Chief Reader Report on Student Responses:

2024 AP® Precalculus Free-Response Questions

• Number of Students Scored	184,394		
• Number of Readers	481		
• Score Distribution	Exam Score	N	%At
	5	47,767	25.9
	4	44,036	23.9
	3	47,678	25.9
	2	26,836	14.6
	1	18,077	9.8
• Global Mean	3.42		

The following comments on the 2024 free-response questions for AP® Precalculus were written by the Chief Reader, Michael Boardman of Pacific University. They give an overview of each free-response question and of how students performed on the question, including typical student errors. General comments regarding the skills and content that students frequently have the most problems with are included. Some suggestions for improving student preparation in these areas are also provided. Teachers are encouraged to attend a College Board workshop to learn strategies for improving student performance in specific areas.

Question 1

Topic: Function Concepts

	Max. Points:	Mean Score:
1A1	1	0.63
1A2	1	0.61
1B1	1	0.51
1B2	1	0.48
1C1	1	0.59
1C2	1	0.15

Overall Mean: 2.95

What were the responses to this question expected to demonstrate?

This question assesses knowledge of and skill with specific function concepts from the course framework. It involves functions that are presented in different representations, one given graphically and the other given analytically.

- To complete Part A(i), one must identify from the graph the output of the function f when 3 is the input (Skill 2.A) and use it to compute the value of the composition $g(f(3))$. (LO 2.7.A, EK 2.7.A.2)
- In Part A(ii) the response must demonstrate an ability to identify from the graph of the function f inputs that produce the specific output 1 (Skill 2.A, LO 2.8.A, EK 2.8.A.2). In this case, there were three such inputs: $x = -3$, $x = 0$, and $x = 3$.
- In Part B(i) one must do similar work as in A(ii) but with the analytically presented function g. A graphing calculator is used to solve an equation to find all domain values that produce an output of 2. (Skill 1.A, LO 2.13.A, EK 2.13.A.2)
- Part B(ii) asks for the right end behavior of g, an exponential decay function. The response requires the use of proper limit notation in stating $\lim_{x \to \infty} g(x) = 0$. (Skill 3.A, LO 2.3.A, EK 2.3.A.5)
- Part C requires (i) determining that the function f does not have an inverse function (Skill 1.C, LO 2.8.A, EK 2.8.A.1) and (ii) giving reasoning for this answer (Skill 3.C, LO 2.8.A, EK 2.8.A.1). The reasoning must be specific to the function f, such as stating that the output 1 comes from the multiple inputs $x = -3$, $x = 0$, and $x = 3$.

How well did the responses address the course content related to this question? How well did the responses integrate the skill(s) required on this question?

- Part A(i): Most responses indicated facility with interpreting the graph of the function f, and an understanding of composition.
- Part A(ii): Most responses indicated an ability to distinguish between inputs and outputs on the graph of f.

- Part B(i): Responses generally showed an understanding of solving an exponential equation using a graphing calculator.
- Part B(ii): Responses often had a correct end behavior but had incorrect limit notation.
- Part C(i): Responses to this part were often difficult to interpret. A response of "*f* does not have an inverse" or "*f* is not invertible" would have sufficed, but many responses included phrases such as "*f* is not an inverse," which has different meaning from the expected response. It may be that the respondents understood the concept of inverse but had difficulty expressing it in words.
- Part C(ii): Many responses included correct general statements regarding when a function does or does not have an inverse. To earn the point, the response was required to include evidence from the function *f* supporting the conclusion that *f* is not invertible. This evidence was only provided sometimes.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses That Demonstrate Understanding*
• Part A(ii): Not knowing that a particular output may come from multiple inputs	• Identifying each of $x = -3$, $x = 0$, and $x = 3$ as producing the output 1
• Part B(i): Not presenting an answer that is accurate to three places after the decimal point. This can result from rounding or truncating intermediate computations or from not reporting enough decimal places in the final answer.	• $x = 1.057$
• Part B(ii): Difficulty with the limit notation	• $\lim_{x \to \infty} g(x) = 0$
• Part C(i): Difficulty clearly stating that *f* does not have an inverse function	• The function *f* does not have an inverse function.
• Part C(ii): Difficulty giving a clear reason that *f* does not have an inverse function	• For the function *f*, the output 1 comes from multiple inputs, $x = -3$, $x = 0$, and $x = 3$.

Based on your experience at the AP Reading with student responses, what advice would you offer teachers to help them improve student performance on the exam?

