AP® Statistics

Sample Student Responses and Scoring Commentary

Inside:

Free-Response Question 5

☑ **Scoring Guidelines**

☑ **Student Samples**

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Question 5: Multi-Focus   4 points

General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).

- The model solution represents an ideal response to each section of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

Model Solution	Scoring
(a) $P(11+ \text{ months} \cap \text{ majority regular cards})$ $= \dfrac{71 + 76 + 112}{500}$ $= \dfrac{259}{500}$ $= 0.518$	**Essentially correct (E)** if the response satisfies the following two components: 1. Provides the correct probability 2. Shows work for the correct probability **Partially correct (P)** if the response satisfies only one of the two components required for E. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- An arithmetic or transcription error in a response can be ignored if correct work is shown.
- A correct fraction, decimal, or percentage may satisfy component 1.
- To satisfy component 2 work must demonstrate how the numerator of 259 was computed. This may be accomplished by showing the addition or clearly indicating the appropriate summands on the table.
- A response that satisfies components 1 and 2 for the probability that the collector has been collecting baseball cards for 11 or more months and has a majority of *rare* baseball cards may be scored P.
- A specific probability statement is not required, but if correctly given should be considered a positive in holistic scoring.

Model Solution	Scoring
(b) $P(\text{majority regular cards} \mid \text{fewer than 6 months})$ $= \dfrac{P(\text{majority regular cards} \cap \text{fewer than 6 months})}{P(\text{fewer than 6 months})}$ $= \dfrac{\frac{80}{500}}{\frac{91}{500}}$ $= \dfrac{80}{91}$ $= 0.879$	**Essentially correct (E)** if the response satisfies the following two components: 1. Provides the correct probability 2. Shows work for the correct probability **Partially correct (P)** if the response satisfies only one of the two components required for E. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- An arithmetic or transcription error in a response can be ignored if correct work is shown.

- A response of $\frac{80}{91}$ satisfies both components 1 and 2.

- A response that satisfies components 1 and 2 for the probability that given a randomly selected collector has been collecting baseball cards for fewer than six months, they have a majority of *rare* baseball cards may be scored P.

- A specific probability statement is not required, but if correctly given should be considered a positive in holistic scoring.

Model Solution	Scoring
(c) (i) Michelle should conduct a chi-square test for independence between months collecting baseball cards and majority card status for all baseball card collectors at the convention. (ii) H_0: There is not an association between months collecting baseball cards and majority card status for all baseball card collectors at the convention. H_a: There is an association between months collecting baseball cards and majority card status for all baseball card collectors at the convention. *OR* H_0: Months collecting cards and majority card status are independent for all baseball card collectors at the convention. H_a: Months collecting cards and majority card status are not independent for all baseball card collectors at the convention.	**Essentially correct (E)** if the response satisfies the following three components: 1. In part (c-i) the response identifies a chi-square test for independence by name 2. In part (c-ii) the response states the correct null hypothesis to imply there is not an association (relationship) *AND* states the correct alternative hypothesis to imply there is an association (relationship) 3. In part (c-ii) the response provides sufficient context for at least one of the hypotheses by including reference to both "months collecting" and "majority card status" **Partially correct (P)** if the response satisfies only two of the three components required for E *OR* if the response identifies a "chi-square test for homogeneity" in part (c-i) by name or formula *AND* components 2 and 3 are consistent with the chi-square test for homogeneity. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes

- A response that states merely "chi-square" or "chi-square test," without specifying independence, does not satisfy component 1. However, a response that states "chi-square test of association" does satisfy component 1.
- Any discussion of the degrees of freedom for the test should be ignored in scoring.
- If the hypotheses do not explicitly state the population of interest, it should be assumed that the population is all baseball card collectors at the convention. However, if the response clearly references the sample, component 2 is not satisfied.

