2024

AP® Precalculus

Sample Student Responses and Scoring Commentary

Inside:

Free-Response Question 3

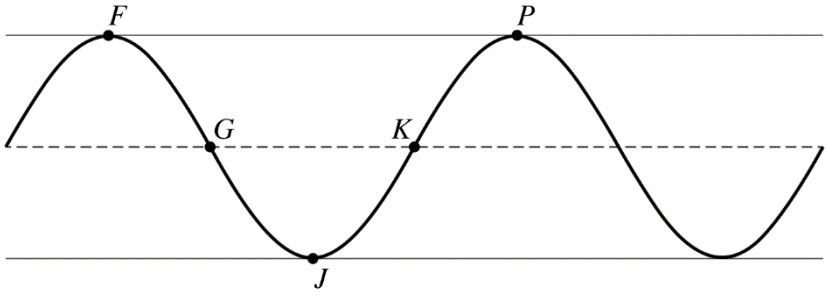
- ✓ **Scoring Guidelines**
- ✓ **Student Samples**
- ✓ **Scoring Commentary**

Question 3: Modeling a Periodic Context
Part B: Graphing calculator not allowed — 6 points

Note: Figure not drawn to scale.

The tire of a car has a radius of 9 inches, and a person rolls the tire forward at a constant rate on level ground, as shown in the figure. Point W on the edge of the tire touches the ground at time $t = \frac{1}{2}$ second. The tire completes a full rotation, and the next time W touches the ground is at time $t = \frac{5}{2}$ seconds. The maximum height of W above the ground is 18 inches. As the tire rolls, the height of W above the ground periodically increases and decreases.

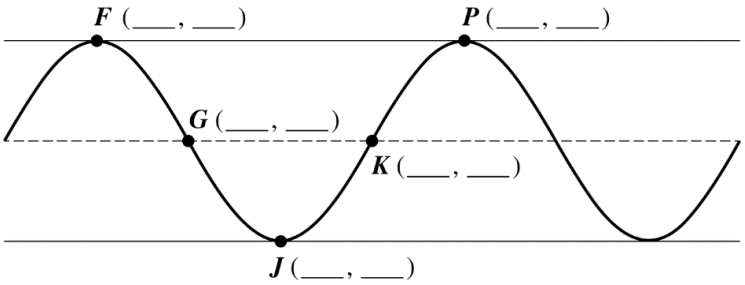
The sinusoidal function h models the height of point W above the ground, in inches, as a function of time t, in seconds.

	Model Solution	Scoring	
(A)	The graph of h and its dashed midline for two full cycles is shown. Five points, F, G, J, K, and P, are labeled on the graph. No scale is indicated, and no axes are presented. Determine possible coordinates $(t, h(t))$ for the five points: F, G, J, K, and P.  		
	F has coordinates $\left(\frac{3}{2}, 18\right)$. G has coordinates $(2, 9)$. J has coordinates $\left(\frac{5}{2}, 0\right)$. K has coordinates $(3, 9)$. P has coordinates $\left(\frac{7}{2}, 18\right)$. OR	$h(t)$-coordinates	**1 point**
		t-coordinates	**1 point**

F has coordinates $\left(-\frac{1}{2}, 18\right)$. G has coordinates $(0, 9)$. J has coordinates $\left(\frac{1}{2}, 0\right)$. K has coordinates $(1, 9)$. P has coordinates $\left(\frac{3}{2}, 18\right)$. Note: t-coordinates will vary. A correct set of coordinates for one full cycle of h as pictured is acceptable.	

Scoring notes:

- No supporting work is required.
- $h(t)$-coordinates and/or t-coordinates may appear in a list. Negative t-coordinates are acceptable.
- t-coordinates must be $-\frac{1}{2} + 2k, 0 + 2k, \frac{1}{2} + 2k, 1 + 2k, \frac{3}{2} + 2k$, for a specific integer k.
- If the graph is used to record coordinates, that work is scored. In this case, other work is considered scratchwork and is not scored. Use of the graph is not required.