- Students need facility with composition of functions where the functions are represented in different ways (i.e., graphically, numerically, analytically).
- It is important for students to have practice solving equations that involve a function presented graphically.

- Teachers should help students develop skills with using their graphing calculators to evaluate functions, find real zeros of functions, and find numerical solutions to equations in one variable. Emphasize that the General Instructions for the free-response section indicate that “any decimal approximations reported in your work should be accurate to three places after the decimal point.”
- Students should experience how rounding intermediate computations from their graphing calculator can change the accuracy of a final answer. Emphasize to students to avoid rounding intermediate computations on the way to the final result.
- Exposure to and practice with limit notation is crucial (Topics 1.6, 1.7, 1.9, 1.10, 2.3, and 2.11), both for performance on the AP Precalculus Exam and as preparation for calculus.
- Students need practice with writing clear, unambiguous responses to questions asking for explanations or reasoning (such as in Part C).

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- Teachers may want to have more class time devoted to AP Daily videos and AP Classroom Topic Questions from Topic 2.7: Composition of Functions and Topic 2.8: Inverse Functions. Using class time to work on language and processes from these two topics will help students tighten their communication and understanding. As teachers move to Unit 3 and Topic 3.9: Inverse Trigonometric Functions, students should revisit the concepts from Topics 2.7 and 2.8.
- When students encounter limit notation in Topics 1.6, 1.7, 1.9, 1.10, 2.3, and 2.11, they should practice with the AP Classroom Topic Questions and observe the AP Daily videos to develop more familiarity with the notation and the required components.

Question 2

Topic: Modeling a Non-Periodic Context

	Max. Points:	Mean Score:
2A1	1	0.58
2A2	1	0.36
2B1	1	0.58
2B2	1	0.39
2B3	1	0.04
2C1	1	0.08

Overall Mean: 2.03

What were the responses to this question expected to demonstrate?

This question assesses several skills and essential knowledge statements from the course framework in a non-periodic context: sales each day of a particular video game. Daily sales of the video game are given on two days, the first day of sales and 91 days later. A logarithmic function of the form $G(t) = a + b\ln(t + 1)$ is used to model daily sales.

- In Part A(i) a response should display two equations that use the data from the question stem and that can be used to find both a and b. The equations come directly from $G(0) = 40$ and $G(91) = 76$.
- In Part A(ii) a response should give the values of a and b, accurate to three places after the decimal point. Both points in Part A assess Skill 1.C, LO 2.14.A, and EK 2.14.A.2.
- In Part B(i) a response should use the data from the question stem to demonstrate the computation of the average rate of change of daily sales over an interval (Skill 1.B, LO 1.3.A, EK 1.3.A.3). A decimal approximation to this average rate of change should be presented.
- In Part B(ii) the average rate of change computed in (i) is used to estimate the daily sales on day 50, a day between the two given data points. (Skill 3.B, LO 1.14.C, EK 1.14.C.1)
- In Part B(iii) a response must explain why, for days between 0 and 91, estimates of daily sales using the average rate of change in (i) are all less than what the model G predicts. A response is expected to indicate that the estimates come from the secant line on the interval and that the graph of function G is concave down on the interval. (Skill 3.C, LO 1.14.C, EK 1.14.C.1)
- In Part C a response is to explain why the error in the model G increases after $t = 91$ (Skill 3.C, LO 2.6.B, EK 2.6.B.2). This explanation should include that model G and actual sales agree at $t = 91$, but that G increases while daily sales decrease after $t = 91$.

How well did the responses address the course content related to this question? How well did the responses integrate the skill(s) required on this question?

- Part A(i): Most responses had correct equations. Some responses incorporated the units of thousands and had correct equations with these values (e.g., 40,000 and 76,000).

