

### **Chief Reader Report on Student Responses:** 2024 AP<sup>®</sup> Statistics Free-Response Questions

Number of Students Scored	252,914			
<ul> <li>Number of Readers</li> </ul>	1,350			
Score Distribution	Exam Score	Ν	%At	
	5	44,310	17.5	
	4	55,156	21.8	
	3	56,891	22.5	
	2	40,240	15.9	
	1	56,317	22.3	
• Global Mean	2.96			

The following comments on the 2024 free-response questions for AP<sup>®</sup> Statistics were written by the Chief Reader, Dr. Barb Barnet, Ph.D. They give an overview of each free-response question and of how students performed on the question, including typical student errors. General comments regarding the skills and content that students frequently have the most problems with are included. Some suggestions for improving student preparation in these areas are also provided. Teachers are encouraged to attend a College Board workshop to learn strategies for improving student performance in specific areas.

#### Task: Inference Max Score: 4 Mean Score: 1.61

#### What were the responses to this question expected to demonstrate?

The primary goals of the question were to assess a student's ability to (1) identify an appropriate procedure for conducting a hypothesis test for a two-sample z-test for a difference of population proportions; (2) identify the correct hypotheses for conducting a two-sample z-test for a difference of population proportions; (3) check the conditions for the hypothesis test for a two-sample z-test for a difference of population proportions; (4) calculate the test statistic and *p*-value for a two-sample z-test for a difference of population proportions; (5) compare the *p*-value to a significance level to make a decision regarding the hypotheses; and (6) determine an appropriate conclusion for a hypothesis test for a two-sample z-test for a difference of population proportions.

This question primarily assesses skills in skill category 1: Selecting Statistical Methods, skill category 3: Using Probability and Simulation, and skill category 4: Statistical Argumentation. Skills required for responding to this question include (1.E) Identify an appropriate inference method for significance tests, (1.F) Identify null and alternative hypotheses, (3.E) Calculate a test statistic and find a *p*-value, provided conditions for inference are met, (4.C) Verify that inference procedures apply in a given situation, and (4.E) Justify a claim using a decision based on significance tests.

This question covers content from Unit 6: Inference for Categorical Data: Proportions of the course framework in the AP Statistics Course and Exam Description. Refer to topics 6.10 and 6.11 and learning objectives VAR-6.H, VAR-6.I, VAR-6.J, VAR-6.K, and DAT-3.D.

### How well did the responses address the course content related to this question? How well did the responses integrate the skill(s) required on this question?

This question was scored in three sections. The first section includes statements of the null and alternative hypotheses and identification of the appropriate hypothesis test. The second section includes verifying conditions for the test identified in the first section and calculating the value of the test statistic and corresponding *p*-value. The third section includes the conclusion for the test identified in the first section.

- In section 1 most responses indicated that a two-sample *z*-test should be used, although many responses did not complete the identification by stating that it was for the difference in population proportions. In some responses an incorrect formula for the test statistic was given, contradicting a correct identification of the test by name. Most responses indicated that the null hypothesis involved the equality of two quantities and that the alternative hypothesis was two-sided. Some responses did not set up hypotheses in terms of proportions or used non-standard notation. A somewhat common error was an incomplete description of parameters through lack of sufficient context for either the populations or groups being compared.
- In section 2 most responses made attempts to verify conditions. Many responses did not clearly indicate that two random samples were taken—one from each population. Most responses addressed the large counts condition by comparing observed numbers of successes and failures for both groups to a standard criterion (e.g., 5 or 10). Relatively few responses addressed this condition by calculating the expected number of successes and failures using the combined (pooled) proportion, which is the correct method for a hypothesis test for the difference in proportions. Some responses incorrectly compared counts to 30. Most responses correctly reported the value of the

*z*-statistic and *p*-value. Some responses used a confidence interval to conduct the test. A few responses used a chi-square test for homogeneity. Both these procedures are reasonable approaches for this question.

• In section 3 most responses made an explicit comparison between a *p*-value and the stated  $\alpha = 0.05$  and included a correct decision to fail to reject the null hypothesis. Some responses used incorrect, definitive language (e.g., "there is no evidence for the alternative hypothesis" or "there is no difference in the proportions") in stating a conclusion. Most responses had a conclusion stated (correctly) in terms of the alternative hypothesis. Many responses had insufficient context in the conclusion, lacking clear language about populations, proportions, the groups being compared, or the variable of interest.

Common Misconceptions/Knowledge Gaps	Responses that Demonstrate Understanding		
• Does not refer to population proportions in the statement of hypotheses (e.g., uses $\hat{p}_{\rm Y}$ and $\hat{p}_{\rm O}$ symbols, refers to means, or states hypotheses in words and never refers to population proportions, or uses non-standard notation without defining the notation).	<ul> <li>Let p<sub>Y</sub> represent the proportion of all 18- to 55-year-old exercise center members who would be interested in online fitness classes, and let p<sub>O</sub> represent the proportion of all 56 years and older exercise center members who would be interested in online fitness classes.</li> <li>The null hypothesis is H<sub>0</sub>: p<sub>Y</sub> - p<sub>O</sub> = 0, and the alternative hypothesis is H<sub>a</sub>: p<sub>Y</sub> - p<sub>O</sub> ≠ 0.</li> </ul>		
<ul> <li>Provides no definition (or incomplete definition) of parameters used in the hypotheses.</li> </ul>	• Let $p_{\rm Y}$ represent the proportion of all 18- to 55-year- old exercise center members who would be interested in online fitness classes, and let $p_{\rm O}$ represent the proportion of all 56 years and older exercise center members who would be interested in online fitness classes.		
• Refers to "2 sample <i>z</i> -test" without reference to a difference in proportions.	• An appropriate test procedure is a two-sample <i>z</i> -test for the difference in population proportions.		
<ul> <li>Verifying conditions:</li> <li>Does not indicate that there are two independent random samples.</li> <li>Does not check that it is reasonable to assume that the individuals within each sample can be viewed as independent even though the sampling is done without replacement (10% condition).</li> </ul>	<ul> <li>Verifying conditions:</li> <li>The first condition for applying the test is that the data are gathered from independent random samples. We are told that a random sample of 170 members from the 18- to 55-year-old age group was taken and a random sample of 230 members from the 56 years and older age group was taken.</li> <li>The second condition is that observations are independent. We are told that there are several</li> </ul>		

