

2024



AP[®] Physics C: Electricity and Magnetism

Free-Response Questions Set 2

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS	
Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^8$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}$ mol ⁻¹	Universal gravitational constant, $G = 6.67 \times 10^{-11}$ (N·m ²)/kg ²
Universal gas constant, $R = 8.31$ J/(mol·K)	Acceleration due to gravity at Earth's surface, $g = 9.8$ m/s ²
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	
1 unified atomic mass unit,	$1 \text{ u} = 1.66 \times 10^{-27}$ kg = 931 MeV/c ²
Planck's constant,	$h = 6.63 \times 10^{-34}$ J·s = 4.14×10^{-15} eV·s
	$hc = 1.99 \times 10^{-25}$ J·m = 1.24×10^3 eV·nm
Vacuum permittivity,	$\epsilon_0 = 8.85 \times 10^{-12}$ C ² /(N·m ²)
Coulomb's law constant, $k = 1/(4\pi\epsilon_0) = 9.0 \times 10^9$ (N·m ²)/C ²	
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}$ (T·m)/A
Magnetic constant, $k' = \mu_0/(4\pi) = 1 \times 10^{-7}$ (T·m)/A	
1 atmosphere pressure,	$1 \text{ atm} = 1.0 \times 10^5$ N/m ² = 1.0×10^5 Pa

UNIT SYMBOLS	meter, m	mole, mol	watt, W	farad, F
	kilogram, kg	hertz, Hz	coulomb, C	tesla, T
	second, s	newton, N	volt, V	degree Celsius, °C
	ampere, A	pascal, Pa	ohm, Ω	electron volt, eV
	kelvin, K	joule, J	henry, H	

PREFIXES		
Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES							
θ	0°	30°	37°	45°	53°	60°	90°
$\sin \theta$	0	1/2	3/5	$\sqrt{2}/2$	4/5	$\sqrt{3}/2$	1
$\cos \theta$	1	$\sqrt{3}/2$	4/5	$\sqrt{2}/2$	3/5	1/2	0
$\tan \theta$	0	$\sqrt{3}/3$	3/4	1	4/3	$\sqrt{3}$	∞

The following assumptions are used in this exam.

- I. The frame of reference of any problem is inertial unless otherwise stated.
- II. The direction of current is the direction in which positive charges would drift.
- III. The electric potential is zero at an infinite distance from an isolated point charge.
- IV. All batteries and meters are ideal unless otherwise stated.
- V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$$v_x = v_{x0} + a_x t$$

$$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$$

$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{J} = \int \vec{F} dt = \Delta\vec{p}$$

$$\vec{p} = m\vec{v}$$

$$|\vec{F}_f| \leq \mu |\vec{F}_N|$$

$$\Delta E = W = \int \vec{F} \cdot d\vec{r}$$

$$K = \frac{1}{2} m v^2$$

$$P = \frac{dE}{dt}$$

$$P = \vec{F} \cdot \vec{v}$$

$$\Delta U_g = mg\Delta h$$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$$

$$I = \int r^2 dm = \sum mr^2$$

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$$

$$v = r\omega$$

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$

$$K = \frac{1}{2} I \omega^2$$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

a = acceleration
 E = energy
 F = force
 f = frequency
 h = height
 I = rotational inertia
 J = impulse
 K = kinetic energy
 k = spring constant
 ℓ = length
 L = angular momentum
 m = mass
 P = power
 p = momentum
 r = radius or distance
 T = period
 t = time
 U = potential energy
 v = velocity or speed
 W = work done on a system
 x = position
 μ = coefficient of friction
 θ = angle
 τ = torque
 ω = angular speed
 α = angular acceleration
 ϕ = phase angle

$$\vec{F}_s = -k\Delta\vec{x}$$

$$U_s = \frac{1}{2} k (\Delta x)^2$$

$$x = x_{max} \cos(\omega t + \phi)$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_s = 2\pi \sqrt{\frac{m}{k}}$$

