2024

# AP® Precalculus

## Sample Student Responses and Scoring Commentary

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**Free-Response Question 2**

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## Question 2: Modeling a Non-Periodic Context
## Part A: Graphing calculator required — 6 points

On the initial day of sales ($t = 0$) for a new video game, there were $40$ thousand units of the game sold that day. Ninety-one days later ($t = 91$), there were $76$ thousand units of the game sold that day.

The number of units of the video game sold on a given day can be modeled by the function $G$ given by $G(t) = a + b\ln(t+1)$, where $G(t)$ is the number of units sold, in thousands, on day $t$ since the initial day of sales.

| | Model Solution | Scoring |
|---|---|---|
| **(A)** | (i) Use the given data to write two equations that can be used to find the values for constants $a$ and $b$ in the expression for $G(t)$.<br>(ii) Find the values for $a$ and $b$ as decimal approximations. | |
| | (i) Because $G(0) = 40$ and $G(91) = 76$, two equations to find $a$ and $b$ are<br>$a + b\ln(0+1) = 40$<br>$a + b\ln(91+1) = 76$. | Two equations **1 point** |
| | (ii) $a = 40 - b\ln 1 = 40$<br>$b = \dfrac{(76-40)}{\ln 92} = 7.961451$<br>$G(t) = 40 + 7.961\ln(t+1)$ | Values of $a$ and $b$ **1 point** |

**General Scoring Notes for Question 2 Parts (A), (B), and (C):**

- Decimal approximations must be correct to three places after the decimal point by rounding or truncating. Decimal values of $0$ in final digits need not be reported ($2.000 = 2.00 = 2.0 = 2$).
- A **decimal presentation error** occurs when a response is complete and correct, but the answer is reported to fewer digits than required.
- The first decimal presentation error in Question 2 does not earn the point. For each additional part of Question 2 that requires a decimal approximation and contains a decimal presentation error, the response is eligible to earn the point.

**Scoring notes:**

- The first point is earned for presenting two equations involving $a$ and $b$ that use the given input-output pairs.
- The second point is earned for correct values of $a$ and $b$ with or without supporting work. If correct values are identified, work should be ignored.
- The second point is earned for correct values of $a$ and $b$ presented as either stand-alone values OR in an expression for $G(t)$.
- A response is eligible to earn both points with a correct translation to "thousands." Use of $40{,}000$ and $76{,}000$ results in values of $a = 40{,}000$ and $b = 7961$.
- A response that does not earn either point in Part (A) is eligible for **partial credit** in Part (A) if the response has one correct equation in the presence of two equations involving $a$ and $b$ AND one correct value. Partial credit response is scored **1-0** in Part (A).

**(B)** (i) Use the given data to find the average rate of change of the number of units of the video game sold, in thousands per day, from $t = 0$ to $t = 91$ days. Express your answer as a decimal approximation. Show the computations that lead to your answer.

(ii) Use the average rate of change found in (i) to estimate the number of units of the video game sold, in thousands, on day $t = 50$. Show the work that leads to your answer.

(iii) Let $A_t$ represent the estimate of the number of units of the video game sold, in thousands, using the average rate of change found in (i). For $A_{50}$, found in (ii), it can be shown that $A_{50} < G(50)$. Explain why, in general, $A_t < G(t)$ for all $t$, where $0 < t < 91$.

| | | |
|---|---|---|
| (i) $\frac{G(91) - G(0)}{91 - 0} = \frac{(76 - 40)}{91} = 0.395604$<br>The average rate of change is $0.396$ (or $0.395$) thousand units per day. | Average rate of change | **1 point** |
| (ii) The average rate of change is $r = \frac{G(91) - G(0)}{91 - 0} = 0.395604$.<br>The secant line between point $(0, G(0))$ and point $(91, G(91))$ is given by $y = y_1 + r(x - x_1)$, where $(x_1, y_1)$ can be either one of the points.<br><br>Estimates using the average rate of change are given by<br>$y = G(0) + r(t - 0)$<br>OR<br>$y = G(91) + r(t - 91)$.<br>Both of these produce the same estimate.<br><br>For $t = 50$,<br>$y = 40 + r(50 - 0) = 59.780$<br>OR<br>$y = 76 + r(50 - 91) = 59.780$.<br><br>The number of units sold on day $t = 50$ was approximately $59.780$ thousand. | Estimate using average rate of change | **1 point** |
| (iii) The estimate $A_t$ is the $y$-coordinate of a point on the secant line that passes through $(0, G(0))$ and $(91, G(91))$. Because the graph of $G$ is concave down on the interval $(0, 91)$, this secant line is below the graph of $G$ on the interval $(0, 91)$. Therefore, the estimate $A_t$ is less than the value of $G(t)$ for all $t$ on the interval $(0, 91)$. | Answer with explanation | **1 point** |