<ul> <li>Does not verify that all four of the expected counts</li> </ul>	thousand exercise center members in each of the
are greater than 5 or 10 to check the approximate	18- to 55-year-old and the 56 years and older age
normality of the sampling distribution of $\hat{p}_{\rm Y} - \hat{p}_{\rm O}$ .	groups. It is reasonable to assume that 170 is less
	than 10% of the total number of members in the
	18- to 55-year-old age group and 230 is less than
	10% of the total number of members in the
	56 years and older age group.
	• The third condition is that the sampling distribution
	of the test statistic is approximately normal. The
	pooled estimate of the proportion of members
	interested in online fitness classes is
	51+79 0.225 TH is a full
	$p_{\rm c} = \frac{1}{170 + 230} = 0.325$ . The estimates of the
	expected counts are $170(0.325) = 55.25$ ,
	$170(1 - 0.325) = 114.75, \ 230(0.325) = 74.75,$
	and $230(1 - 0.325) = 155.25$ . Because all four of
	these counts are greater than 10, the condition is
	satisfied.
• Computes an incorrect value for the test statistic or	
<i>p</i> -value.	• Using the pooled estimate of the proportion of exercise
<i>p</i> -value. OR	• Using the pooled estimate of the proportion of exercise members interested in online fitness classes,
<ul> <li><i>p</i>-value.</li> <li>OR</li> <li>Provides an incorrect formula and correct z-test statistic</li> </ul>	• Using the pooled estimate of the proportion of exercise members interested in online fitness classes, $\hat{p}_c = 0.325$ , the value of the test statistic is:
<ul> <li>Provides an incorrect formula and correct <i>z</i>-test statistic (using a calculator)</li> </ul>	• Using the pooled estimate of the proportion of exercise members interested in online fitness classes, $\hat{p}_c = 0.325$ , the value of the test statistic is:
<ul> <li>Provides an incorrect formula and correct <i>z</i>-test statistic (using a calculator).</li> </ul>	• Using the pooled estimate of the proportion of exercise members interested in online fitness classes, $\hat{p}_c = 0.325$ , the value of the test statistic is: 0.3 - 0.3435
<ul> <li>Provides an incorrect formula and correct <i>z</i>-test statistic (using a calculator).</li> <li>OR</li> <li>Uses the sample propertions instead of the peoled</li> </ul>	• Using the pooled estimate of the proportion of exercise members interested in online fitness classes, $\hat{p}_c = 0.325$ , the value of the test statistic is: $z = \frac{0.3 - 0.3435}{\sqrt{0.225(1 - 0.225)}} \sqrt{\frac{1}{1 + 1}} \approx -0.918$
<ul> <li>Provides an incorrect formula and correct <i>z</i>-test statistic (using a calculator).</li> <li>OR</li> <li>Uses the sample proportions instead of the pooled proportion in calculation of the standard error.</li> </ul>	• Using the pooled estimate of the proportion of exercise members interested in online fitness classes, $\hat{p}_c = 0.325$ , the value of the test statistic is: $z = \frac{0.3 - 0.3435}{\sqrt{0.325(1 - 0.325)}\sqrt{\frac{1}{170} + \frac{1}{230}}} \approx -0.918$
<ul> <li>Provides an incorrect formula and correct <i>z</i>-test statistic (using a calculator).</li> <li>OR</li> <li>Uses the sample proportions instead of the pooled proportion in calculation of the standard error.</li> </ul>	• Using the pooled estimate of the proportion of exercise members interested in online fitness classes, $\hat{p}_c = 0.325$ , the value of the test statistic is: $z = \frac{0.3 - 0.3435}{\sqrt{0.325(1 - 0.325)}\sqrt{\frac{1}{170} + \frac{1}{230}}} \approx -0.918$
<ul> <li>OR</li> <li>Provides an incorrect formula and correct <i>z</i>-test statistic (using a calculator).</li> <li>OR</li> <li>Uses the sample proportions instead of the pooled proportion in calculation of the standard error.</li> <li>OR</li> </ul>	• Using the pooled estimate of the proportion of exercise members interested in online fitness classes, $\hat{p}_c = 0.325$ , the value of the test statistic is: $z = \frac{0.3 - 0.3435}{\sqrt{0.325(1 - 0.325)}\sqrt{\frac{1}{170} + \frac{1}{230}}} \approx -0.918$ The <i>p</i> -value is $2 \cdot P(z < -0.918) \approx 0.359$ .
<ul> <li>or p-value.</li> <li>OR</li> <li>Provides an incorrect formula and correct <i>z</i>-test statistic (using a calculator).</li> <li>OR</li> <li>Uses the sample proportions instead of the pooled proportion in calculation of the standard error.</li> <li>OR</li> <li>Provides the <i>p</i>-value for a one-sided test.</li> </ul>	• Using the pooled estimate of the proportion of exercise members interested in online fitness classes, $\hat{p}_c = 0.325$ , the value of the test statistic is: $z = \frac{0.3 - 0.3435}{\sqrt{0.325(1 - 0.325)}\sqrt{\frac{1}{170} + \frac{1}{230}}} \approx -0.918$ The <i>p</i> -value is $2 \cdot P(z < -0.918) \approx 0.359$ .
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<ul> <li>or p-value.</li> <li>OR</li> <li>Provides an incorrect formula and correct <i>z</i>-test statistic (using a calculator).</li> <li>OR</li> <li>Uses the sample proportions instead of the pooled proportion in calculation of the standard error.</li> <li>OR</li> <li>Provides the <i>p</i>-value for a one-sided test.</li> <li>States a conclusion that implies that the null hypothesis has been proven rather than stating there is not</li> </ul>	<ul> <li>Using the pooled estimate of the proportion of exercise members interested in online fitness classes, <i>p</i><sub>c</sub> = 0.325, the value of the test statistic is: <i>z</i> = <u>0.3 - 0.3435</u> <u>√0.325(1 - 0.325)</u>√<u>1</u>/<u>170</u> + <u>1</u>/<u>230</u> ≈ -0.918     </li> <li>The <i>p</i>-value is 2·<i>P</i>(<i>z</i> &lt; -0.918) ≈ 0.359.     </li> <li>Because the <i>p</i>-value is greater than α = 0.05, we do not have convincing statistical evidence that the</li> </ul>
<ul> <li>or p-value.</li> <li>OR</li> <li>Provides an incorrect formula and correct <i>z</i>-test statistic (using a calculator).</li> <li>OR</li> <li>Uses the sample proportions instead of the pooled proportion in calculation of the standard error.</li> <li>OR</li> <li>Provides the <i>p</i>-value for a one-sided test.</li> <li>States a conclusion that implies that the null hypothesis has been proven rather than stating there is not convincing statistical evidence to support the alternative</li> </ul>	<ul> <li>Using the pooled estimate of the proportion of exercise members interested in online fitness classes, <i>p</i><sub>c</sub> = 0.325, the value of the test statistic is: <i>z</i> = <u>0.3 - 0.3435</u> <u>√0.325(1 - 0.325)</u>√<u>1</u>/<u>170</u> + <u>1</u>/<u>230</u> ≈ -0.918     </li> <li>The <i>p</i>-value is 2·<i>P</i>(<i>z</i> &lt; -0.918) ≈ 0.359.     </li> <li>Because the <i>p</i>-value is greater than α = 0.05, we do not have convincing statistical evidence that the proportion of 18- to 55-year-old exercise center</li> </ul>
<ul> <li>or p-value.</li> <li>OR</li> <li>Provides an incorrect formula and correct <i>z</i>-test statistic (using a calculator).</li> <li>OR</li> <li>Uses the sample proportions instead of the pooled proportion in calculation of the standard error.</li> <li>OR</li> <li>Provides the <i>p</i>-value for a one-sided test.</li> <li>States a conclusion that implies that the null hypothesis has been proven rather than stating there is not convincing statistical evidence to support the alternative hypothesis.</li> </ul>	<ul> <li>Using the pooled estimate of the proportion of exercise members interested in online fitness classes, <i>p</i><sub>c</sub> = 0.325, the value of the test statistic is: <i>z</i> = <u>0.3 - 0.3435</u> <u>√0.325(1 - 0.325)</u>√<u>170</u> + <u>1230</u> ≈ -0.918     </li> <li>The <i>p</i>-value is 2·<i>P</i>(<i>z</i> &lt; -0.918) ≈ 0.359.     </li> <li>Because the <i>p</i>-value is greater than α = 0.05, we do not have convincing statistical evidence that the proportion of 18- to 55-year-old exercise center members who are interested in online fitness classes is</li> </ul>
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<ul> <li>or boundary of the for the test statistic of <i>p</i>-value.</li> <li>OR</li> <li>Provides an incorrect formula and correct <i>z</i>-test statistic (using a calculator).</li> <li>OR</li> <li>Uses the sample proportions instead of the pooled proportion in calculation of the standard error.</li> <li>OR</li> <li>Provides the <i>p</i>-value for a one-sided test.</li> <li>States a conclusion that implies that the null hypothesis has been proven rather than stating there is not convincing statistical evidence to support the alternative hypothesis.</li> <li>OR</li> <li>States a conclusion in terms of only the null hypothesis</li> </ul>	<ul> <li>Using the pooled estimate of the proportion of exercise members interested in online fitness classes, <i>p</i><sub>c</sub> = 0.325, the value of the test statistic is: <i>z</i> = <u>0.3 - 0.3435</u> <u>√0.325(1 - 0.325)</u>√<u>1</u>/<u>170</u> + <u>1</u>/<u>230</u> ≈ -0.918     </li> <li>The <i>p</i>-value is 2·<i>P</i>(<i>z</i> &lt; -0.918) ≈ 0.359.     </li> <li>Because the <i>p</i>-value is greater than α = 0.05, we do not have convincing statistical evidence that the proportion of 18- to 55-year-old exercise center members who are interested in online fitness classes is different from the proportion of 56 years and older exercise members who are interested in online fitness</li> </ul>
<ul> <li>or computes an incorrect value for the test statistic of <i>p</i>-value.</li> <li>OR</li> <li>Provides an incorrect formula and correct <i>z</i>-test statistic (using a calculator).</li> <li>OR</li> <li>Uses the sample proportions instead of the pooled proportion in calculation of the standard error.</li> <li>OR</li> <li>Provides the <i>p</i>-value for a one-sided test.</li> <li>States a conclusion that implies that the null hypothesis has been proven rather than stating there is not convincing statistical evidence to support the alternative hypothesis.</li> <li>OR</li> <li>States a conclusion in terms of only the null hypothesis without stating that there is not convincing evidence to a statistical evidence to support the alternative hypothesis.</li> </ul>	<ul> <li>Using the pooled estimate of the proportion of exercise members interested in online fitness classes, <i>p</i><sub>c</sub> = 0.325, the value of the test statistic is: <i>z</i> = <u>0.3 - 0.3435</u> <u>√0.325(1 - 0.325)</u>√<u>1 + 1</u>/(170) + 1/(230) ≈ -0.918     </li> <li>The <i>p</i>-value is 2·<i>P</i>(<i>z</i> &lt; -0.918) ≈ 0.359.     </li> <li>Because the <i>p</i>-value is greater than α = 0.05, we do not have convincing statistical evidence that the proportion of 18- to 55-year-old exercise center members who are interested in online fitness classes is different from the proportion of 56 years and older exercise members who are interested in online fitness classes.     </li> </ul>