$$T_p = 2\pi \sqrt{\frac{\ell}{g}}$$

$$|\vec{F}_G| = \frac{Gm_1 m_2}{r^2}$$

$$U_G = -\frac{Gm_1 m_2}{r}$$

ELECTRICITY AND MAGNETISM

$$|\vec{F}_E| = \frac{1}{4\pi\epsilon_0} \left| \frac{q_1 q_2}{r^2} \right|$$

$$\vec{E} = \frac{\vec{F}_E}{q}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$E_x = -\frac{dV}{dx}$$

$$\Delta V = -\int \vec{E} \cdot d\vec{r}$$

$$V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$$

$$U_E = qV = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

$$\Delta V = \frac{Q}{C}$$

$$C = \frac{\kappa\epsilon_0 A}{d}$$

$$C_p = \sum_i C_i$$

$$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$$

$$I = \frac{dQ}{dt}$$

$$U_C = \frac{1}{2} Q\Delta V = \frac{1}{2} C(\Delta V)^2$$

$$R = \frac{\rho\ell}{A}$$

$$\vec{E} = \rho\vec{J}$$

$$I = Nev_d A$$

$$I = \frac{\Delta V}{R}$$

$$R_s = \sum_i R_i$$

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

$$P = I\Delta V$$

A = area
 B = magnetic field
 C = capacitance
 d = distance
 E = electric field
 \mathcal{E} = emf
 F = force
 I = current
 J = current density
 L = inductance
 ℓ = length
 n = number of loops of wire per unit length
 N = number of charge carriers per unit volume
 P = power
 Q = charge
 q = point charge
 R = resistance
 r = radius or distance
 t = time
 U = potential or stored energy
 V = electric potential
 v = velocity or speed
 ρ = resistivity
 Φ = flux
 κ = dielectric constant

$$\vec{F}_M = q\vec{v} \times \vec{B}$$

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{\ell} \times \hat{r}}{r^2}$$

$$\vec{F} = \int I d\vec{\ell} \times \vec{B}$$

$$B_s = \mu_0 n I$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -L \frac{dI}{dt}$$

$$U_L = \frac{1}{2} L I^2$$

ADVANCED PLACEMENT PHYSICS C EQUATIONS

GEOMETRY AND TRIGONOMETRY

Rectangle

$$A = bh$$

Triangle

$$A = \frac{1}{2}bh$$

Circle

$$A = \pi r^2$$

$$C = 2\pi r$$

$$s = r\theta$$

Rectangular Solid

$$V = \ell wh$$

Cylinder

$$V = \pi r^2 \ell$$

$$S = 2\pi r \ell + 2\pi r^2$$

Sphere

$$V = \frac{4}{3}\pi r^3$$

$$S = 4\pi r^2$$

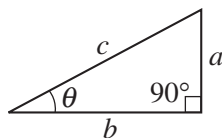
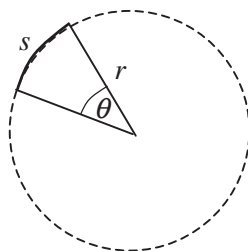
Right Triangle

$$a^2 + b^2 = c^2$$

$$\sin \theta = \frac{a}{c}$$

$$\cos \theta = \frac{b}{c}$$

$$\tan \theta = \frac{a}{b}$$

 A = area C = circumference V = volume S = surface area b = base h = height ℓ = length w = width r = radius s = arc length θ = angle

CALCULUS

$$\frac{df}{dx} = \frac{df}{du} \frac{du}{dx}$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a \cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a \sin(ax)$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax)$$

VECTOR PRODUCTS

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$|\vec{A} \times \vec{B}| = AB \sin \theta$$

Begin your response to **QUESTION 1** on this page.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