**Scoring notes:**

- Supporting work is required in (i) and (ii).
- The first point is earned for a correct decimal approximation in the presence of a quotient that uses the given data values. Units are not needed and are ignored if presented.
- Eligibility for the second point:
    - If a response earned the point in (i) without a decimal presentation error, then an estimate in the range $[59.750, 59.805]$ earns the second point in the presence of supporting work.
    - If a response in (i) has a decimal presentation error, the reported value in (i) as the average rate of change can be used to arrive at an estimate in (ii). To earn the second point, the estimate in (ii) must be consistent with both the reported value in (i) and the endpoint used in the supporting work in (ii).
    - If a response in (i) is incorrect, the reported value in (i) as the average rate of change can be used to arrive at an estimate in (ii). To earn the second point, the estimate in (ii) must be consistent with both the reported value in (i) and the endpoint used in the supporting work in (ii).
- The final number in (ii) may be reported as $59$ thousand or $60$ thousand provided the supporting work has a correct decimal approximation for the estimate.
- A response is eligible to earn both points with a correct translation to "thousands."
    - Use of $40{,}000$ and $76{,}000$ results in an answer of $395.604$ in (i).
    - If a response earned the point in (i) without a decimal presentation error, then an estimate in the range $[59{,}750,\ 59{,}805]$ earns the second point in the presence of supporting work.
    - If a response in (i) has a decimal presentation error, the reported value in (i) as the average rate of change can be used to arrive at an estimate in (ii). To earn the second point, the estimate in (ii) must be consistent with both the reported value in (i) and the endpoint used in the supporting work in (ii).
    - If a response in (i) is incorrect, the reported value in (i) as the average rate of change can be used to arrive at an estimate in (ii). To earn the second point, the estimate in (ii) must be consistent with both the reported value in (i) and the endpoint used in the supporting work in (ii).
- A response that does not earn either point in Part (B) (i) and Part (B) (ii) is eligible for **partial credit** in Part (B) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **1-0** in Part (B)(i)/(ii).

| **First Column** | **Second Column** |
|---|---|
| A correct quotient that uses the given data values that is not expressed as a decimal approximation | A correct estimate in (ii) that does not include supporting work |
| A correct quotient that uses the given data values and has a decimal presentation error | Correct supporting work in (ii) that does not provide an estimate |
| A correct average rate of change in (i) that does not include supporting work | |

- To earn the third point, the reasoning must include:
  - The graph of $G$ is concave down OR the rate of change of $G$ is decreasing
  - A reference to the use of a secant line on $0 < t < 91$ OR the use of a linear function with reference to endpoints $0$ and $91$ that provide the placement of the line

**(C)** The makers of the video game reported that daily sales of the video game decreased each day after $t = 91$. Explain why the error in the model $G$ increases after $t = 91$.

| | | |
|---|---|---|
| On day $t = 91$, the output for daily sales and $G(91)$ are the same. For $t > 91$, daily sales are decreasing and $G$ is increasing. Therefore, the absolute value of the difference between the actual daily sales and the daily sales predicted by $G$ is increasing each day for $t > 91$. | Answer with reason | **1 point** |

**Scoring notes:**

- To earn the point, the reasoning must include an implicit or explicit connection between the "function model is increasing" and "daily sales are decreasing."

**Total for question 2 6 points**

2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 part (A) on this page.

Response for question 2(A)

(i)

$40 = a + b\ln(0+1)$

$76 = a + b\ln(91+1)$

(ii)

$a = 40 - b\ln(1)$

$b\ln(1) = 0$

$76 = 40 - b\ln(1) + b\ln(92)$

$36 = b(-\ln(1) + \ln(92))$

$b = \frac{36}{-\ln(1)+\ln(92)}$

$b \doteq 7.9614$

~~$a = 40$~~

$a = 40$

**Answer QUESTION 2 parts (B) and (C) on this page.**

**Response for question 2(B)**

(i) Average rate of change $= \frac{f(t_2) - f(t_1)}{t_2 - t_1} = \frac{76 - 40}{91 - 0} \approx 0.3956$

The average rate of change is about 0.3956 thousand units of video game sold per day.

(ii) $\frac{f(t_2) - f(t_1)}{t_2 - t_1} = \frac{76 - f(50)}{91 - 50} \approx 0.3956$

$f(50) \approx 59.7802$

About 60 thousand units of the video game were sold on day $t = 50$.

(iii) $A_t < G(t)$ for all of $t$ where $0 < t < 91$ because the secant line approximation ($A_t$) increases at a constant rate while $G(t)$ is concave down and increasing at a decreasing rate. This means that for $0 < t < 91$, all approximations using $A_t$ will be less than $G(t)$ as the secant line lies below the curve of $G(t)$.