Some teaching tips:

• Encourage students to clearly define all parameters in the context of the problem, and not provide contradicting definitions (e.g., stating  $p_1 = \frac{51}{70}$  as a sample proportion, and then using  $p_1$  later in the statement of hypotheses).

- When labeling parameters, encourage students to use labels that more clearly distinguish between the two groups, (e.g.,  $p_{\rm Y}$  and  $p_{\rm O}$  instead of  $p_1$  and  $p_2$ ).
- Encourage students to clearly communicate the correct procedure. (This is more often correct when communicated in words and not via formulas or calculator notation.)
- In checking conditions, encourage students to spell out the words "random sample" and not use abbreviations such as "random," "RS," etc.
- Encourage students to verify all three conditions for both samples in words, not just using check marks.
- Encourage students to conduct a significance test when they are asked to determine if the data provide convincing evidence for a claim, rather than a confidence interval.
- Encourage students to avoid calculator syntax when calculating a *p*-value.
- Encourage students to make a direct comparison of the *p*-value to a (provided) alpha level and use the result of that comparison to support their conclusion.
- Encourage students to clearly communicate that the data provide (or don't provide) convincing evidence to support the <u>alternative</u> hypothesis (and not just state that there is no evidence to reject the null hypothesis).
- Encourage students to provide complete context in their conclusion, including the population, parameters, groups, and variable of interest.

- The AP Statistics Course and Exam Description (CED), effective Fall 2020, includes instructional resources for AP Statistics teachers to develop students' broader skills. Please see pages 226, 230, and 232 of the CED for examples of key questions and instructional strategies designed to develop skill 1.E, identify an appropriate inference method for significance tests, skill 1.F, identify an appropriate inference method for significance tests, skill 1.F, identify an appropriate inference are met, skill 4.C, verify that inference procedures apply in a given situation, and skill 4.E, justify a claim using a decision based on significance tests.
- A table of representative instructional strategies, including definitions and explanations of each, is included on pages 213–223 of the CED. The strategy "Build the Model Solution," for example, may be helpful in developing students' abilities to use precise language when justifying a claim based on a significance test.
- AP Classroom provides four videos focused on the content and skills to answer this question.
  - The daily video 1 for topic 6.10 discusses how to state the null and alternative hypotheses for a two-sample *z*-test for a difference of population proportions (see VAR-6.H.2 and VAR-6.H.3). The key takeaways from this video that were relevant to this question are identifying the hypotheses, including the two different versions found in the CED, and defining the parameter for this significance test that students may need extra practice on.
  - The daily video 2 for topic 6.10 explores how to identify the procedure and check the conditions for performing a two-sample *z*-test (See VAR-6.I.1 and VAR-6.J.1). The key takeaway from this video explains to students how to calculate the combined (pooled) proportions of successes,  $\hat{p}_c$ , which is used to check whether the shape of the sampling distribution is approximately normal for this significance test.
  - The daily video 1 for topic 6.11 describes how to calculate a test statistic and *p*-value for a two-sample *z*-test for a difference of population proportions (see VAR-6.K.1). The key takeaway of the video shows students how the combined (pooled) proportions of successes,  $\hat{p}_{c}$ , is used when calculating a test statistic for this significance test.
  - The daily video 2 for topic 6.11 demonstrates how to interpret the *p*-value and state a conclusion for a two-sample *z*-test for a difference of population proportions (see DAT-3.D.1 and DAT-3.D.2). A key takeaway from this video explains to students what the *p*-value means in terms of the test statistic.

- The daily video 3 for topic 7.5 demonstrates how to complete a significance test for matched pair (See VAR-6.H.1, VAR-6.H.2, VAR-6.H.3, VAR-6.I.1, VAR-6.J.1, VAR-6.K.1, DAT-3.D.1, and DAT-3.D.2). The key takeaway for the video helps students understand to use an alpha level of 0.05 when no significance level is stated.
- AP Classroom also provides topic questions for formative assessment of topics 6.10 and 6.11, as well as access to the question bank, which is a searchable database of past AP Questions on this topic.
- The Online Teacher Community features many resources shared by other AP Statistics teachers. For example, to locate resources to give your students practice verifying conditions, try entering the keywords "Pooling for two sample *z*-tests" in the search bar, then selecting the drop-down menu for "Resource Library." When you filter for "Classroom-Ready Materials," you may find worksheets, data sets, practice questions, and guided notes, among other resources.

#### Task: Exploring Data Max Score: 4 Mean Score: 2.05

#### What were the responses to this question expected to demonstrate?

The primary goals of this question were to assess a student's ability to (1) use summarized data from a description to complete two segmented bar graphs for two different schools; (2) determine whether an administrator's conclusion is correct based on the previously constructed two segmented bar graphs and description; and (3) use information presented in a mosaic plot to determine the larger proportion and the larger number of large bottles for two different high schools.

This question primarily assesses skills in skill category 2: Data Analysis, skill category 3: Using Probability and Simulation, and skill category 4: Statistical Argumentation. Skills required for responding to this question include (2.B) Construct numerical or graphical representations of distributions, (2.D) Compare distributions or relative positions of points within a distribution, (3.A) Determine relative frequencies, proportions, or probabilities using simulation, and (4.B) Interpret statistical calculations and findings to assign meaning or assess a claim or calculations.