- A response that does not earn either point in Part (A) is eligible for **partial credit** in Part (A) if the response meets one of the following criteria. Partial credit response is scored **0-1** in Part (A).
  - All 5 points in the form $(h(t), t)$ with correct input values and correct output values swapped
  - 3 correct points out of the 5 points
  - All 5 points $(t, h(t))$ meet these requirements:
    - t-coordinates in arithmetic sequence with $\Delta t = \frac{1}{2}$
    - $h(t)$-coordinates are such that
      (1) F and P have same $h(t)$-coordinate
      (2) G and K have same $h(t)$-coordinate, which is less than $h(t)$-coordinate of F and P
      (3) Difference in $h(t)$-coordinates for F and G equals difference in $h(t)$-coordinates for G and J

(B) The function h can be written in the form $h(t) = a\sin(b(t+c)) + d$. Find values of constants a, b, c, and d.

$h(t) = a\sin(b(t+c)) + d$ $a = 9$ $\frac{2\pi}{b} = 2$, so $b = \frac{2\pi}{2} = \pi$ $c = -1$ $d = 9$ $h(t) = 9\sin(\pi(t-1)) + 9$ OR $a = -9$ $\frac{2\pi}{b} = 2$, so $b = \frac{2\pi}{2} = \pi$ $c = 0$ $d = 9$ $h(t) = -9\sin(\pi t) + 9$ Note: Based on horizontal shifts and reflections, there are other correct forms for $h(t)$.	Vertical transformations: Values of a and d **1 point** Horizontal transformations: Values of b and c **1 point**

Scoring notes:

- No supporting work is required.
- Points are earned for correct values in a list OR for correct values in an expression for $h(t)$. Only one of these answer presentations is required.
- If the answer box is used to record values, that work is scored. In this case, other work is considered scratchwork and is not scored. Use of the answer box is not required.

$a =$ _____

$b =$ _____

$c =$ _____

$d =$ _____

- Other correct values of c:
  - $h(t) = 9\sin(\pi(t+c)) + 9 \Rightarrow c = -1 + 2k$, for any integer k
  - $h(t) = -9\sin(\pi(t+c)) + 9 \Rightarrow c = 0 + 2k$, for any integer k
- Full credit for Part (B) is possible based on the correct use of an imported response from Part (A) that meets these criteria:
  - $a \neq 1$, $b \neq 1$, $d \neq 0$, and if $a > 0$, then $c \neq 0$
  - All 5 points $(t, h(t))$ meet these requirements:
    - t-coordinates in arithmetic sequence with $\Delta t = \frac{1}{2}$
    - $h(t)$-coordinates are such that
      (1) F and P have same $h(t)$-coordinate
      (2) G and K have same $h(t)$-coordinate, which is less than $h(t)$-coordinate of F and P
      (3) Difference in $h(t)$-coordinates for F and G equals difference in $h(t)$-coordinates for G and J

- A response that does not earn either point in Part (B) is eligible for **partial credit** in Part (B) if the response meets one of the following criteria. Partial credit response is scored **1-0** in Part (B).
  - Values of a and b [Values of a and b could be $\pm$]
  - Values of b and d [Value of b could be $\pm$]
  - Response uses $h(t) = a\cos(b(t+c)) + d$ with values as follows:
    - $a = 9$; $b = \pi$; $c = -\frac{3}{2} + 2k$, for a specific integer k; $d = 9$
    - $a = -9$; $b = \pi$; $c = -\frac{1}{2} + 2k$, for a specific integer k; $d = 9$

(C) Refer to the graph of h in part (A). The t-coordinate of K is t_1, and the t-coordinate of P is t_2.

(i) On the interval (t_1, t_2), which of the following is true about h ?

a. h is positive and increasing.
b. h is positive and decreasing.
c. h is negative and increasing.
d. h is negative and decreasing.

(ii) Describe how the rate of change of h is changing on the interval (t_1, t_2).