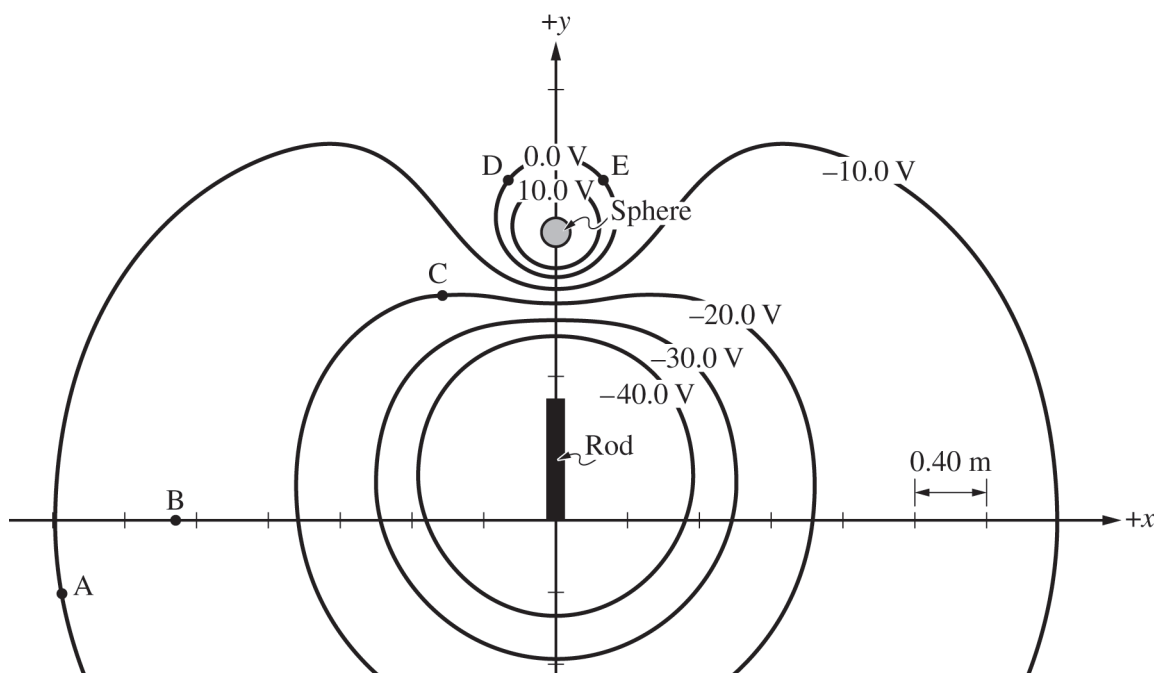


Figure 1

1. A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0 \text{ nC}$. The rod and sphere are held at rest on the y -axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x -axis and on the y -axis are 0.40 m apart.
- (a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the 0.0 V equipotential line.

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Continue your response to **QUESTION 1** on this page.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of $C \rightarrow E \rightarrow D \rightarrow A$. The test charge is held momentarily at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work W_{ED} done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of W_{CE} . If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a “0” in the corresponding columns, as appropriate.

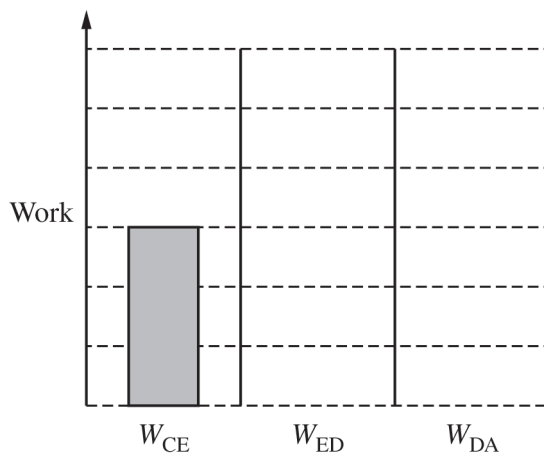


Figure 2

ii. **Calculate** the approximate magnitude of the x -component of the electric field at Position B.

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Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position C. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

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Continue your response to **QUESTION 1** on this page.

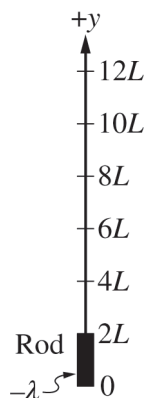


Figure 3

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the y -axis in the orientation shown in Figure 3. Position P (not shown) is located on the y -axis a distance y_P from the origin, where $y_P > 2L$.

(d) The electric potential V_P at y_P is $V_P = -k\lambda \ln\left(\frac{y_P}{y_P - 2L}\right)$.

i. Using integral calculus, **derive** the expression for V_P provided.

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Continue your response to **QUESTION 1** on this page.

- ii. On Figure 4, **sketch** a graph of the y -component E_y of the electric field resulting from the rod as a function of y in the region $2L < y < 12L$.