This question covers content from Unit 1: Exploring One-Variable Data, Unit 2: Exploring Two-Variable Data, and Unit 4: Probability, Random Variables, and Probability Distributions of the course framework in the AP Statistics Course and Exam Description. Refer to topics, 1.4, 2.2, and 4.3, and learning objectives UNC-1.C, UNC-1.E, UNC-1.P, VAR-4.A, and VAR-4.B.

- In part (a) most responses completed the segmented bar graphs correctly using the key provided. However, a number of responses misunderstood the sentence "For the middle school students, the proportion of bottles sold was equal for all three sizes" to mean that the proportions of bottles sold by the middle school students were equal to the proportions of bottles sold by the elementary students for all three sizes.
- In part (b) most responses accurately indicated that the administrator's conclusion was incorrect. However, many responses did not provide sufficient correct mathematical support to verify that the administrator's claim was incorrect. Most responses provided context by including "elementary," "middle," and "bottles."
- In part (c-i) most responses correctly identified High School A as the school that sold a greater proportion of large bottles. Most responses based their reasoning on the heights of the appropriate rectangles in the mosaic plot by identifying the relative frequencies of 0.7 and 0.6. In part (c-ii) most students identified High School B as the school that sold the greater number of large bottles. However, many responses did not support their choice based on the area of the rectangles depicting large bottles in the mosaic plot.

Common Misconceptions/Knowledge Gaps	Responses that Demonstrate Understanding	
• In part (a) responses partitioned the middle school segmented bar graph with segments of widths 0.5 for small bottles, 0.3 for medium bottles, and 0.2 for large bottles.	• The middle school segmented bar graph was partitioned to create three equal areas.	
• In part (a) responses partitioned the middle school segmented bar graph with segments of widths 0.3 for small bottles, 0.3 for medium bottles and 0.4 for large bottles.	• The middle school segmented bar graph was partitioned to create three equal areas.	
• In part (b) for mathematical support, many responses only provided "middle school students sold 3 times as many bottles."	• The middle school students sold three times as many bottles as the elementary students. Therefore, the middle school students sold more small bottles because $\left(\frac{1}{3}\right)(3x) > 0.5x$ where $x =$ the number of bottles sold by the elementary school students.	
• In part (b) responses indicated that the middle school students sold twice as many small bottles as the elementary students sold without sufficient mathematical support.	• Because the middle school students sold three times as many bottles as the elementary school students, the middle school students sold twice as many small bottles as the elementary school students. Let $x =$ the number of bottles sold by the elementary school students. $\left(\frac{1}{3}\right)(3x) > 0.5x$ , and $x > 0.5x$ .	
• In part (b) responses indicated that the middle school students sold as many small bottles as the total number of bottles sold by the elementary students sold without sufficient mathematical support.	• Because the middle school students sold three times as many bottles as the elementary school students, the middle school students sold as many small bottles as the total number of bottles that the elementary students sold. Let $x =$ the number of bottles sold by elementary school students. $\left(\frac{1}{3}\right)(3x) > 0.5x$ .	
• In part (c-i) some responses indicated High School A students sold a greater proportion of large bottles without justification.	• High School A. The proportion of large bottles sold by High School A students is 0.7, which is greater than the proportion of large bottles sold by High School B students, which is 0.6.	
• In part (c-i) some responses reported a greater frequency of large bottles sold rather than a greater relative frequency of large bottles sold.	• In part (c-i) High School A students sold a greater relative frequency of large bottles.	

•	In part (c-ii) responses refer to the width of the rectangles on the mosaic plot as opposed to the areas of the rectangles for the large bottles.	•	High School B. The area of the rectangle for large bottles on the mosaic plot for High School B is larger than the area of the rectangle for large bottles at High School A, so students at High School B sold more large bottles.
•	In part (c-ii) responses refer to the area of the entire rectangle representing each school as opposed to the large bottle areas of the rectangles on the mosaic plot.	•	High School B. The area of the rectangle for large bottles on the mosaic plot for High School B is larger than the area of the rectangle for large bottles at High School A, so students at High School B sold more large bottles.

- Remind students to review how to construct and interpret segmented bar graphs and mosaic plots.
- Have a student describe a graph to another student who will draw the graph. Compare the original graph to the one drawn and discuss the differences.
- Remind students to give a direct answer when asked a direct question before attempting to support their response.
- Remind students to include context when writing their responses.
- Remind students to show their work when asked to provide mathematical justification. In some cases, creating an example helps make a justification more concrete.
- Have students practice writing a statement to support an argument with statistical justification.
- Emphasize the difference between number (count, frequency) and proportion (relative frequency).
- Remind students that using the stem of the prompt is helpful but may not be sufficient to earn credit.

- The AP Statistics Course and Exam Description (CED), effective Fall 2020, includes instructional resources for AP Statistics teachers to develop students' broader skills. Please see pages 227–229 of the CED for examples of key questions and instructional strategies designed to develop skill 2.B, construct numerical or graphical representations of distributions, and skill 2.D, compare distributions or relative positions of points within a distribution, skill 3.A, determine relative frequencies, proportions, or probabilities using simulation or calculations, and skill 4.B (page 232), interpret statistical calculations and findings to assign meaning or assess a claim.
- A table of representative instructional strategies, including definitions and explanations of each, is included on pages 213–223 of the CED. The strategy "Create Representations," for example, may allow students to have a deeper understanding of the differences in qualitative graphs.
- AP Classroom provides four videos focused on the content and skills to answer this question.
  - The daily video 1 for topic 2.2 explores how to construct segmented bar graphs (see UNC-1.P.1). The key takeaway from this video that is relevant to this question is the description of taking relative frequencies and creating segmented bar graphs (part (a)).
  - The daily video 2 for topic 1.4 discusses how different graphical representations can help us compare multiple sets of categorical data (see UNC-1.E.1). The key takeaway from this video that is relevant to this question discusses how comparing relative frequencies between the two segmented bar graphs can be used to justify the claims given in the question (part (b)).

- The daily video 1 for topic 4.3 explains how probabilities are calculated (see VAR-4.A.2). A key takeaway from this video explores how a student can write a probability as a fraction and as a decimal (c) for comparing data.
- AP Classroom also provides topic questions for the formative assessment of topics 1.4, 2.2, and 4.3, as well as access to the question bank, which is a searchable database of past AP Questions on this topic.
- The Online Teacher Community features many resources shared by other AP Statistics teachers. For example, to locate resources to give your students practice discussing distributions of histograms, try entering the keywords "mosaic plots" in the search bar, then selecting the drop-down menu for "Resource Library." When you filter for "Classroom-Ready Materials," you may find worksheets, data sets, practice questions, and guided notes, among other resources.

#### Task: Sampling Max Score: 4 Mean Score: 1.30

#### What were the responses to this question expected to demonstrate?

The primary goals of the question were to assess a student's ability to (1) identify an observational study with justification that no treatments were imposed; (2) describe a correct procedure for randomly assigning the use of the autopilot feature to the observational units in a completely randomized design; and (3) indicate the change to the original study that would allow the results to be generalized to all Model D cars in the club.

This question primarily assesses skills in skill category 1: Selecting Statistical Methods and skill category 4: Statistical Argumentation. Skills required for responding to this question include (1.B) Identify key and relevant information to answer a question or solve a problem, (1.C) Describe an appropriate method for gathering and representing data, (4.B) Interpret statistical calculations and findings to assign meaning or assess a claim, and (4.E) Justify a claim using a decision based on significance tests.