**Response for question 2(C)**

The error in model G increases after $t = 91$ because it extrapolates the model derived from the data to a time outside the timeframe of the data where the trend of the daily sales may change. As seen in the report, sales begin to decrease after $t = 91$ while G would still predict increasing sales.

2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 part (A) on this page.

Response for question 2(A)

(i)

$40 = a + b\ln(1)$ (1)

$76 = a + b\ln(92)$ (1)

(ii)

$a + b\ln(1) = 40$

$a + b(0) = 40$

$a = 40$

$76 = 40 + b\ln(92)$

$36 = b\ln(92)$

$b = 7.961$

$a = 40$

$b \approx 7.961$

2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

**Answer QUESTION 2 parts (B) and (C) on this page.**

**Response for question 2(B)**

(i)

$$\frac{G(t_2) - G(t_1)}{t_2 - t_1} = \frac{76 - 40}{91 - 0} = \boxed{0.396 \text{ thousand games each day}}$$

(ii)

$$40 + (0.396)(50 - 0) = 40 + 50(0.396) = \boxed{19.8}$$

~~$40 + \left(\frac{36}{91}\right)(50 - 0) = \left(\frac{36}{91}\right)50 = 19.78$~~

(iii) A + ∠ G(t) because model G is a logarithmic graph. Logarithmic graphs have outputs that increase additively for inputs that increase multiplicatively. Since the average rate of change is made only from t=0 and t=91, toward t=91 the logarithmic function increases less over each equal-interval input compared to earlier. This makes the secant line between t=91 and t=0 fall under G(t) for t=0 to t=91.

**Response for question 2(C)**

The error in the model G increases after t=91 because model G is formed with the maximum value of 91, Any further values are an extrapolation.

2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

**Answer QUESTION 2 part (A) on this page.**

**Response for question 2(A)**

(i) $40{,}000 = a + b\ln(t+1)$

$a + b\ln(1)$

$a + b0$

$40{,}000 = a$

$40{,}000 = a + b\ln(0+1)$

$76{,}000 = 40{,}000 + b\ln(91+1$

$-40{,}000$

$\frac{36{,}000}{\ln(92)} = \frac{b\ln(92)}{\ln(92)}$

$7961.4514 = b$

$76{,}000 = 40{,}000 + b\ln(93)$

(ii)

$40{,}000 = a + b\ln(t+1)$

$a + b\ln(1)$

$a + b(0)$

$40{,}000 = a$

$76{,}000 = 40{,}000 + b\ln(91+1)$

$-40{,}000 \quad -40{,}000$

$\frac{36{,}000}{\ln(92)} = \frac{b\ln(92+1}{\ln(92)}$

$7961.451401 = b$

2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

**Answer QUESTION 2 parts (B) and (C) on this page.**

**Response for question 2(B)**

(i) $\frac{76{,}000 - 40{,}000}{91 - 0}$ = 395.604 = ROC

(ii) $40{,}000 + (395.604 \cdot 50) = 59780$

(iii) The reason G(t) is greater than $A_t$ is because $A_t$ is a linear model while G(t) is an exponential model.

**Response for question 2(C)**

The error happen because it was a log graph

# Question 2: Modeling a Non-Periodic Context

**Note:** Student samples are quoted verbatim and may contain spelling and grammatical errors.

## Overview

This question assesses several skills and essential knowledge statements from the course framework in a non-periodic context: sales each day of a particular video game. Daily sales of the video game are given on two days, the first day of sales and 91 days later. A logarithmic function of the form $G(t) = a + b\ln(t+1)$ is used to model daily sales.

- In Part A(i) a response should display two equations that use the data from the question stem and that can be used to find both *a* and *b*. The equations come directly from $G(0) = 40$ and $G(91) = 76$.
- In Part A(ii) a response should give the values of *a* and *b*, accurate to three places after the decimal point. Both points in Part A assess Skill 1.C, LO 2.14.A, and EK 2.14.A.2.
- In Part B(i) a response should use the data from the question stem to demonstrate the computation of the average rate of change of daily sales over an interval (Skill 1.B, LO 1.3.A, EK 1.3.A.3). A decimal approximation to this average rate of change should be presented.
- In Part B(ii) the average rate of change computed in (i) is used to estimate the daily sales on day 50, a day between the two given data points. (Skill 3.B, LO 1.14.C, EK 1.14.C.1)
- In Part B(iii) a response must explain why, for days between 0 and 91, estimates of daily sales using the average rate of change in (i) are all less than what the model *G* predicts. A response is expected to indicate that the estimates come from the secant line on the interval and that the graph of function *G* is concave down on the interval. (Skill 3.C, LO 1.14.C, EK 1.14.C.1)
- In Part C a response is to explain why the error in the model *G* increases after $t = 91$ (Skill 3.C, LO 2.6.B, EK 2.6.B.2). This explanation should include that model *G* and actual sales agree at $t = 91$, but that *G* increases while daily sales decrease after $t = 91$.

