

**2024**

**AP**<sup>®</sup>



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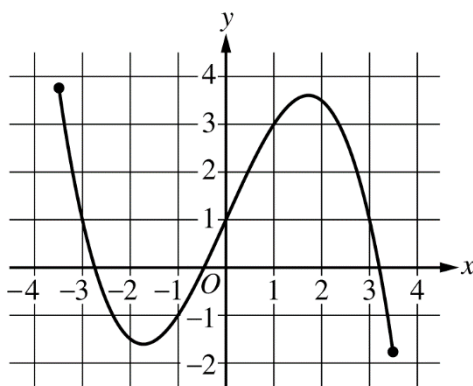
# **AP<sup>®</sup> Precalculus**

## **Sample Student Responses and Scoring Commentary**

### **Inside:**

#### **Free-Response Question 1**

- ✓ **Scoring Guidelines**
- ✓ **Student Samples**
- ✓ **Scoring Commentary**

**Question 1: Function Concepts****Part A: Graphing calculator required****6 points**Graph of  $f$ 

The figure shows the graph of the function  $f$  on its domain of  $-3.5 \leq x \leq 3.5$ . The points  $(-3, 1)$ ,  $(0, 1)$ , and  $(3, 1)$  are on the graph of  $f$ . The function  $g$  is given by  $g(x) = 2.916 \cdot (0.7)^x$ .

**Model Solution****Scoring**

- (A) (i) The function  $h$  is defined by  $h(x) = (g \circ f)(x) = g(f(x))$ . Find the value of  $h(3)$  as a decimal approximation, or indicate that it is not defined.
- (ii) Find all values of  $x$  for which  $f(x) = 1$ , or indicate that there are no such values.

(i) $h(3) = g(f(3)) = g(1) = 2.041$	Value	<b>1 point</b>
(ii) From the graph, $f(x) = 1$ when $x = -3$ , $x = 0$ , and $x = 3$ .	Values	<b>1 point</b>

**General Scoring Notes for Question 1 Parts (A), (B), and (C):**

- Decimal approximations must be correct to three places after the decimal point by rounding or truncating. Decimal values of 0 in final digits need not be reported ( $2.000 = 2.00 = 2.0 = 2$ ).
- A **decimal presentation error** occurs when a response is complete and correct, but the answer is reported to fewer digits than required.
- The first decimal presentation error in Question 1 does not earn the point. For each additional part of Question 1 that requires a decimal approximation and contains a decimal presentation error, the response is eligible to earn the point.

**Scoring notes:**

- The first point is earned for a correct decimal approximation of 2.041.
- The second point does not require supporting work.

- A response that does not earn either point in Part (A) is eligible for **partial credit** in Part (A) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **0-1** in Part (A).

First Column	Second Column
Correct value in (i) that is not expressed as a decimal approximation	Only one correct value in (ii) with no incorrect values included
Correct value in (i) with a decimal presentation error	Only two correct values in (ii) with no incorrect values included

- (B)** (i) Find all values of  $x$ , as decimal approximations, for which  $g(x) = 2$ , or indicate that there are no such values.
- (ii) Determine the end behavior of  $g$  as  $x$  increases without bound. Express your answer using the mathematical notation of a limit.

(i) $g(x) = 2 \Rightarrow 2.916(0.7)^x = 2$ $x = 1.057$	Value	<b>1 point</b>
(ii) As $x$ increases without bound, the output values of $g$ get arbitrarily close to 0. Therefore, $\lim_{x \rightarrow \infty} g(x) = 0$ .	End behavior with limit notation	<b>1 point</b>

**Scoring notes:**

- The first point is earned for a correct decimal approximation of 1.057. No incorrect values may be included.
- The second point requires a correct limit statement with four components: “lim,” “ $x \rightarrow \infty$ ,” the function  $g$ , and 0. Examples that earn the point include:
  - $\lim_{x \rightarrow \infty} g(x) = 0$  OR  $\lim_{x \rightarrow \infty} g = 0$
  - $\lim_{x \rightarrow \infty} g(x) \rightarrow 0$  OR  $\lim_{x \rightarrow \infty} g \rightarrow 0$
  - $\lim_{x \rightarrow \infty} g(x) \ 0$  OR  $\lim_{x \rightarrow \infty} g \ 0$

If the response includes an additional, complete limit statement (e.g.,  $\lim_{x \rightarrow -\infty} g(x) = \infty$ ), the value of the limit must be correct.