This question covers content from Unit 3: Collecting Data and Unit 7: Inference for Quantitative Data: Means of the course framework in the AP Statistics Course and Exam Description. Refer to topics 3.2, 3.3, 3.5, 3.7, and 7.5, and learning objectives DAT-2.A, DAT-2.C, DAT-3.F, VAR-3.B, VAR-3.C, and VAR-3.E.

- In part (a) most responses correctly chose observational study and attempted to justify the choice. Many responses correctly supported the choice of observational study with a statement noting the lack of an imposed treatment. However, many responses did not include context. Many responses substituted words for "treatment" that lacked clarity.
- In part (b) most responses attempted to describe a random assignment process that could be used for a completely randomized design. However, many responses left out components of implementing the random assignment process so that the result did not ensure that every possible random assignment was equally likely. Many responses incorrectly used a random assignment process that did not result in each treatment (autopilot or not using autopilot) being assigned to exactly 35 days.
- In part (c) most responses recognized that more cars are necessary to generalize the conclusion to the population of interest (Model D cars of members in the club). However, many students did not completely describe the population of interest or indicate that random sampling is required.

Common Misconceptions/Knowledge Gaps		Responses that Demonstrate Understanding	
•	In part (a) some responses did not use precise language. For example, some responses did not use the word "treatment" or say that treatments were not imposed when justifying their answer. A response attempting to use a phrase or word similar to the word "treatment" often failed to meet the criteria for credit.	•	Observational study. The investigation does not have a treatment imposed on the car owners.
•	In part (a) many responses did not include context.	•	The car owners were not randomly assigned a car model.
•	In part (b) some responses did not sufficiently describe how to implement the random assignment of days to treatment groups. For example, some responses that used slips of paper did not mix the slips of paper or did not indicate that sampling was done without replacement.	•	Label each day uniquely from 1 to 70. Write the numbers 1 to 70 on equal-sized slips of paper and put them in a hat. Mix the slips. Randomly draw 35 unique slips of paper without replacement; on the days corresponding to these numbers, James will drive using the autopilot feature. On the remaining 35 days, James will drive without using the autopilot feature.
•	In part (b) when using a random number generator, some responses did not indicate the range of numbers to enter into the random number generator. For example, some responses state that the first 35 unique numbers from a random generator will be assigned to driving with the autopilot feature.	•	Label each day uniquely from 1 to 70. Using a random number generator, generate 35 unique integers from 1 to 70. Assign the days corresponding to these 35 numbers to driving with the autopilot feature and the other 35 days to driving without the autopilot feature.
•	In part (b) some responses did not create a random process that results in every possible random assignment being equally likely. For example, "label the days from 1 to 70 and have James toss a coin to decide whether James will use the autopilot feature on even numbered days or odd numbered days." This results in a process where two consecutive days could never have the same treatment.	•	Beginning with day 1, use a random number generator to select an integer from 1 to 70 to assign to day 1. For day 2, use a random number generator to select an integer from 1 to 70, without repeating a previously chosen integer, to assign to day 2. Continue this process for all 70 days. On the days assigned an even number, have James drive using the autopilot feature. On the days assigned an odd number, have James drive without the autopilot feature.

•	In part (b) some responses do not create a random process that results in the requirements described in the prompt. For example, "each day toss a coin to determine the driving method. If the outcome is heads, use the autopilot feature. If the outcome is tails, do not use the autopilot feature." This method may not result in each treatment being assigned to 35 days or will require a stopping rule (once a treatment reaches 35 days use the other treatment for the remainder of the days). The stopping rule approach does not result in an equally likely chance for every possible random assignment. The last two days are much more likely to have the same treatment.	•	Number the days from 1 to 70. Place 35 red and 35 blue marbles in a box and mix them well. Select a marble without replacement and if it is blue, then on day 1, James will drive with the autopilot feature, if it is red he will drive without the autopilot feature. Continue selecting marbles without replacement and assigning treatments to days (blue equals autopilot, red equals no autopilot) in this fashion until all 70 days have a treatment assigned.
•	In part (c) many responses do not fully identify the population of interest or the need for random selection.	•	In order to generalize, James should randomly select a sample of Model D cars from those owned by members of the club.

- Require students to provide answers in context to provide clear communication. When working with a prompt, take the time to fully discuss the context.
- Have students practice describing statistical processes such as random assignment. Have peers read the response to see if they could carry out the written process step-by-step and obtain the desired random assignment. Have students check for details including, mixing slips of paper, selecting random numbers without replacement, and including the boundaries for a random number generator.
- Discuss the difference between random assignment and random selection. With random assignment, a cause-andeffect statement can be made and with random selection from a population, the results can be generalized to that population.
- Discuss different statistical designs, such as the completely randomized design, the randomized block design, and the matched pairs design. Remind the students to closely read the stem of the problem to see if a type of design is specified.

### What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

• The AP Statistics Course and Exam Description (CED), effective Fall 2020, includes instructional resources for AP Statistics teachers to develop students' broader skills. Please see page 225 of the CED for examples of key questions and instructional strategies designed to develop skill 1.B, identify key and relevant information to answer a question or solve a problem, and skill 1.C, describe an appropriate method for gathering and representing data as well as page 232 for skill 4.B, interpret statistical calculations and findings to assign meaning or assess a claim and skill 4.E, justify a claim using a decision based on significance tests.

- A table of representative instructional strategies, including definitions and explanations of each, is included on pages 213–223 of the CED. The strategy "Peer Critique," for example, may be helpful in developing students' abilities to describe a correct procedure that could be used for creating a completely randomized design. The scoring guidelines have different examples that are broken down that could help guide students' learning.
- AP Classroom provides four videos focused on the content and skills to answer this question.
  - The daily video 1 for topic 3.2 demonstrates how to determine the type of study being conducted (see DAT-2.A.3 and DAT-2.A.4). The key takeaway from this video is how to identify an observational study in part (a).
  - The daily video 1 for topic 3.3 introduces students to what a simple random sample (SRS) is (see DAT-2.C.2). The key takeaway from this video helps students understand that each individual has an equal chance of being selected (part (b)). The scoring guidelines for this question have defined examples in the model solution and additional notes that can help explain what is required when describing a completely randomized design.
  - The daily video 1 for topic 3.7 explores what results can be interpreted from a well-designed experiment (See VAR-3.E.4). The key takeaway from this video is to help students understand that if the experimental units used in an experiment are representative of the population, then the results of an experiment can be generalized to the population of subjects like the ones in the study (part (c)).
- AP Classroom also provides topic questions for formative assessment of topics 3.2, 3.3, 3.5, 3.7, and 7.5, as well as access to the question bank, which is a searchable database of past AP Questions on this topic.
- The Online Teacher Community features many resources shared by other AP Statistics teachers. For example, to locate resources to give your students practice verifying conditions, try entering the keywords "random assignment" in the search bar, then selecting the drop-down menu for "Resource Library." When you filter for "Classroom-Ready Materials," you may find worksheets, data sets, practice questions, and guided notes, among other resources.

#### Task: Probability Max Score: 4 Mean Score: 1.30

#### What were the responses to this question expected to demonstrate?

The primary goals of the question were to assess a student's ability to (1) calculate the mean of a geometric distribution; (2) calculate the standard deviation of a geometric distribution; (3) calculate a probability for a geometric distribution; (4) calculate a probability for a geometric distribution using a previous calculated probability; (5) calculate the mean of a geometric distribution using previous calculated probabilities; and (6) interpret the mean of a geometric distribution using previous calculated probabilities.

This question primarily assesses skills in skill category 3: Using Probability and Simulation, and skill category 4: Statistical Argumentation. Skills required for responding to this question include (3.A) Determine relative frequencies, proportions, or probabilities using simulation or calculations, (3.B) Determine parameters for probability distributions, and (4.B) Interpret statistical calculations and findings to assign meaning or assess a claim.