- Part A(ii): Many responses with correct equations in A(i) solved correctly for the values of a and b.
- Part B(i): Most responses showed the setup for the requested average rate of change and presented a correct decimal approximation for this value.
- Part B(ii): Many responses showed complete work, finding the y-coordinate of the point with t-coordinate 50 along the secant line from the point $(0, 40)$ to the point $(91, 76)$.
- Part B(iii): Few responses communicated an understanding that because the graph of $y = G(x)$ is concave down on the interval $0 < t < 91$, the secant line that passes through the point $(0, G(0))$ and the point $(91, G(91))$ is below the graph of $y = G(x)$.
- Part C: Few responses communicated an understanding that the error in the model increases after $t = 91$ because the model G and daily sales agree at $t = 91$ and for $t > 91$, the model G increases and the daily sales decrease.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses That Demonstrate Understanding*
• Part A(ii): Some responses had difficulty with algebraically solving an equation involving a logarithm. However, work was not required in this part because the equation can be solved using the graphing calculator without algebraic manipulation. Also, some responses did not present an answer that is accurate to three places after the decimal point. This can result from rounding or truncating intermediate computations or from not reporting enough decimal places in the final answer.	• Use of the graphing calculator to obtain $a = 40$ and $b = 7.961$
• Part B(i): Not knowing that the average rate of change of a function on an interval is the same as slope of the secant line between the corresponding points on the graph	• Average rate of change $= \dfrac{G(91) - G(0)}{91 - 0} = 0.395604$
• Part B(iii): Some responses had difficulty distinguishing between how a function is changing and how the rate of change of a function is changing. Many responses included statements such as “the rate [of change] of a linear function grows at a constant rate.” Although this statement is technically true (this rate of change is changing at a rate of 0, which is constant), it can indicate a belief that the rate of change is changing at a constant, nonzero rate.	• The graph of G is concave down on $0 < t < 91$, so the secant line on this interval is below the graph of G. Therefore, A_t, the y-coordinate of a point on the secant line, is less than $G(t)$.

• Part C: Difficulty giving a clear explanation of why error in the model G grows after $t = 91$	• Daily sales on day 91 and $G(91)$ are the same. For $t > 91$, daily sales decrease while G is increasing. Thus, the error in the model G is increasing.

Based on your experience at the AP Reading with student responses, what advice would you offer teachers to help them improve student performance on the exam?

- Precise use of mathematical notation is crucial. Students should have practice seeing correct answers that are accompanied by work that contains imprecise use of notation, such as expressions connected with "=" when the expressions are not truly equal to each other. This will help students recognize characteristics of using precise notation.
- Students should have practice writing clear, unambiguous responses to questions that involve explanations or reasoning. Teach students to be precise with their mathematical language, and model using the language in the course framework throughout the course. This practice can include exposure to a collection of responses, some that are vague (e.g., lots of uses of "it"), some that are logically confusing, and some that are correct. This will help students recognize characteristics of good responses.
- Teachers should help students develop skills with using their graphing calculator. These include producing graphs and tables, solving equations, calculating regressions (when applicable), and performing computations. Emphasize that the General Instructions for the free-response section indicate that "any decimal approximations reported in your work should be accurate to three places after the decimal point." Students should have practice using their graphing calculator to store information such as computed values for constants, functions they are working with, and any intermediate values. Computations with the graphing calculator that use the stored information help to maintain as much precision as possible and ensure the desired accuracy in final answers.
- Students should experience how rounding intermediate computations from their graphing calculator can change the accuracy of a final answer. Emphasize to students to avoid rounding intermediate computations on the way to the final result.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- Because students struggle with communication in this type of question, teachers may want to use the AP Classroom question bank to build custom practice exams with questions that focus on Skills 3.A and 3.C. This provides opportunities for students to see well-constructed reasoning.
- When students take each of the three AP Precalculus Practice Exams, teachers may want to use class time to discuss students' responses to Question 2, Parts (B)(iii) and (C), and develop activities that have students refine their written communication to be more precise. The Practice Exams are available on both the AP Course Audit site and in AP Classroom.
- Use the task verbs from page 151 of the *AP Precalculus Course and Exam Description* in your instruction starting from day 1 of the course so that students receive multiple opportunities to demonstrate the appropriate methods and reasoning that they'll be expected to show on the AP Exam.

Question 3

Topic: Modeling a Periodic Context

	Max. Points:	Mean Score:
3A1	1	0.71
3A2	1	0.39
3B1	1	0.60
3B2	1	0.20
3C1	1	0.68
3C2	1	0.38

Overall Mean: 2.96

What were the responses to this question expected to demonstrate?

This question assesses several skills and essential knowledge statements from the course framework in the periodic context of a rolling tire of radius 9 inches. A sinusoidal function h will be used to model the distance from the ground to a fixed point on the tire. Enough information is given to determine that this distance is 0 at times $t = \frac{1}{2}$ and $t = \frac{5}{2}$, and is not 0 between those times.