(i) Choice a.	Function behavior	**1 point**
(ii) Because the graph of h is concave down on the interval (t_1, t_2), the rate of change of h is decreasing on the interval (t_1, t_2).	Change in rate of change	**1 point**

Scoring notes:

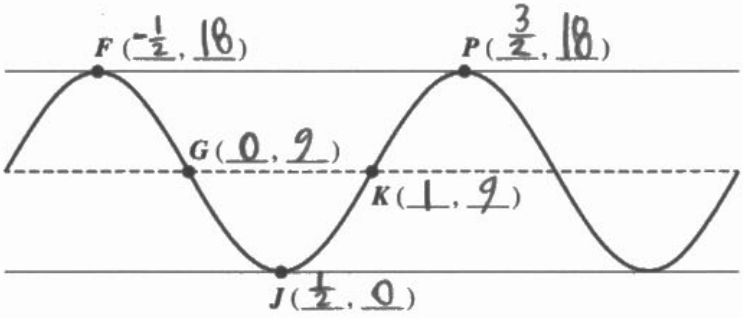
- No supporting work is required.
- The first point is earned for a correct answer of "a" OR "positive and increasing." If both the letter choice and written description are included, the written description is scored.
- To earn the second point, "decreasing" OR "function h is increasing at a decreasing rate" is acceptable. If concavity of the graph of h is referenced, it must be correct.
- The second point is not earned for a response that only includes "the graph of h is concave down."
- A response with a statement that the rate of change of h is decreasing at an increasing (or decreasing) rate does not earn the second point. Analysis to make such a conclusion requires calculus.
- The second point is not earned for a response that states "increasing at a decreasing rate" without a subject. The implied subject is "the rate of change of h."
- The second point cannot be earned if there are any errors in Part (C) (ii).

Total for question 3 **6 points**

3 3 3 3 3 NO CALCULATOR ALLOWED 3 3 3 3 3

Answer QUESTION 3 part (A) on this page.

Response for question 3(A)



3 3 3 3 3 NO CALCULATOR ALLOWED 3 3 3 3 3

Answer QUESTION 3 parts (B) and (C) on this page.

Response for question 3(B)

$h(t) = a\sin(b(t+c)) + d$

$h(t) = -9\sin(\pi(t+0)) + 9$

$a = -9$

$b = \pi$

$c = 0$

$d = 9$

$p = \frac{2\pi}{b}$

$b = \frac{2\pi}{2}$

$b = \pi$

$p = 2$

Response for question 3(C)

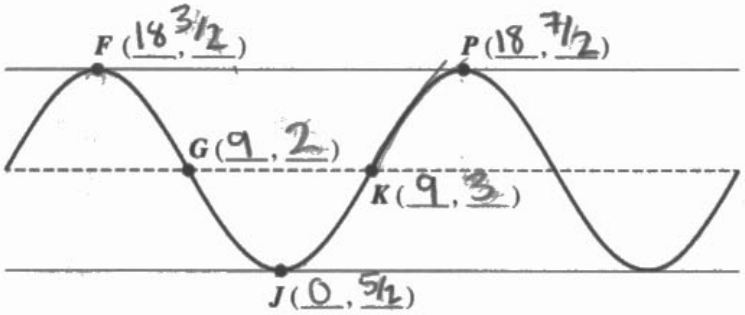
(i) h is positive and increasing

(ii) The rate of change of h is decreasing on the interval (t_1, t_2).

3 3 3 3 3 NO CALCULATOR ALLOWED 3 3 3 3 3

Answer QUESTION 3 part (A) on this page.

Response for question 3(A)



3 3 3 3 3 NO CALCULATOR ALLOWED 3 3 3 3 3

Answer QUESTION 3 parts (B) and (C) on this page.