Model Solution	Scoring
(d) Because the p-value of 0.0075 is less than any reasonable α level such as 0.05 or 0.10, the null hypothesis should be rejected. The data provide convincing statistical evidence that there is an association between months collecting baseball cards and majority card status for all baseball card collectors at the convention.	**Essentially correct (E)** if the response satisfies the following two components: 1. Provides correct comparison of the p-value to alpha (p-value is less than alpha) *AND* provides a correct decision about the null and/or alternative hypothesis 2. States a conclusion, in context, consistent with and in terms of the stated alternative hypothesis in part (c) using nondefinitive language **Partially correct (P)** if the response satisfies only one of the two components required for E. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes

- Because no alpha value is explicitly given, a response may satisfy component 1 by stating that the p-value is small *AND* providing a correct decision about the null and/or alternative hypothesis.
- An explicit decision about the null hypothesis is not required to satisfy component 1.
- If an explicit decision is stated and the conclusion is inconsistent with the decision, component 1 is not satisfied.
- The decision part of component 1 may be satisfied by implying the decision within the conclusion statement (sufficient evidence/insufficient evidence for the alternative hypothesis).
- To satisfy the context in component 2 the response must include discussion of both "months spent collecting baseball cards" and "which type of card is the majority in the collection."
- If the response omits hypotheses in part (c), assume the correct alternative hypothesis is provided when scoring component 2.
- If the response states incorrect hypotheses in part (c), components 1 and 2 may be satisfied by either referring to the stated alternative hypothesis or referring to the inference question.
- Examples of nondefinitive language in component 2 include "evidence to accept the alternative," "there is evidence for the alternative," "there is not sufficient evidence for the alternative," and "Michelle should conclude that her belief is correct."
- Examples of definitive language in component 2 include "proves the null," "proves the alternative," "accepts the alternative," "there is not evidence for the alternative," "no evidence for the alternative," "therefore the alternative is true," and "there is an association."
- If components 1 and/or 2 are satisfied and the response provides an incorrect interpretation of the p-value, the score is lowered from E to P or P to I.

Scoring for Question 5 Each essentially correct (E) part counts as 1 point, and each partially correct (P) part counts as ½ point.	Score
Complete Response	**4**
Substantial Response	**3**
Developing Response	**2**
Minimal Response	**1**

If a response is between two scores (for example, 2 ½ points), use a holistic approach to decide whether to score up or down, depending on the strength of the response and quality of the communication.

Question 5

Begin your response to **QUESTION 5** on this page.

5. Baseball cards are trading cards that feature data on a player's performance in baseball games. Michelle is at a national baseball card collector's convention with approximately 20,000 attendees. She notices that some collectors have both regular cards, which are easily obtained, and rare cards, which are harder to obtain. Michelle believes that there is a relationship between the number of months a collector has been collecting baseball cards and whether the majority of the cards (cards appearing more often) in their collection are regular or rare. She obtains information from a random sample of 500 baseball card collectors at the convention and records how many full months they have been collecting baseball cards and whether the majority of the cards in their card collection are regular or rare. Her results are displayed in a two-way table.

Majority Type of Baseball Cards and Months of Collecting Baseball Cards

	Fewer Than 6 Months	6 - 10 Months	11 - 15 Months	16 - 20 Months	21 or More Months	Total
Has a Majority of Regular Baseball Cards	80	84	71	76	112	423
Has a Majority of Rare Baseball Cards	11	16	9	6	35	77
Total	91	100	80	82	147	500

(a) If one collector from the sample is selected at random, what is the probability that the collector has been collecting baseball cards for 11 or more months and has a majority of regular baseball cards? Show your work.

$$P(11^+ \text{ months} \cap \text{regular}) = \frac{71 + 76 + 112}{500} = .518$$

(b) Given that a randomly selected collector from the sample has been collecting baseball cards for fewer than 6 months, what is the probability the collector has a majority of regular baseball cards? Show your work.

$$P\left(\text{regular} \mid \frac{\text{less than}}{6 \text{ months}}\right) = \frac{P\left(\text{regular} \cap \frac{\text{less than}}{6 \text{ months}}\right)}{P(\text{less than } 6 \text{ months})}$$

$$= \frac{80/500}{91/500} = \frac{.16}{.182} = .879$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5261/10

Question 5

Continue your response to **QUESTION 5** on this page.

