2x^2) - \log_{10}(x^9) \\
 & \log_{10}(8x^5 \cdot 2x^2) - \log_{10}(x^9) \\
 & \log_{10}\left(\frac{8x^5 \cdot 2x^2}{x^9}\right) = \log_{10}\left(\frac{16x^7}{x^9}\right) \\
 & \log_{10}\left(\frac{16}{x^2}\right)
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad & k(x) = \left(\frac{1 - \sin^2 x}{\sin x}\right) \sec x \\
 & = \left(\frac{\cos^2 x}{\sin x}\right) \left(\frac{1}{\cos x}\right) \\
 & = \frac{\cos x \cancel{\cos x}}{\sin x \cancel{\cos x}} = \frac{\cos x}{\sin x} = \cot x = \boxed{\frac{1}{\tan x}}
 \end{aligned}$$

Response for question 4(C)

$$\theta = \cos^{-1}(\tan(2x))$$

$$\cos(\theta) = \tan(2x)$$

$$1 = \tan(2x)$$

$$\tan^{-1}(1) = 2x$$

$$\frac{\pi}{4}, \frac{3\pi}{4} = \frac{2x}{2}$$

$$x = \frac{\pi}{8}, \frac{3\pi}{8}$$

$$1 = \frac{\cancel{2\sin x} \cos x}{\cos^2 x - \cancel{2\sin^2 x}}$$

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NO CALCULATOR ALLOWED

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Answer QUESTION 4 part (A) on this page.

Response for question 4(A)

(i) $10 = e^{x+3}$

$x = \ln 10 - 3$

$\ln 10 = x + 3$

(ii)

$\frac{\pi}{4} = \arcsin \frac{x}{2}$

$\sin \frac{\pi}{4} = \frac{x}{2}$

$\frac{\sqrt{2}}{2} = \frac{x}{2}$

$x = \sqrt{2}$

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NO CALCULATOR ALLOWED

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Answer QUESTION 4 parts (B) and (C) on this page.

Response for question 4(B)

(i)

$$\log_{10} 8x^5 + \log_{10} 2x^2 - 9 \log_{10} x$$

$$\log_{10} \frac{(8x^5)(2x^2)}{x^9}$$

$$\frac{10}{9} \log_{10} \frac{16x}{x}$$

(ii)

$$\left(\frac{1 - \sin^2 x}{\sin x} \right) \sec x$$

$$\frac{\cos^2 x}{\sin x} \cdot \frac{\cos x}{\cos x} = \tan x$$

Response for question 4(C)

$$0 = \cos^{-1}(\tan(2x))$$

$$\cos(0) = \tan 2x$$

$$1 = \tan 2x$$

$$\tan^{-1}(1) = 2x$$

$$\frac{\pi}{8} = x$$

$$\frac{\pi}{4} = 2x$$

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Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 4: Symbolic Manipulations

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

This question assesses facility with symbolic manipulation of exponential, logarithmic, trigonometric, and inverse trigonometric functions. Symbolic manipulation is an important theme in the course framework.

- In Part A an exponential function and an inverse trigonometric function are given analytically. Each of these functions is used in an equation that is to be solved, one in part (i) and the other in part (ii). In Part A(i) a response should present the work and solution to an equation involving an exponential expression (Skill 1.A, LO 2.13.A, EK 2.13.A.1). In Part A(ii) a response should present the work and solution to an equation involving an inverse trigonometric expression (Skill 1.A, LO 3.10.A, EK 3.10.A.1).
- In Part B two functions are given—function j that involves several terms with logarithm base 10 and function k that involves trigonometric expressions. A response should rewrite the expression for each function in a specified way. In Part B(i) a response is to use rules of logarithms to rewrite $j(x)$ so that its expression involves only one term of the form $\log_{10}(\text{expression})$ (Skill 1.B, LO 2.12.A, EK 2.12.A.1, EK 2.12.A.2). In Part B(ii) a response is to use trigonometric identities to rewrite $k(x)$ so that its expression has one term involving only $\tan x$ (Skill 1.B, LO 3.12.A, EK 3.12.A.1).
- In Part C a function involving the composition of an inverse trigonometric function and a trigonometric function is given—specifically, $m(x) = \cos^{-1}(\tan(2x))$. A response, showing the work leading to the answer, is to determine all inputs to m that yield an output of 0 (Skill 1.A, LO 3.10.A, EK 3.10.A.1). There are infinitely many such input values. The first point is earned for presenting one of the input values, while the second point is earned for presenting the entire collection of input values.