Response for question 3(B)

$a = 9$
$b = \pi/2$
$c = +1$
$d = 9$

$a=9 \quad a\sin(b(t+c))+d$

$\frac{2\pi}{4} = \frac{\pi}{2} \qquad \frac{2\pi}{\frac{\pi}{2}}$

$2 \times 4 = 8$

~~sin~~

Response for question 3(C)

(i)

a, h is positive and increasing

(ii)

~~From t_1 to t_2 the rate of change is concave down and decreasing.~~

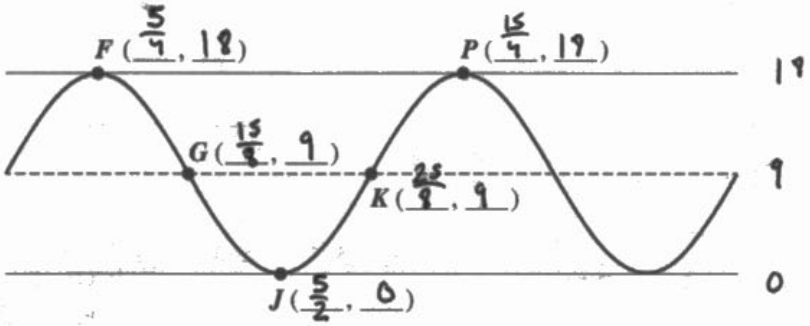
From t_1 to t_2 the rate of change is concave down and decreasing.

3 3 3 3 3 NO CALCULATOR ALLOWED 3 3 3 3 3

Answer QUESTION 3 part (A) on this page.

Response for question 3(A)

Time
rotations



3 3 3 3 3 NO CALCULATOR ALLOWED 3 3 3 3 3

Answer QUESTION 3 parts (B) and (C) on this page.

Response for question 3(B)

$a \sin(b(t+c)) + d$

$a = 9$

$p = \frac{2\pi}{b}$

$\frac{5}{2} = \frac{2\pi}{b}$

$\frac{5b}{5} = \frac{4\pi}{5}$

$b = \frac{4\pi}{5}$

$d = 18$

$9 \sin\left(\frac{4\pi}{5}\left(t + \frac{5}{9}\right)\right) + 18$

$c = \frac{5}{9}$

$a =$ 9

$b =$ $\frac{4\pi}{5}$

$c =$ $\frac{5}{9}$

$d =$ 18

Response for question 3(C)

(i) on the interval (t_1, t_2), h is positive and increasing.

(ii) the AROCs are decreasing b/c on that interval (t_1, t_2) the graph is concave down.

Question 3: Modeling a Periodic Context

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

This question assesses several skills and essential knowledge statements from the course framework in the periodic context of a rolling tire of radius 9 inches. A sinusoidal function h will be used to model the distance from the ground to a fixed point on the tire. Enough information is given to determine that this distance is 0 at times $t = \frac{1}{2}$ and $t = \frac{5}{2}$, and is not 0 between those times.

- In Part A a generic sinusoidal graph is given, without scale or axes. Five points are labeled with letters. The response should give appropriate t-coordinates (Skill 2.B, LO 3.7.A, EK 3.7.A.1, EK 3.7.A.3) and $h(t)$ -coordinates (Skill 2.B, LO 3.7.A, EK 3.7.A.2) for these points based on the context.
- In Part B it is indicated that $h(t) = a\sin(b(t + c)) + d$. The response should present valid values of the four parameters a, b, c, and d (Skill 1.C, LO 3.6.A, EK 3.6.A.6). Finding these values demonstrates an understanding of amplitude, period, phase shift, and vertical shift.
- In Part C(i) a response is to indicate which of four choices accurately describes the behavior of the function h on an interval between two specific points from the graph in Part A (Skill 2.A, LO 1.1.A, EK 1.1.A.3). On this interval, the function h is positive and increasing.
- In part C(ii) a response should state the behavior of the rate of change of h on the same interval (Skill 3.A, LO 1.1.B, EK 1.1.B.4). In this case, because the graph of h is concave down on the interval, the rate of change is decreasing.

Sample: 3A
Part A Point 1: 1
Part A Point 2: 1
Part B Point 1: 1
Part B Point 2: 1
Part C Point 1: 1
Part C Point 2: 1

Total Score: 6

Part A (first point) (0–1 points): 1

The response earned the point with correct $h(t)$ -coordinates for all five points on the graph.