This question covers content from Unit 4: Probability, Random Variables, and Probability Distributions in the AP Statistics Course and Exam Description. Refer to topics 4.5, 4.6, 4.8, and 4.12, and learning objectives UNC-3.E, UNC-3.F, VAR-4.A, VAR-4.E, VAR-5.C, and VAR-5.D.

- In part (a) most responses correctly calculated the mean and standard deviation of the geometric distribution, showing the correct value of p = 0.08 substituted into each of the formulas. In part (a-i) some responses inappropriately rounded the mean to an integer or noted that there couldn't be half of a geode. In part (a-ii) some responses wrote the formula correctly but then miscalculated it by taking the square root of both the numerator and denominator rather than just the numerator.
- In part (b-i) most responses correctly calculated the probability that Conrad finds a red crystal in the third geode that he opens. In part (b-i) many responses used calculator function syntax, some with labeled inputs and some without. However, in part (b-ii), most responses incorrectly calculated the probability that Conrad either finds a red crystal in the fourth geode he opens or stops opening geodes. Instead, most responses calculated the probability that Conrad finds a red crystal in the fourth geode he opens or stops opening geodes. Instead, most responses calculated the probability that Conrad finds a red crystal in the fourth geode he opens but did not account for Conrad stopping at four geodes without finding a red crystal.
- In part (c) most responses either correctly calculated the mean of the distribution consistent with the values calculated in part (b) or calculated the mean of the correct probability distribution. In part (c-ii) most responses interpreted the mean, including the concept of an average or mean and the context of the number of geodes opened, but did not include the concept of repeating the selection process over a long period of time.

Common Misconceptions/Knowledge Gaps	Responses that Demonstrate Understanding	
• In part (a-i) many responses rounded the value of the mean to an integer, demonstrating a misunderstanding of means/expected values.	• $\mu = E(G) = \frac{1}{0.08} = 12.5$ geodes	
• In part (a-ii) many responses wrote the formula correctly but made errors in calculating the standard deviation.	• $\sigma_{\rm G} = \frac{\sqrt{1 - 0.08}}{0.08} \approx 11.99 \text{ geodes}$	
• In part (b-i) many responses used calculator function syntax without identifying the parameters.	• $P(Y = 3) = (0.92)^2 (0.08) = 0.067712$ OR geompdf ( $p = 0.08, x = 3$ ) = 0.067712	
• In part (b-ii) most responses calculated the incorrect probability, computing the probability of Conrad finding a red crystal in the fourth geode without accounting for the probability of not finding a crystal and stopping upon opening the fourth geode.	• $P(Y = 4) = 1 - P(Y = 1 \text{ or } 2 \text{ or } 3)$ = 1 - (0.08 + 0.0736 + 0.067712) = 0.778688 OR $P(Y = 4) = (0.92)^4 + (0.92)^3 (0.08) = 0.778688$	
• In part (c-i) many responses calculated the mean from a probability distribution which did not sum to 1. Other responses used unacceptable calculator syntax (e.g. 1-VAR STATS(L1, L2)) instead of showing appropriate work using the formula.	• $\mu = E(Y)$ $\approx (1)(0.08) + (2)(0.0736) + (3)(0.0677) + (4)(0.778688)$ $\approx 0.08 + 0.1472 + 0.2031 + 3.1148$ $\approx 3.545$ geodes	
• In part (c-ii) many responses did not include an interpretation of the mean of the distribution that included both the concepts of repeating the selection process over a long period of time and the concept that this value is the average of the random variable. Some responses incorrectly identified the context as the number of red crystals found.	• The mean of 3.545 geodes is the average number of geodes that result from many, many trials of opening randomly selected geodes and counting the number opened until either a red geode is found, or the fourth geode is opened.	

- Encourage your students to communicate clearly, use correct statistical notation, and show formulas with correct values plugged in.
- If you let your students use calculator syntax, require them to label all inputs or define all the parameters and boundary values.
- Have students reflect on an answer to determine if the value is valid, such as noticing that a probability distribution that does not sum to one is not valid.
- Have students interpret responses to numerical values, not only the results of statistical tests, and explain what these values represent in the context of the problem or data set.
- Have students look at a plot of the probability distribution to understand that the mean/expected value is the center of the distribution, and it does not have to be an integer value.
- Have students conduct a probability experiment, such as rolling a die many times, to show that the mean/expected value is the result of a long run process.

- The AP Statistics Course and Exam Description (CED), effective Fall 2020, includes instructional resources for AP Statistics teachers to develop students' broader skills. Please see pages 229, 230, and 232 of the CED for examples of key questions and instructional strategies designed to develop skill 3.A, determine relative frequencies, proportions, or probabilities using simulation or calculations, skill 3.B, determine parameters for probability distributions, and skill 4.B, interpret statistical calculations and findings to assign meaning or assess a claim.
- A table of representative instructional strategies, including definitions and explanations of each, is included on pages 213–223 of the CED. The strategy "Odd One Out" for example, may be helpful in developing students' abilities to distinguish between binomial and geometric distributions.
- AP Classroom provides four videos focused on the content and skills to answer this question.
  - The daily video 1 for topic 4.12 shows how to calculate probabilities and parameters for a geometric distribution (see UNC-3.E.2 and UNC-3.F.1). The key takeaways from this video show how to solve a probability by hand and with the calculator (parts (a-i, a-ii, b-i, and b-ii)).
  - The daily video 1 for topic 4.8 demonstrates how to calculate and interpret the mean of a distribution (see VAR-5.C.2 and VAR-5.D.1). The key takeaway from this video explains the terminology for interpreting the mean of a distribution and what is expected in part (c).
- AP Classroom also provides topic questions for formative assessment of topics 4.5, 4.6, 4.8, and 4.12, as well as access to the question bank, which is a searchable database of past AP Questions on this topic.
- The Online Teacher Community features many resources shared by other AP Statistics teachers. For example, to locate resources to give your students practice verifying conditions, try entering the keywords "geometric distributions" in the search bar, then selecting the drop-down menu for "Resource Library." When you filter for "Classroom-Ready Materials," you may find worksheets, data sets, practice questions, and guided notes, among other resources.

#### Task: Multi-Focus Max Score: 4 Mean Score: 2.26

#### What were the responses to this question expected to demonstrate?

The primary goals of the question were to assess a student's ability to (1) calculate a probability from a two-way table; (2) calculate a conditional probability from a two-way table; (3) identify an appropriate procedure for conducting a hypothesis test for a chi-square test of independence; (4) identify the correct hypotheses for conducting a chi-square test of independence; (5) compare the *p*-value to a significance level to make a decision regarding the hypotheses; and (6) determine an appropriate conclusion for a chi-square test of independence.

This question primarily assesses skills in category 1: Selecting Statistical Methods, skill category 3: Using Probability and Simulation, and skill category 4: Statistical Argumentation. Skills required for responding to this question include (1.E) Identify an appropriate inference method for significance tests, (1.F) Identify null and alternative hypotheses, (3.A) Determine relative frequencies, proportions, or probabilities using simulation or calculations, and (4.E) Justify a claim using a decision based on significance tests.

This question covers content from Unit 4: Probability, Random Variables, and Probability Distributions and Unit 8: Inference for Categorical Data: Chi-Square of the course framework in the AP Statistics Course and Exam Description. Refer to topics 4.5, 4.6, 8.5, and 8.6, and learning objectives DAT-3.L, VAR-4.D, VAR-4.E, VAR-8.1, and VAR-8.J.