- In Part A a generic sinusoidal graph is given, without scale or axes. Five points are labeled with letters. The response should give appropriate t-coordinates (Skill 2.B, LO 3.7.A, EK 3.7.A.1, EK 3.7.A.3) and $h(t)$-coordinates (Skill 2.B, LO 3.7.A, EK 3.7.A.2) for these points based on the context.
- In Part B it is indicated that $h(t) = a\sin(b(t + c)) + d$. The response should present valid values of the four parameters a, b, c, and d (Skill 1.C, LO 3.6.A, EK 3.6.A.6). Finding these values demonstrates an understanding of amplitude, period, phase shift, and vertical shift.
- In Part C(i) a response is to indicate which of four choices accurately describes the behavior of the function h on an interval between two specific points from the graph in Part A (Skill 2.A, LO 1.1.A, EK 1.1.A.3). On this interval, the function h is positive and increasing.
- In part C(ii) a response should state the behavior of the rate of change of h on the same interval (Skill 3.A, LO 1.1.B, EK 1.1.B.4). In this case, because the graph of h is concave down on the interval, the rate of change is decreasing.

How well did the responses address the course content related to this question? How well did the responses integrate the skill(s) required on this question?

- Part A: Many responses indicated an ability to translate the contextual information to points on a sinusoidal graph.
- Part B(i): Most responses that earned the point for $h(t)$-coordinates in Part A (i) went on to present correct values for the amplitude and vertical shift (values of a and d).

- Part B(ii): Although many responses presented a correct value for b (based on the periodic behavior in the context), few presented a correct value c.
- Part C(i): Most responses identified the correct behavior of the graph of h on the specified interval.
- Part C(ii): Few responses indicated an understanding of how the rate of change of h was changing on the specified interval.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses That Demonstrate Understanding*
• Part A: Difficulty identifying mathematical information from a verbal context	• Providing correct coordinates for the five points labeled on the graph
• Part B(ii): Difficulty identifying a phase shift in a sinusoidal function	• One correct form is $h(t) = 9\sin(\pi(t + -1)) + 9$ or $h(t) = 9\sin(\pi(t - 1)) + 9$.
• Part C(ii): Some responses had difficulty distinguishing between how a function is changing and how the rate of change of a function is changing. Many responses included statements indicating that "the rate of change of h is increasing at an decreasing rate." The second part of this statement (at a decreasing rate) requires calculus knowledge to conclude.	• The rate of change of h is decreasing.

Based on your experience at the AP Reading with student responses, what advice would you offer teachers to help them improve student performance on the exam?

- Students need extensive practice with contextual scenarios throughout the AP Precalculus course.
- Identifying amplitude, midline/vertical shift, period, and phase shift within a periodic context is important.
- Students should have practice writing clear, unambiguous responses to questions that involve explanations or reasoning. Teach students to be precise with their mathematical language, and model using the language in the course framework throughout the course. This practice can include exposure to a collection of responses, some that are vague (e.g., lots of uses of "it"), some that are logically confusing, and some that are correct. This will help students recognize characteristics of good responses.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- The AP Daily videos, AP Classroom Topic Questions, and AP Precalculus Practice Exams provide many opportunities for students to practice modeling a periodic context. In addition, the AP Exam practice videos also spend time leading students through the process of sinusoidal modeling.
- As students study Topics 1.2, 1.3, and 1.4, teachers should use care to attend to the language in the course framework and have students practice that language throughout all units and topics of the course. This is especially important as students practice Skill 3.A and describe function behavior as it relates to rates of change and rates of rates of change. Using the AP Classroom topic questions and other questions from the question bank that indicate concavity as the topic could strengthen student understanding for both precalculus and future work in calculus.
- Use the task verbs from page 151 of the *AP Precalculus Course and Exam Description* in your instruction starting from day 1 of the course so that students receive multiple opportunities to demonstrate the appropriate methods and reasoning that they'll be expected to show on the AP Exam.

Question 4

Topic: Symbolic Manipulations

	Max. Points:	Mean Score:
4A1	1	0.37
4A2	1	0.33
4B1	1	0.16
4B2	1	0.24
4C1	1	0.15
4C2	1	0.04

Overall Mean: 1.28

What were the responses to this question expected to demonstrate?

This question assesses facility with symbolic manipulation of exponential, logarithmic, trigonometric, and inverse trigonometric functions. Symbolic manipulation is an important theme in the course framework.