Part A (second point) (0–1 points): 1

The response earned the point with correct t-coordinates for all five points on the graph. Note these values are those in the second set of coordinates from the model solution in the scoring guidelines.

Part B (first point) (0–1 points): 1

The response earned the point for correct values of a and d. The response presents these values in the provided answer box. The response also presents a complete expression for $h(t)$ that is consistent with the values in the answer box. This is not required and, correct or incorrect, does not affect the point when a complete answer is given in the provided box.

Question 3: Modeling a Periodic Context (continued)

Part B (second point) (0–1 points): 1

The response earned the point for correct values of b and c. The response presents these values in the provided answer box. The response also presents a complete expression for $h(t)$ that is consistent with the values in the answer box. This is not required and, correct or incorrect, does not affect the point when a complete answer is given in the provided box.

Part C (i) (0–1 points): 1

The response earned the point with "h is positive and increasing."

Part C (ii) (0–1 points): 1

The response earned the point with "The rate of change of h is decreasing on the interval (t_1, t_2)."

Question 3: Modeling a Periodic Context (continued)

Sample: 3B
Part A Point 1: 0
Part A Point 2: 1
Part B Point 1: 1
Part B Point 2: 1
Part C Point 1: 1
Part C Point 2: 0

Total Score: 4

Part A (first point) (0–1 points): 0

The response did not earn the point because the values of $h(t)$-coordinates for the five points on the graph are incorrect.

Part A (second point) (0–1 points): 1

The response did not initially earn the point because the *t*-coordinates are incorrect. However, the response earned the point by meeting one of the conditions for partial credit in Part A listed in the scoring guidelines. All 5 points are in the form $(h(t), t)$ with correct input values and correct output values swapped. Therefore, Part A is scored 0 for the first point and 1 for the second point.

Part B (first point) (0–1 points): 1

The response earned the point for correct values of *a* and *d* in the provided answer box.

Part B (second point) (0–1 points): 1

The response earned the point for correct values of *b* and *c* in the provided answer box.

Part C (i) (0–1 points): 1

The response *"a*, *h* is positive and increasing" earned the point. Either the letter "*a*" or the phrase "*h* is positive and increasing" would have earned the point.

Part C (ii) (0–1 points): 0

The response did not earn the point because the claim "the rate of change is concave down" requires a calculus-based justification, which is not included. If the statement had included only "decreasing" as the response, it would have earned the point.

Question 3: Modeling a Periodic Context (continued)

Sample: 3C
Part A Point 1: 1
Part A Point 2: 0
Part B Point 1: 0
Part B Point 2: 0
Part C Point 1: 1
Part C Point 2: 1

Total Score: 3

Part A (first point) (0–1 points): 1

The response earned the point with correct $h(t)$ -coordinates for the five points on the graph.

Part A (second point) (0–1 points): 0

The response did not earn the point. The *t*-coordinates are not of the form $-\frac{1}{2}+2k$, $0+2k$, $\frac{1}{2}+2k$, $1+2k$, $\frac{3}{2}+2k$, for a specific integer *k*.

Part B (first point) (0–1 points): 0

The response did not earn the point. The presented value of *d* in the provided answer box is incorrect. The value of *a* in the provided answer box is correct. Because $\Delta t \neq \frac{1}{2}$, the response is not eligible to use imported values from Part A. The response also presents a complete expression for $h(t)$. This is not required and, correct or incorrect, does not affect the point when a complete answer is given in the provided box.

Part B (second point) (0–1 points): 0

The response did not earn the point because the values of *b* and *c* in the provided answer box are incorrect. The response also presents a complete expression for $h(t)$. This is not required and, correct or incorrect, does not affect the point when a complete answer is given in the provided box.

Part C (i) (0–1 points): 1

The response earned the point because the response correctly states that "*h* is positive and increasing." The use of the letter "*a*" for the answer choice is not required.

Part C (ii) (0–1 points): 1

The response earned the point with the statement "the AROC's are decreasing … the graph is concave down."