Sample: 4A**Part A Point 1: 1****Part A Point 2: 1****Part B Point 1: 1****Part B Point 2: 1****Part C Point 1: 1****Part C Point 2: 1****Total Score: 6****Part A (i) (0–1 points): 1**

The response earned the point for presenting the correct answer with supporting work. Line 3 is sufficient for supporting work.

Part A (ii) (0–1 points): 1

The response earned the point for presenting the correct answer with supporting work. Line 4 is sufficient for supporting work. The question requires knowledge of $\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ (OR $\frac{1}{\sqrt{2}}$).

Question 4: Symbolic Manipulations (continued)**Part B (i) (0–1 points): 1**

The response earned the point with a correct answer that is presented according to the instructions given and bullet points 2 and 3 in the directions, together with supporting work. Line 2 is sufficient for supporting work. Although use of parentheses around the log argument is encouraged, the absence of parentheses in line 3 does not affect earning the point.

Part B (ii) (0–1 points): 1

The response earned the point with a correct answer together with supporting work. Line 2 is sufficient for supporting work.

Part C (i) (0–1 points): 1

The response earned the point by declaring that $x = \frac{\pi}{8}$ is among the answers, together with supporting work.

Part C (ii) (0–1 points): 1

The response earned the point for a correct answer. The “ $\forall k \in \mathbb{Z}$ ” (or similar statement such as for any k , where k is an integer) is not required.

Question 4: Symbolic Manipulations (continued)**Sample: 4B****Part A Point 1: 1****Part A Point 2: 1****Part B Point 1: 1****Part B Point 2: 1****Part C Point 1: 1****Part C Point 2: 0****Total Score: 5****Part A (i) (0–1 points): 1**

The response earned the point by presenting the correct answer with supporting work.

Part A (ii) (0–1 points): 1

The response earned the point for presenting the correct answer with supporting work. If the response had omitted one of lines 2 or 3 (but not both), there still would have been sufficient supporting work. The question requires knowledge of $\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ (OR $\frac{1}{\sqrt{2}}$).

Part B (i) (0–1 points): 1

The response earned the point for a correct answer that is presented according to the instructions and bullet points 2 and 3 in the directions, together with supporting work.

Part B (ii) (0–1 points): 1

The response earned the point for a correct answer with supporting work.

Part C (i) (0–1 points): 1

The response earned the point by including a solution of $x = \frac{\pi}{8}$ with supporting work.

Part C (ii) (0–1 points): 0

The response did not earn the point because it presents an incorrect value, $x = \frac{3\pi}{8}$, and does not present a general solution.

Question 4: Symbolic Manipulations (continued)**Sample: 4C****Part A Point 1: 1****Part A Point 2: 1****Part B Point 1: 0****Part B Point 2: 0****Part C Point 1: 1****Part C Point 2: 0****Total Score: 3****Part A (i) (0–1 points): 1**

The response earned the point for presenting the correct answer with supporting work.

Part A (ii) (0–1 points): 1

The response earned the point for presenting the correct answer with supporting work. The question requires knowledge of $\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ (OR $\frac{1}{\sqrt{2}}$).

Part B (i) (0–1 points): 0

The response did not earn the point because the final answer is incorrect. The first expression on line 2 is a correct expression, but it was not correctly presented according to bullets 2 and 3 of the directions.

Part B (ii) (0–1 points): 0

The response did not earn the point because the answer in the final step is incorrect.

Part C (i) (0–1 points): 1

The response earned the point by declaring that $x = \frac{\pi}{8}$ is one of the solutions together with supporting work.

Part C (ii) (0–1 points): 0

The response did not earn the point. No general solution is presented.