- In part (a) the majority of responses provided the correct probability with appropriate work shown. Some responses incorrectly computed the joint probability using the formula for independent events while other responses incorrectly computed a conditional probability. Probability statements were included in some responses and, while not required, showed strong communication.
- In part (b) most responses provided the correct probability with work shown. Many of the correct responses used a ratio of counts directly from the table. Some of the correct responses used the conditional probability formula to perform the calculation. Probability statements were included in some responses and, while not required, showed strong communication.
- In part (c-i) some responses correctly identified the chi-square test of independence. Many responses identified the test as chi-square but then either didn't specify which chi-square test or identified the test incorrectly as either a test of homogeneity or a test for goodness of fit. Some responses identified the incorrect test. In part (c-ii) most responses stated correct null and alternative hypotheses regardless of whether the test identified in part (c-i) was correct. Almost all responses provided sufficient context. Occasionally, the response incorrectly reversed the hypotheses, stating the alternative hypothesis as the null and the null as the alternative. Additionally, some responses included an alternative hypothesis that implied causality.
- In part (d) most responses provided the correct comparison of the *p*-value to a reasonable alpha (most often choosing alpha equal to 0.05), provided the correct decision, and stated the correct conclusion in context using non-definitive language. Some responses failed to make an explicit comparison of the *p*-value to alpha. Some responses failed to state the conclusion in terms of the alternative hypothesis. Almost all responses did not mention the population to which the results could be generalized.

Common Misconceptions/Knowledge Gaps	Responses that Demonstrate Understanding	
• In part (a) responses incorrectly used the formula for independent events to compute a joint probability when the events are not known to be independent.	• $P(11+ \text{ months AND majority regular cards})$ = $\left(\frac{71+76+112}{500}\right) = \frac{259}{500} = 0.518$	
• In part (c-i) some responses identified an incorrect test.	• Chi-square test of independence.	
• In part (c-ii) some responses provided an alternative hypothesis that suggested that one variable causes the other, such as, "The type of majority cards depends on the months spent collecting cards."	<ul> <li>H<sub>0</sub>: The months spent collecting cards, and the majority type of cards are independent.</li> <li>H<sub>a</sub>: The months spent collecting cards and the majority type of cards are not independent.</li> </ul>	
• In part (d) some responses failed to explicitly compare the <i>p</i> -value to a reasonable value of alpha.	• Because the <i>p</i> -value of 0.0075 is less than any reasonable alpha level, the null hypothesis should be rejected.	
• In part (d) some responses failed to state the conclusion in terms of the alternative hypothesis or stated the conclusion using definitive language.	• The data provide convincing statistical evidence that there is a relationship between the number of months spent collecting baseball cards and which type of card is the majority in the collection.	
• In part (d) most responses failed to reference the population of interest in the conclusion.	• The data provide convincing statistical evidence that there is a relationship between the number of months spent collecting baseball cards and which type of card is the majority in the collection for all baseball card collectors at the convention.	

## Based on your experience at the AP<sup>®</sup> Reading with student responses, what advice would you offer teachers to help them improve student performance on the exam?

- Emphasize the importance of showing work in probability calculations as good communication.
- Discuss the difference between independent and non-independent events and how that impacts probability calculations.
- When dealing with bivariate data, help students distinguish between categorical and quantitative data and how this affects the choice of significance tests.
- Provide opportunities to match-inference procedures to scenarios.
- Use language from the problem stem to inform how to write context.
- Encourage students to use the words "greater than" or "less than" instead of the symbols.

- Require students to include the population of interest in their hypotheses and conclusions.
- Enforce that inference conclusions are in terms of the alternative hypothesis.
- Help students understand why non-definitive language is important in inferential conclusions.
- Give students the opportunity to grade conclusions so they can differentiate complete and incorrect conclusion statements.
- Don't include an interpretation of the *p*-value as part of the hypothesis test procedure.
- Provide a concise response that directly answers the question.

- The AP Statistics Course and Exam Description (CED), effective Fall 2020, includes instructional resources for AP Statistics teachers to develop students' broader skills. Please see pages 226, 229, and 232 of the CED for examples of key questions and instructional strategies designed to develop skill 1.E, identify an appropriate inference method for significance tests, 1.F, identify an appropriate inference method for significance tests, skill 3.A, determine relative frequencies, proportions, or probabilities using simulation or calculations, and skill 4.E, justify a claim using a decision based on significance tests.
- A table of representative instructional strategies, including definitions and explanations of each, is included on pages 213–223 of the CED. The strategy "Ask the Expert," for example, may be helpful for students to think of how the content is aligned. This FRQ has Units 4 and 8 linked together and students may create new probabilities based on the two-way table.
- AP Classroom provides four videos focused on the content and skills to answer this question.
  - The daily video 2 for topic 4.6 demonstrates how to properly calculate the probability of the union of two events (see VAR-4.E.4). The key takeaway from this video is how to calculate the union of probabilities when the two events are mutually exclusive (part (a)).
  - The daily video for topic 4.5 explains how to solve a conditional probability (see VAR-4.D.1). The key takeaway from this video helps students understand how to calculate these types of probabilities (part (b)) using a two-way table like the one given in the problem.
  - The daily video 1 for topic 8.5 explores how to identify the procedure (See VAR-8.J.2) and the hypotheses (See VAR-8.I.2) for a chi-square test for independence. The key takeaway from this video is how to identify which chi-square test and hypotheses are needed based on the number of categorical variables and populations. This video will help students with part (c).
  - The daily video 2 for topic 8.6 illustrates how to conclude the significance test using the *p*-value and level of significance (see DAT-3.L.1) and justify a claim (see DAT-3.L.2) for a chi-square test for independence. The key takeaway from this video explains to students the rules of rejection and the correct wording for the conclusion in part (d).
- AP Classroom also provides topic questions for formative assessment of topics 4.5, 4.6, 8.5, and 8.6, as well as access to the question bank, which is a searchable database of past AP Questions on this topic.
- The Online Teacher Community features many resources shared by other AP Statistics teachers. For example, to locate resources to give your students practice verifying conditions, try entering the keywords "conditional probability" in the search bar, then selecting the drop-down menu for "Resource Library." When you filter for "Classroom-Ready Materials," you may find worksheets, data sets, practice questions, and guided notes, among other resources.

#### Task: Investigative Task Max Score: 4 Mean Score: 1.80

#### What were the responses to this question expected to demonstrate?

The primary goals of the question were to assess a student's ability to (1) identify an appropriate procedure for conducting a one-sample *t*-interval for a population mean; (2) identify the parameter of interest for conducting a one-sample *t*-interval for a population mean; (3) describe the shape of the distribution of a sample using summary statistics for a sample of whistle prices; (4) use the  $1.5 \times IQR$  rule to determine outliers from summary statistics for a sample of whistle prices; (5) calculate Pearson's coefficient of skewness using summary statistics for a sample of whistle prices; (6) locate the value of the calculated Pearson's coefficient of skewness on a graph; (7) conclude on the shape of the distribution for a sample of whistle prices based on the interpretation of the graph; and (8) explain if the normality condition for the one-sample *t*-interval is met.

This question primarily assesses skills in skill category 1: Selecting Statistical Methods, skill category 2: Data Analysis, and skill category 4: Statistical Argumentation. Skills required for responding to this question include (1.D) Identify an appropriate inference method for confidence intervals, (2.A) Describe data presented numerically or graphically, (2.B) Construct numerical or graphical representations of distributions, (2.C) Calculate summary statistics, relative positions of points within a distribution, correlation, and predicted response, (4.B) Interpret statistical calculations and findings to assign meaning or assess a claim, and (4.C) Verify that inference procedures apply in a given situation.