(c) Michelle believes there is a relationship between the number of months spent collecting baseball cards and which type of card is the majority in the collection (regular or rare).

(i) Name the hypothesis test Michelle should use to investigate her belief. Do not perform the hypothesis test.

chi-squared test for independence

(ii) State the appropriate null and alternative hypotheses for the hypothesis test you identified in (c-i). Do not perform the hypothesis test.

H_0: there is no relationship between months spent collecting baseball cards and which type of card is majority in collection

H_A: there is a relationship between months spent collecting baseball cards and which type of card is majority in collection

(d) After completing the hypothesis test described in part (c), Michelle obtains a *p*-value of 0.0075. Assuming the conditions for inference are met, what conclusion should Michelle make about her belief? Justify your response. $\alpha = .05$

$P = .0075 < .05$ $\therefore$ reject H_0

Michelle should reject H_0, finding significant evidence at $\alpha = .05$ level that there is a relationship between months spent collecting baseball cards and which type of card is majority in the collection

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5261/11

Question 5

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5. Baseball cards are trading cards that feature data on a player's performance in baseball games. Michelle is at a national baseball card collector's convention with approximately 20,000 attendees. She notices that some collectors have both regular cards, which are easily obtained, and rare cards, which are harder to obtain. Michelle believes that there is a relationship between the number of months a collector has been collecting baseball cards and whether the majority of the cards (cards appearing more often) in their collection are regular or rare. She obtains information from a random sample of 500 baseball card collectors at the convention and records how many full months they have been collecting baseball cards and whether the majority of the cards in their card collection are regular or rare. Her results are displayed in a two-way table.

Majority Type of Baseball Cards and Months of Collecting Baseball Cards

	Fewer Than 6 Months	6 - 10 Months	11 - 15 Months	16 - 20 Months	21 or More Months	Total
Has a Majority of Regular Baseball Cards	80	84	71	76	112	423
Has a Majority of Rare Baseball Cards	11	16	9	6	35	77
Total	91	100	80	82	147	500

(a) If one collector from the sample is selected at random, what is the probability that the collector has been collecting baseball cards for 11 or more months and has a majority of regular baseball cards? Show your work.

P(collecting for 11+ months ∩ majority of regular Baseball cards) = $\frac{259}{309}$

$\frac{71 + 76 + 112}{80 + 82 + 147} = \frac{259}{309}$

(b) Given that a randomly selected collector from the sample has been collecting baseball cards for fewer than 6 months, what is the probability the collector has a majority of regular baseball cards? Show your work.

P(majority of regular baseball cards | collecting for less than 6 months) = $\frac{80}{91}$

Collecting for 6 months total : 91

~~Majority of~~

Page 10   **GO ON TO THE NEXT PAGE.**

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5261/10

Question 5

Continue your response to **QUESTION 5** on this page.

(c) Michelle believes there is a relationship between the number of months spent collecting baseball cards and which type of card is the majority in the collection (regular or rare).

(i) Name the hypothesis test Michelle should use to investigate her belief. Do not perform the hypothesis test.

Chi-Squared Test of Independence

(ii) State the appropriate null and alternative hypotheses for the hypothesis test you identified in (c-i). Do not perform the hypothesis test.

H_0: There is no association between the number of months spent collecting baseball cards & the type of card is in the majority

H_A: There is an association between the number of months spent collecting baseball cards & the type of card in the majority.

(d) After completing the hypothesis test described in part (c), Michelle obtains a *p*-value of 0.0075. Assuming the conditions for inference are met, what conclusion should Michelle make about her belief? Justify your response.

$P = .0075$

a is assumed $.05$

$p < a$, reject H_0

We have significant evidence at $a = .05$ to suggest that there is a relationship between the number of months spent collecting baseball cards & the type of card is the majority in the collection

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Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5261/11

Question 5

Begin your response to **QUESTION 5** on this page.