- A response that does not earn either point in Part (B) is eligible for **partial credit** in Part (B) if the response has one criteria from the first column AND one criteria from the second column. Partial credit response is scored **1-0** in Part (B).

First Column	Second Column
Correct answer in (i) that is not expressed as a decimal approximation	Correct end behavior statement in (ii) without use of limit notation
Correct value in (i) with a decimal presentation error	Correct end behavior statement in (ii) with incorrect limit notation
	Correct limit statement in (ii) that is missing “ $x \rightarrow \infty$ ”

- (C) (i) Determine if  $f$  has an inverse function.  
(ii) Give a reason for your answer based on the definition of a function and the graph of  $y = f(x)$ .

(i) $f$ does not have an inverse function on its domain of $-3.5 \leq x \leq 3.5$ .	Answer	<b>1 point</b>
(ii) There are output values of $f$ that are not mapped from unique input values; for example, $f(-3) = f(0) = f(3) = 1$ .	Reason	<b>1 point</b>

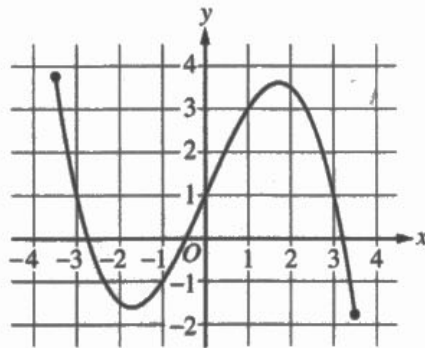
**Scoring notes:**

- The first point is earned for a correct answer.
- Both points may be earned in (ii) provided there is no incorrect response in (i).
- The second point requires an implicit or explicit reference to the definition of a function AND support for the reason by referencing specific function values.
- A response such as “ $f$  does not have an inverse function because  $f(-3) = f(0) = 1$ ” OR “ $f$  does not have an inverse function because there are two input values mapped to 1” earns both points.
- A response such as “ $f$  is not one-to-one” OR “ $f$  fails the horizontal line test” OR “There are output values that are not mapped from unique input values” is not sufficient to earn the second point.
- The second point cannot be earned if there are any errors in Part (C) (ii).
- A response that indicates that  $f$  **has** an inverse function in Part (C) (i) without a reason in Part (C) (i) combined with a response in Part (C) (ii) that provides both the correct answer and a correct reason is scored **0-1**.

**Total for question 1      6 points**

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Answer QUESTION 1 part (A) on this page.

Graph of  $f$ 

Response for question 1(A)

(i)  $f(3) = 1$

$$g(1) = 2.916 \cdot 0.7^1$$

$$= 2.041$$

(ii)

$x = -3, 0, 3$

$x = -3 \quad x = 0 \quad x = 3$

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Answer QUESTION 1 parts (B) and (C) on this page.

Response for question 1(B)

(i)

$$2.916 \cdot 0.7^x = 2$$

$$0.7^x = 0.686$$

$$\log_{0.7} 0.686 = x$$

$$x = 1.057$$

(ii)

$$\lim_{x \rightarrow \infty} g(x) = 0$$

Response for question 1(C)

(i) function  $f$  does not have an inverse.  
No

(ii) Function  $f(x)$  has multiple input values with the same output value. For example,  $(-3, 1)$ ,  $(0, 1)$  and  $(3, 1)$ . So,  $f^{-1}$  would have multiple output values for one input value. For example,  $(1, -3)$ ,  $(1, 0)$  and  $(1, 3)$ .