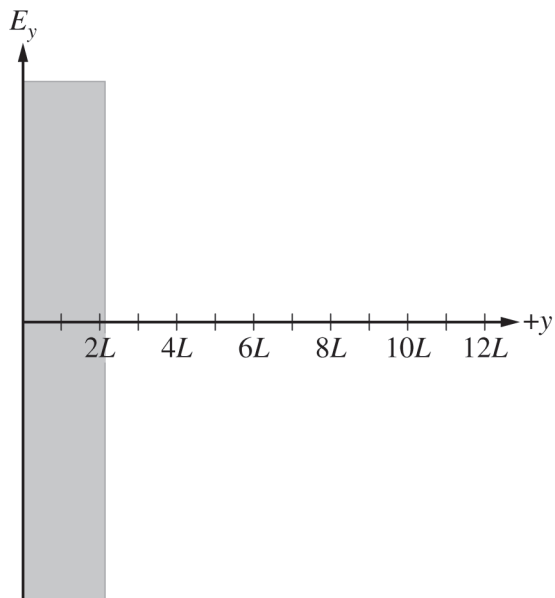


Figure 4

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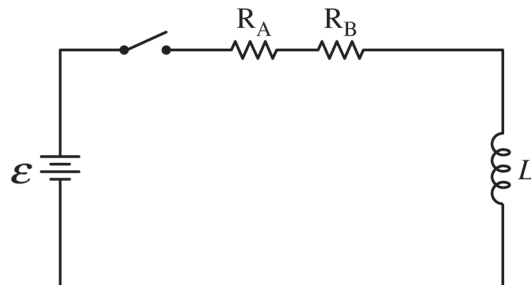


Figure 1

2. Students are asked to determine the resistance R of identical resistors R_A and R_B . The resistors are connected in series with each other, a battery of known emf \mathcal{E} , an inductor of known inductance L , and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that increases with time to determine R .

(a)

- i. On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that increases with time.



Voltmeter Symbol

- ii. **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for R using a measured quantity that increases with time. Provide enough detail so that another student could replicate the experiment.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

(b)

i. On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

- **Label** the quantities graphed on the vertical and horizontal axes.
- **Sketch** a line or curve that represents the expected trend of the collected data.
- **Label** any appropriate intercepts and/or asymptotes in terms of the quantities provided.



Figure 2

ii. **Describe** how the information from the graph in part (b)(i) would be used to determine the experimental value for R .

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

- (c) Starting with an appropriate application of Kirchhoff's loop rule, **derive**, but do NOT solve, a differential equation that can be used to determine the current I in the inductor at time t after the switch is closed. Express your answer in terms of R , \mathcal{E} , L , t , and physical constants, as appropriate.

After reaching steady state, the absolute value of the potential difference across R_A is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across R_A is $|\Delta V_2|$.

- (d) **Indicate** whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.

_____ $|\Delta V_2| > |\Delta V_1|$ _____ $|\Delta V_2| < |\Delta V_1|$ _____ $|\Delta V_2| = |\Delta V_1|$

Justify your answer.