This question covers content from Unit 1: Exploring One-Variable Data, Unit 2: Exploring Two-Variable Data, and Unit 7: Inference for Quantitative Data: Means of the course framework in the AP Statistics Course and Exam Description. Refer to topics 1.6, 1.7, 1.8, 2.2, 7.2, 7.3, and 7.5, and learning objectives UNC-1.H, UNC-1.K, UNC-1.M, UNC-4.O, UNC-4.P, and UNC-4.S.

- In part (a-i) many responses did not correctly indicate the appropriate inference procedure. Of the responses that did not correctly indicate the appropriate inference procedure, many indicated a wrong procedure, such as a "test," and others did not provide a complete response (e.g., *t*-interval instead of one-sample *t*-interval). In part (a-ii) many responses did not provide a complete description of the parameter; some did not include reference to the population and others did not include context for the variable. Some responses incorrectly referred to the sample when defining the parameter.
- In part (b-i) most responses correctly indicated the distribution was skewed to the right. However, most responses provided no justification based on the summary statistics and simply described shape, center, spread, and outliers. In part (b-ii) many responses correctly computed the lower and upper outlier criteria and stated there were no outliers. Some responses included wrong formulas or calculation errors.
- In part (c-i) most responses correctly computed the Pearson coefficient of skewness and correctly indicated its value on the graph by marking it with an "X." However, in part (c-ii) many responses placed the "X" in the wrong location on the graph by either placing it on the curve, where the sample size is 0, or somewhere in the middle of the grey section of the graph.

• In part (d) many responses correctly identified the shape of the distribution, provided a reasonable justification, and correctly explained that the normality condition was not satisfied. In part (d-i) some responses did not justify their response. Some responses indicated the distribution was "skewed" in part (d-i) or part (d-ii) instead of "strongly skewed." For responses that incorrectly placed the "X" in the wrong location in part (c), many did not describe the shape consistently with the incorrect placement of the "X." Further, responses that incorrectly identified the shape in part (d-i) often did not completely explain their response to part (d-ii).

# What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	Responses that Demonstrate Understanding	
• In part (a-i) many responses incorrectly stated the appropriate inference procedure was a "test" instead of a confidence interval.	• The inference procedure that should be used to estimate the mean price, in dollars (\$), of this type of whistle at all stores that sell the whistle is a one-sample <i>t</i> -interval for a population mean.	
• In part (a-i) many responses failed to completely state the appropriate inference procedure by leaving out "one-sample," " <i>t</i> ," or "interval."	• The inference procedure that should be used to estimate the mean price, in dollars (\$), of this type of whistle at all stores that sell the whistle is a one-sample <i>t</i> -interval for a population mean.	
• In part (a-ii) many responses did not correctly define the parameter by failing to indicate the parameter is a mean.	• The parameter of interest is the mean whistle price, in dollars (\$), of this type of whistle at all stores that sell the whistle.	
• In part (a-ii) some responses referred to the sample when defining the parameter.	• The parameter of interest is the mean whistle price, in dollars (\$), of this type of whistle at all stores that sell the whistle.	
• In part (b-i) many responses described all characteristics of the distribution instead of justifying the shape based on the summary statistics.	• The distribution of the sample of whistle prices appears slightly skewed to the right, since the mean is slightly higher than the median.	
• In part (b-ii) some responses used an incorrect formula to compute the lower and upper outlier criteria (e.g., added and subtracted 1.5 × IQR from the median).	<ul> <li>Q<sub>1</sub> - 1.5x IQR=4.51 - 1.5(5.475 - 4.51) = 3.0625</li> <li>Q<sub>3</sub> - 1.5x IQR = 5.475 + 1.5(5.475 - 4.51) = 6.9225</li> </ul>	
• In part (c-ii) some responses placed the "X" in the wrong location on the graph.	Sample Size 50 strongly 30 -1.20 -1.00 -0.80 -0.60 -0.40 -0.20 0.00 0.20 0.40 0.60 0.80 1.00 1.20 Pearson's Coefficient of Skewness	

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•	In part (d-i) some responses failed to indicate the shape was strongly skewed.	•	The shape of the distribution of the sample of whistle prices is strongly skewed.
•	In part (d-i) some responses failed to justify their response using the graph or Pearson's coefficient of skewness.	•	Looking at the graph in part (c), for a sample size of 20, and a skewness coefficient of 0.949, this point falls in "the distribution of sample data is considered strongly skewed" region.
•	In part (d-ii) some responses failed to indicate the sample size was less than 30.	•	Julio only has a sample size of 20, which is less than 30.
•	In part (d-ii) some responses indicated the distribution of the sample data was "skewed" but not "strongly skewed."	•	The Pearson's coefficient of skewness indicates the distribution of sample data is strongly skewed.

- Provide students with practice identifying an appropriate inference procedure in words instead of by formula or by using calculator syntax.
- Provide students with practice sets where they describe parameters separately and not as part of a larger procedure such as a hypothesis test.
- Discuss ways shape can be justified using a comparison of summary statistics and discuss the connections between shape and summary statistics.
- Encourage students to read questions carefully and pay attention to specific information they are given. For example, if the question reads "Justify your response using appropriate values from the summary statistics table," the summary statistics in the table must be used.
- Remind students to provide answers in context to provide clear communication. This typically includes the variable name and a description of the population.
- Remind students they should answer the question being asked and then provide a statistical justification for their answer.

- The AP Statistics Course and Exam Description (CED), effective Fall 2020, includes instructional resources for AP Statistics teachers to develop students' broader skills.
  - Please see pages 226, 227, and 232 of the CED for examples of key questions and instructional strategies designed to develop skill 1.D, identify an appropriate inference method for confidence intervals; skill 2.A, describe data presented numerically or graphically, (2.B) Construct numerical or graphical representations of distributions, (2.C) Calculate summary statistics, relative positions of points within a distribution, correlation, and predicted response, (4.B) Interpret statistical calculations and findings to assign meaning or assess a claim, and (4.C) Verify that inference procedures apply in a given situation.

- A table of representative instructional strategies, including definitions and explanations of each, is included on pages 213–223 of the CED. The strategy "Marking the Text," for example, may be helpful for students to think of the investigative task in different ways. This FRQ could be used as an example to help students identify important information in the text and make notes about the interpretation of tasks required and concepts to apply to reach a solution. It will also help students understand the format of the investigative task better.
- AP Classroom provides four videos focused on the content and skills to answer this question.
  - The daily video 2 for topic 7.2 shows how to estimate the value of a population parameter (see UNC-4.O.1). The key takeaway from this video explains how to choose between a confidence interval and a significance test for a population parameter (part (a)). This video also explains the normality conditions for a one-sample *t*-interval for a population mean (see UNC-4.P.1 (b)). This explanation helps students understand how to apply this condition (part (d-ii)).
  - The daily video for topic 1.8 explains to students how the shape of the graph influences the relative relationship of the mean and median (see UNC-1.M.2). The key takeaway from this video helps students understand that when the mean is higher than the median, the distribution of the sample is skewed to the right (part (b-i)).
  - The daily video 2 for topic 1.7 demonstrates how to use the two common methods for determining outliers (See UNC-1.K.1). The key takeaway from this video describes how to correctly report when no outliers are present (part (b-ii)).
- AP Classroom also provides topic questions for formative assessment of topics 1.6, 1.7, 1.8, 2.2, 7.2, 7.3, and 7.5, as well as access to the question bank, which is a searchable database of past AP Questions on this topic.
- The Online Teacher Community features many resources shared by other AP Statistics teachers. For example, to locate resources to give your students practice verifying conditions, try entering the keywords "normality condition" in the search bar, then selecting the drop-down menu for "Resource Library." When you filter for "Classroom-Ready Materials," you may find worksheets, data sets, practice questions, and guided notes, among other resources.