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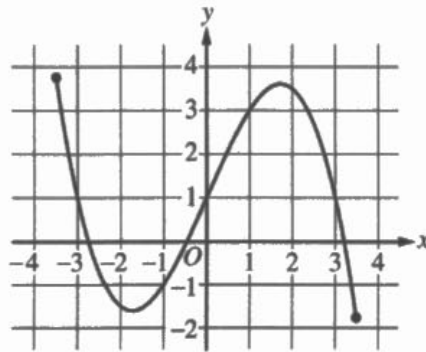
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Answer QUESTION 1 part (A) on this page.

Graph of  $f$ 

Response for question 1(A)

(i)

$$h(3) = g(f(3))$$

$$g(1) = 2.916 \cdot (0.7)$$

$$g(1) = 2.0412$$

$$h(3) = 2.0412$$

(ii)

$$x = -3, 0, 3$$

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Answer QUESTION 1 parts (B) and (C) on this page.

Response for question 1(B)

(i)

$$X = 1.057$$

(ii)

$$\lim_{x \rightarrow \infty} f(x) = 0$$

Response for question 1(C)

(i)

NO

(ii)

f doesn't have an inverse function because it doesn't pass the horizontal line test. In order for a function to have an inverse, it must pass both the horizontal and vertical line test.

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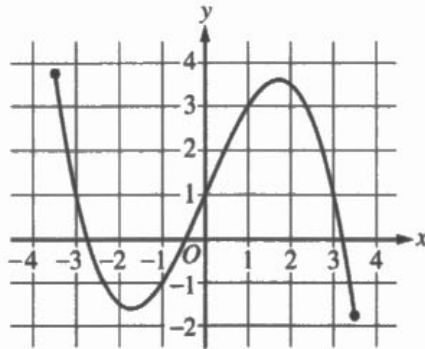
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Answer QUESTION 1 part (A) on this page.

Graph of  $f$ 

Response for question 1(A)

(i)

$$2.916 \cdot (0.7)' = \boxed{2.041}$$

(ii)

$$(-3, 1)$$

$$(0, 1)$$

$$(3, 1)$$

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Answer QUESTION 1 parts (B) and (C) on this page.

Response for question 1(B)

$$(i) \quad 2.916 (0.7)^2 \\ 2.916 (0.49) = \boxed{1.429}$$

(ii)

$$\lim_{x \rightarrow \infty} g(x) = -\infty$$

Response for question 1(C)

(i) No,  $f$  can not have an inverse.

(ii)

To have an inverse function there must be no repeated  $x$ 's or  $y$ 's, in the graph of  $f$  there are repeated  $y$ 's.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

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**Question 1: Function Concepts**

**Note:** Student samples are quoted verbatim and may contain spelling and grammatical errors.

**Overview**

This question assesses knowledge of and skill with specific function concepts from the course framework. It involves functions that are presented in different representations, one given graphically and the other given analytically.

- To complete Part A(i), one must identify from the graph the output of the function  $f$  when 3 is the input (Skill 2.A) and use it to compute the value of the composition  $g(f(3))$ . (LO 2.7.A, EK 2.7.A.2)
- In Part A(ii) the response must demonstrate an ability to identify from the graph of the function  $f$  inputs that produce the specific output 1 (Skill 2.A, LO 2.8.A, EK 2.8.A.2). In this case, there were three such inputs:  $x = -3$ ,  $x = 0$ , and  $x = 3$ .
- In Part B(i) one must do similar work as in A(ii) but with the analytically presented function  $g$ . A graphing calculator is used to solve an equation to find all domain values that produce an output of 2. (Skill 1.A, LO 2.13.A, EK 2.13.A.2)
- Part B(ii) asks for the right end behavior of  $g$ , an exponential decay function. The response requires the use of proper limit notation in stating  $\lim_{x \rightarrow \infty} g(x) = 0$ . (Skill 3.A, LO 2.3.A, EK 2.3.A.5)
- Part C requires (i) determining that the function  $f$  does not have an inverse function (Skill 1.C, LO 2.8.A, EK 2.8.A.1) and (ii) giving reasoning for this answer (Skill 3.C, LO 2.8.A, EK 2.8.A.1). The reasoning must be specific to the function  $f$ , such as stating that the output 1 comes from the multiple inputs  $x = -3$ ,  $x = 0$ , and  $x = 3$ .