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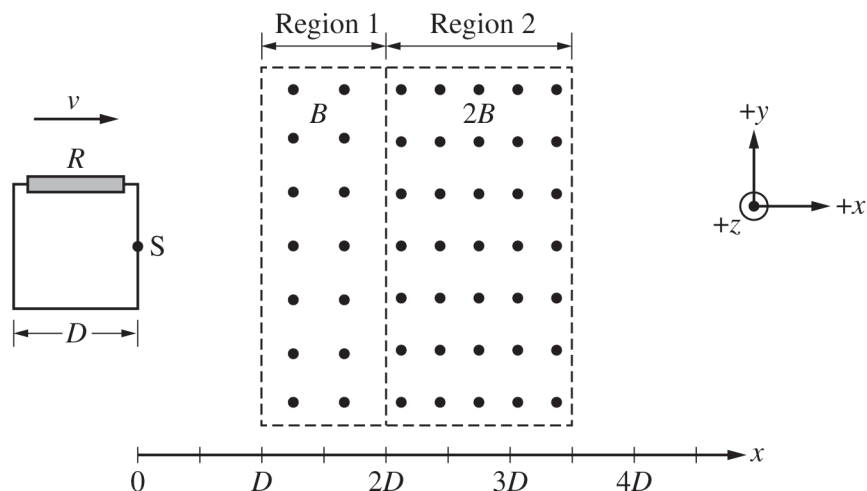
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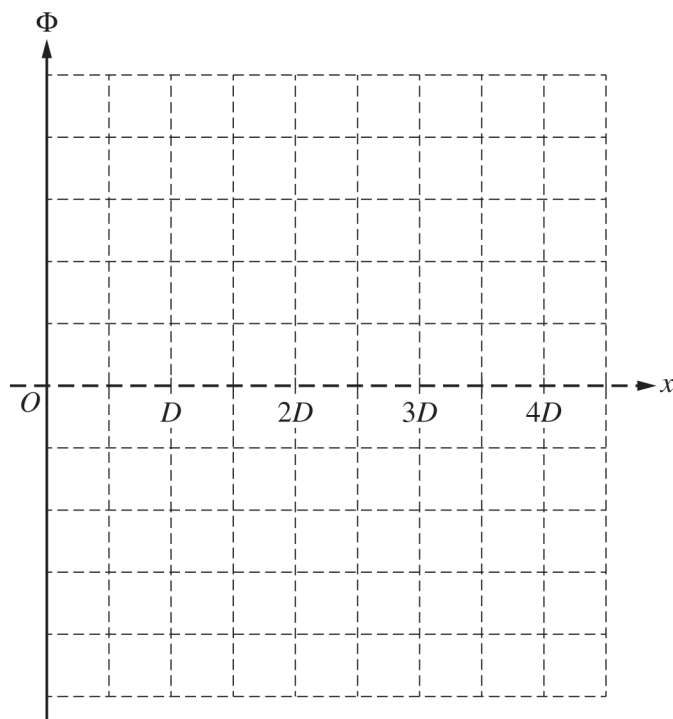
Figure 1

3. A wire is connected to a resistor of resistance R to form a rigid square loop of side length D . An external force is exerted on the loop so that the loop always moves with constant speed v in the $+x$ direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude B that is directed in the $+z$ -direction. Region 1 has boundaries $x = D$ and $x = 2D$. The loop later enters Region 2 of external uniform magnetic field of magnitude $2B$ that is directed in the $+z$ -direction. Region 2 has boundaries $x = 2D$ and $x = 3.5D$. Point S is the midpoint of the leading edge of the loop.

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Continue your response to **QUESTION 3** on this page.

- (a) On the following axes, **sketch** a graph of the magnetic flux Φ through the square loop as a function of the position x of Point S from $x = 0$ to $x = 4.5D$. The $+z$ -direction indicated in Figure 1 corresponds to $+\Phi$.



- (b) Consider the instant when Point S reaches $x = 1.5D$.

i. **Indicate** whether the current I_S that is induced in the square loop when Point S reaches $x = 1.5D$ is clockwise, counterclockwise, or zero.

_____ Clockwise _____ Counterclockwise _____ Zero

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

ii. **Derive** an expression for I_S when Point S reaches $x = 1.5D$. If $I_S = 0$, indicate how the derived expression shows that $I_S = 0$. Express your answer in terms of R , D , v , B , and physical constants, as appropriate.

iii. **Derive** an expression for the power P dissipated by the resistor when Point S reaches $x = 1.5D$. Express your answer in terms of R , D , v , B , and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

The total energy dissipated by the resistor in the square loop as Point S moves from $x = 0$ to $x = 4.5D$ is E_{original} .

The vertical boundary between regions 1 and 2 is now shifted to $x = 2.5D$. After the boundary is shifted, the square loop again moves with speed v in the $+x$ -direction, as shown in Figure 2. The total energy dissipated by the resistor as Point S moves from $x = 0$ to $x = 4.5D$ is E_{new} .