**Sample: 2A**
**Part A Point 1: 1**
**Part A Point 2: 1**
**Part B Point 1: 1**
**Part B Point 2: 1**
**Part B Point 3: 1**
**Part C Point: 1**

**Total Score: 6**

### Part A (i) (0–1 points): 1

The response earned the point for presenting two correct equations involving *a* and *b* that use the given input-output pairs.

### Part A (ii) (0–1 points): 1

The response earned the point for the correct values of *a* and *b* as decimal approximations. Decimal answers must be correct, rounded or truncated, to the third digit past the decimal point. Only the first three decimal places after the decimal point are read for credit. The general instructions for the free-response section include the statement "Unless otherwise specified, any decimal approximations reported in your work should be accurate to three places

## Question 2: Modeling a Non-Periodic Context (continued)

after the decimal point." Supporting work is not required and if included, correct or incorrect, does not affect the point.

### Part B (i) (0–1 points): 1

The response earned the point with the correct decimal approximation in the presence of the correct quotient in line 1.

### Part B (ii) (0–1 points): 1

The response earned the point for having the correct decimal approximation in line 2 on the left, with correct supporting work. The equation "$\frac{76 - f(50)}{91 - 50} = 0.3956$" in line 1 is sufficient work for using the average rate of change to compute the estimate.

### Part B (iii) (0–1 points): 1

The response earned the point for correctly identifying $A_t$ in line 2 as "the secant line approximation" of $G(t)$ on the given interval, along with correctly saying in line 2 "$G(t)$ is concave down."

### Part C (0–1 points): 1

The response earned the point with the statement in lines 3–4 "sales begin to decrease after $t = 91$ while $G$ would still predict increasing sales."

## Question 2: Modeling a Non-Periodic Context (continued)

**Sample: 2B**
**Part A Point 1: 1**
**Part A Point 2: 1**
**Part B Point 1: 1**
**Part B Point 2: 0**
**Part B Point 3: 1**
**Part C Point: 0**

**Total Score: 4**

### Part A (i) (0–1 points): 1

The response earned the point by presenting two correct equations involving $a$ and $b$ that use the given input-output pairs.

### Part A (ii) (0–1 points): 1

The response earned the point for the correct values of $a$ and $b$ as decimal approximations. Decimal answers must be correct, rounded or truncated, to the third digit past the decimal point. Supporting work is not required and if included, correct or incorrect, does not affect the point.

### Part B (i) (0–1 points): 1

The response earned the point with the correct decimal approximation in the presence of a quotient that uses the given data values. The units "thousand games each day" are not required.

### Part B (ii) (0–1 points): 0

This response did not earn the point because the reported estimate is incorrect.

### Part B (iii) (0–1 points): 1

The response earned the point by stating "the logarithmic function increases less over each equal interval input compared to earlier" in lines 5–7 with the reference to the use of a secant line in line 7.

### Part C (0–1 points): 0

The response did not earn the point because a connection between the function model increasing and sales decreasing is not communicated.

## Question 2: Modeling a Non-Periodic Context (continued)

**Sample: 2C**
**Part A Point 1: 0**
**Part A Point 2: 1**
**Part B Point 1: 1**
**Part B Point 2: 1**
**Part B Point 3: 0**
**Part C Point: 0**

**Total Score: 3**

### Part A (i) (0–1 points): 0

The response did not earn the point. Although expressing values in thousands is eligible to earn the point, and the response has the correct values in thousands, one of the declared equations is incorrectly presented as "$76,000 = 40,000 + b\ln(93)$." The right side of this equation should have had $\ln(92)$ rather than $\ln(93)$.

### Part A (ii) (0–1 points): 1

The response earned the point for correct values of $a$ and $b$ in the case of an expression in thousands. In this case, the value $b = 7961$ is sufficient and the added decimal digits as presented in this response are not required.

### Part B (i) (0–1 points): 1

The response earned the point for a correct decimal approximation in the presence of the quotient using the given data values in thousands.

### Part B (ii) (0–1 points): 1

The response earned the point for an estimate reported in thousands in the presence of supporting work.

### Part B (iii) (0–1 points): 0

The response did not earn the point because it does not include a reference to the concavity of the graph of $G$.

### Part C (0–1 points): 0

The response did not earn the point because a connection between the function model increasing and sales decreasing is not included.