**Sample: 1A****Part A Point 1: 1****Part A Point 2: 1****Part B Point 1: 1****Part B Point 2: 1****Part C Point 1: 1****Part C Point 2: 1****Total Score: 6****Part A (i) (0–1 points): 1**

The response earned the point in line 2 with “2.041.” Answers must be correct, rounded or truncated, to the third place after the decimal point. The presence of supporting work, correct or incorrect, does not affect this point. The general instructions for the free-response section include the statement “Unless otherwise specified, any decimal approximations reported in your work should be accurate to three places after the decimal point.”

**Part A (ii) (0–1 points): 1**

The response earned the point in line 1 with all three correct  $x$ -values. This point does not require supporting work.

### Question 1: Function Concepts (continued)

#### Part B (i) (0–1 points): 1

The response earned the point in line 4 with “ $x = 1.057$ .” The presence of supporting work, correct or incorrect, does not affect the point.

#### Part B (ii) (0–1 points): 1

The response earned the point with the correct answer. The limit notation is correct.

#### Part C (i) (0–1 points): 1

The response earned the point with “function  $f$  does not have an inverse” in line 1. The “No” in line 2 would have also earned the point.

#### Part C (ii) (0–1 points): 1

The response earned the point with the first two sentences by implicitly addressing the definition of a function and referencing specific function values from the graph,  $(-3, 1)$  and  $(0, 1)$ . The presence of the additional coordinate point  $(3, 1)$  is correct but is not necessary to earn the point (because two values are presented). The point could also have been earned with the third and fourth sentences.

**Question 1: Function Concepts (continued)****Sample: 1B****Part A Point 1: 1****Part A Point 2: 1****Part B Point 1: 1****Part B Point 2: 0****Part C Point 1: 1****Part C Point 2: 0****Total Score: 4****Part A (i) (0–1 points): 1**

The response earned the point in line 3 with “ $g(1) = 2.0412$ ,” which is correct to three decimal places, rounded or truncated. Only the first three decimal places after the decimal point are read for credit. The correct value declared in line 4 is consistent with the value in line 3. The presence of supporting work, correct or incorrect, does not affect the point.

**Part A (ii) (0–1 points): 1**

The response earned the point with all three correct  $x$ -values. This point does not require supporting work.

**Part B (i) (0–1 points): 1**

The response earned the point with “ $x = 1.057$ .” This point does not require supporting work.

**Part B (ii) (0–1 points): 0**

The response did not earn the point. The response incorrectly refers to the function  $f$  in the limit statement.

**Part C (i) (0–1 points): 1**

The response earned the point with “NO.”

**Part C (ii) (0–1 points): 0**

The response did not earn the point. The response does not reference specific function values from the graph, which is required to earn the point. In line 2, “it doesn’t pass the horizontal line test” is not sufficient.

**Question 1: Function Concepts (continued)****Sample: 1C****Part A Point 1: 1****Part A Point 2: 1****Part B Point 1: 0****Part B Point 2: 0****Part C Point 1: 1****Part C Point 2: 0****Total Score: 3****Part A (i) (0–1 points): 1**

The response earned the point with “ $2.916 \cdot (0.7)^1 = 2.041$ .” The presence of supporting work, correct or incorrect, does not affect the point.

**Part A (ii) (0–1 points): 1**

The response earned the point with the implicit presentation of all three correct  $x$ -values, “ $(-3, 1)$ ,” “ $(0, 1)$ ,” and “ $(3, 1)$ .” This point does not require supporting work.

**Part B (i) (0–1 points): 0**

The response did not earn the point because the response presents an incorrect answer.

**Part B (ii) (0–1 points): 0**

The response did not earn the point. Although the limit notation is correct, the response presents incorrect end behavior.

**Part C (i) (0–1 points): 1**

The response earned the point with “No,  $f$  can not have an inverse.”

**Part C (ii) (0–1 points): 0**

The response did not earn the point. The response did not reference specific function values from the graph, which is required to earn the point.