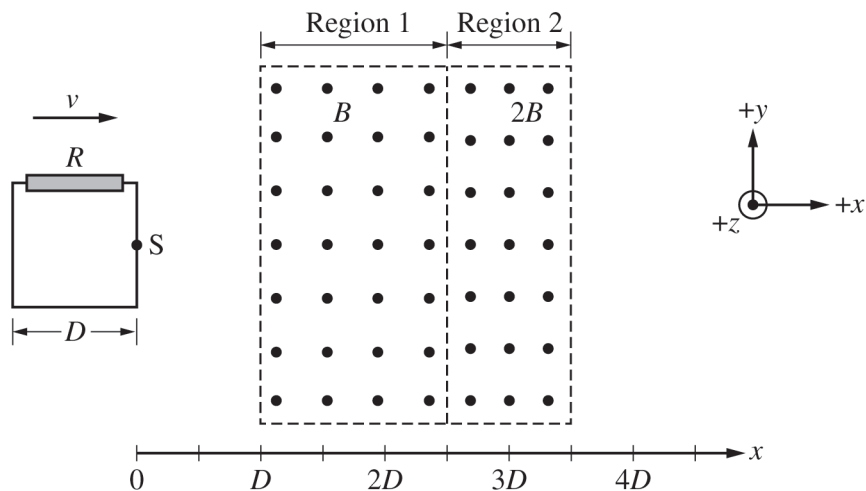


Figure 2

(c) **Indicate** whether E_{new} is greater than, less than, or equal to E_{original} .

_____ $E_{\text{new}} > E_{\text{original}}$ _____ $E_{\text{new}} < E_{\text{original}}$ _____ $E_{\text{new}} = E_{\text{original}}$

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

The original magnetic fields are modified so that the region $D < x < 3.5D$ contains an external uniform magnetic field of magnitude B that is directed in the $+z$ -direction.

A new wire is connected to a resistor of resistance R to form a rigid triangular loop with base length D and height D . An external force is exerted on the loop so that the loop always moves with speed v in the $+x$ -direction, as shown in Figure 3. Point S represents the upper-leading corner of the loop.

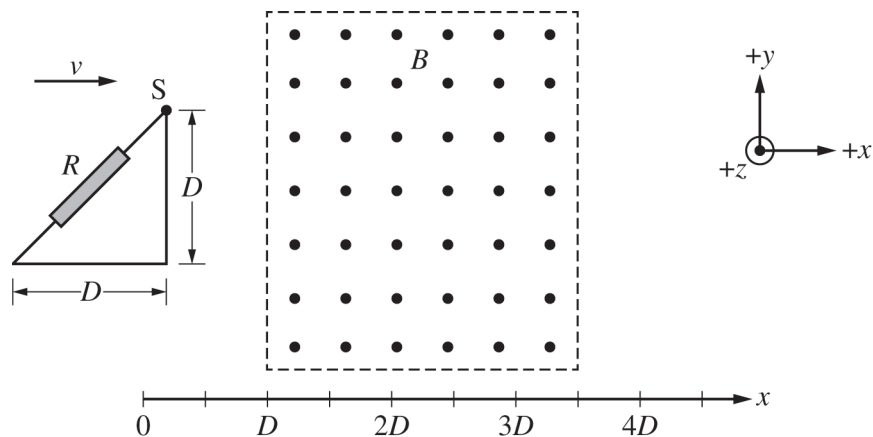
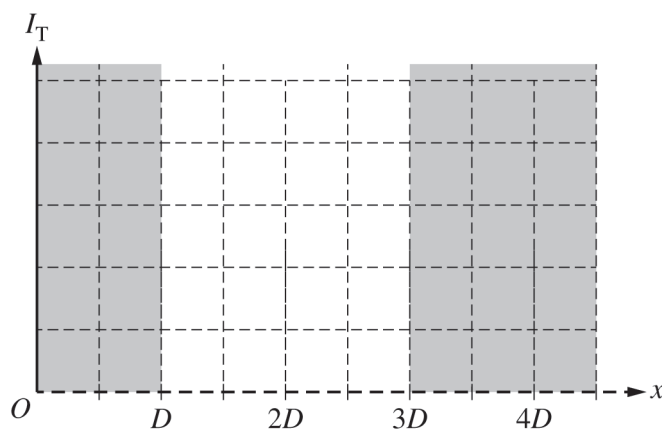


Figure 3

- (d) On the following axes, **sketch** a graph of the induced current I_T in the loop as Point S moves from $x = D$ to $x = 3D$.



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STOP

END OF EXAM