- In Part A an exponential function and an inverse trigonometric function are given analytically. Each of these functions is used in an equation that is to be solved, one in part (i) and the other in part (ii). In Part A(i) a response should present the work and solution to an equation involving an exponential expression (Skill 1.A, LO 2.13.A, EK 2.13.A.1). In Part A(ii) a response should present the work and solution to an equation involving an inverse trigonometric expression (Skill 1.A, LO 3.10.A, EK 3.10.A.1).
- In Part B two functions are given—function j that involves several terms with logarithm base 10 and function k that involves trigonometric expressions. A response should rewrite the expression for each function in a specified way. In Part B(i) a response is to use rules of logarithms to rewrite $j(x)$ so that its expression involves only one term of the form $\log_{10}(\text{expression})$ (Skill 1.B, LO 2.12.A, EK 2.12.A.1, EK 2.12.A.2). In Part B(ii) a response is to use trigonometric identities to rewrite $k(x)$ so that its expression has one term involving only $\tan x$ (Skill 1.B, LO 3.12.A, EK 3.12.A.1).
- In Part C a function involving the composition of an inverse trigonometric function and a trigonometric function is given—specifically, $m(x) = \cos^{-1}(\tan(2x))$. A response, showing the work leading to the answer, is to determine all inputs to m that yield an output of 0 (Skill 1.A, LO 3.10.A, EK 3.10.A.1). There are infinitely many such input values. The first point is earned for presenting one of the input values, while the second point is earned for presenting the entire collection of input values.

How well did the responses address the course content related to this question? How well did the responses integrate the skill(s) required on this question?

- Part A(i): Some responses included sufficient and correct work that led to the presented correct answer.
- Part A(ii): Some responses included the work of converting the equation so that it involved the sine function rather than the arcsine function. These responses usually presented the correct answer.

- Part B(i): Some responses correctly used rules of logarithms but did not necessarily fully rewrite the expression as instructed in the question and in the Free-Response Question 4 Directions.
- Part B(ii): While many responses were able to make use of the Pythagorean identity $1 - \sin^2 x = \cos^2 x$, many did not fully and correctly rewrite the expression so that the expression had a single occurrence of $\tan x$.
- Part C: Few responses included a correct solution to the equation in the domain $[0, 2\pi)$ and even fewer were successful in presenting all real solutions.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

Common Misconceptions/Knowledge Gaps	*Responses That Demonstrate Understanding*
• Part A: Misuse of function notation can indicate a misunderstanding of functions mapping inputs to outputs.	• Correct response with work: $x + 3 = \ln 10$ $x = -3 + \ln 10$
• Part A: Lack of knowledge of the inverse trigonometric functions	• Correct response with work: $\frac{x}{2} = \sin\left(\frac{\pi}{4}\right)$ $x = \sqrt{2}$
• Part B: Difficulty rewriting algebraic expressions involving trigonometric functions	• $\frac{\cos^2 x}{\sin x} \cdot \frac{1}{\cos x} = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$
• Part B: Misunderstanding of $\tan^{-1} x$, thinking that this is eqivalent to $\frac{1}{\tan x}$	• $\frac{\cos x}{\sin x} = \frac{1}{\tan x}$ as final step and answer
• Part C: Lack of knowledge on how to eliminate inverse cosine from the equation	• $\tan(2x) = \cos(0)$ as part of the work

Based on your experience at the AP Reading with student responses, what advice would you offer teachers to help them improve student performance on the exam?

- Symbolic manipulation is an important part of the AP Precalculus course. Students need practice in using the properties of exponents, properties of logarithms, definitions of trigonometric functions, and trigonometric identities in order to rewrite expressions in mathematically equivalent forms. Without the help of technology, students need practice in solving equations with exponential functions, logarithmic functions, trigonometric functions, and inverse trigonometric functions.

- Students should practice showing *all* the steps that lead to their answer when solving an equation or when rewriting an expression.
- Students need practice using fundamental trigonometric identities.
- Students need time and extensive practice to understand inverse trigonometric functions.
- Teachers should share the Free-Response Question 4 Directions with students in advance. Students should practice rewriting numerical and algebraic expressions based on the requirements of these directions. The directions are included in the *AP Precalculus Course and Exam Description*, the AP Precalculus Practice Exams, and in the free-response questions on AP Central and in AP Classroom.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- Throughout the AP Exam, students often demonstrated greater success in conceptual understanding than algebraic manipulation as evidenced by the mean scores in this free-response question. Teachers may want to use the AP Classroom resources for each of the topics that focus on Skills 1.A and 1.B more exhaustively. It may be valuable for teachers to spend time working multiple solution paths to solving equations and rewriting expressions with the students, especially in Topics 1.11, 2.4, 2.12, 2.13, 3.10, and 3.12.