5. Baseball cards are trading cards that feature data on a player's performance in baseball games. Michelle is at a national baseball card collector's convention with approximately 20,000 attendees. She notices that some collectors have both regular cards, which are easily obtained, and rare cards, which are harder to obtain. Michelle believes that there is a relationship between the number of months a collector has been collecting baseball cards and whether the majority of the cards (cards appearing more often) in their collection are regular or rare. She obtains information from a random sample of 500 baseball card collectors at the convention and records how many full months they have been collecting baseball cards and whether the majority of the cards in their card collection are regular or rare. Her results are displayed in a two-way table.

Majority Type of Baseball Cards and Months of Collecting Baseball Cards

	Fewer Than 6 Months	6 - 10 Months	11 - 15 Months	16 - 20 Months	21 or More Months	Total
Has a Majority of Regular Baseball Cards	80	84	71	76	112	423
Has a Majority of Rare Baseball Cards	11	16	9	6	35	77
Total	91	100	80	82	147	500

(a) If one collector from the sample is selected at random, what is the probability that the collector has been collecting baseball cards for 11 or more months and has a majority of regular baseball cards? Show your work.

$$71 + 76 + 112 \rightarrow \frac{259}{309} = \boxed{0.838}$$
$$= 259 \qquad \leftarrow 80 + 82 + 147$$

(b) Given that a randomly selected collector from the sample has been collecting baseball cards for fewer than 6 months, what is the probability the collector has a majority of regular baseball cards? Show your work.

$$\frac{80}{91} = \boxed{0.879}$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5261/10

Question 5

1 sample
2 questions

Continue your response to **QUESTION 5** on this page.

(c) Michelle believes there is a relationship between the number of months spent collecting baseball cards and which type of card is the majority in the collection (regular or rare).

$\alpha = 0.05$

(i) Name the hypothesis test Michelle should use to investigate her belief. Do not perform the hypothesis test.

Michelle should use the homogenity x^2 test. This is because she is obtaining information from one sample and asking two questions.

(ii) State the appropriate null and alternative hypotheses for the hypothesis test you identified in (c-i). Do not perform the hypothesis test.

H_0 = There is no relationship between the number of months spent collecting baseball cards and which type of card is the majority in the collection.

H_a ≠ There is a relationship between the number of months spent collecting baseball cards and which type of card is the majority in the collection.

(d) After completing the hypothesis test described in part (c), Michelle obtains a *p*-value of 0.0075. Assuming the conditions for inference are met, what conclusion should Michelle make about her belief? Justify your response.

(H_0)

We can reject the null since the p-val (0.0075) is less than α (0.05). This means that Michelle can conclude that her belief is reasonable/correct. Because the null was able to be rejected, the alternate hypothesis would be accepted, and the alternate hypothesis states how there is a relationship between the number of months and type of card (majority) in the collection, which was her initial belief.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5261/11

Question 5

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The primary goals of the question were to assess a student's ability to (1) calculate a probability from a two-way table; (2) calculate a conditional probability from a two-way table; (3) identify an appropriate procedure for conducting a hypothesis test for a chi-square test of independence; (4) identify the correct hypotheses for conducting a chi-square test of independence; (5) compare the p-value to a significance level to make a decision regarding the hypotheses; and (6) determine an appropriate conclusion for a chi-square test of independence.

This question primarily assesses skills in category 1: Selecting Statistical Methods, skill category 3: Using Probability and Simulation, and skill category 4: Statistical Argumentation. Skills required for responding to this question include (1.E) Identify an appropriate inference method for significance tests, (1.F) Identify null and alternative hypotheses, (3.A) Determine relative frequencies, proportions, or probabilities using simulation or calculations, and (4.E) Justify a claim using a decision based on significance tests.

This question covers content from Unit 4: Probability, Random Variables, and Probability Distributions and Unit 8: Inference for Categorical Data: Chi-Square of the course framework in the AP Statistics Course and Exam Description. Refer to topics 4.5, 4.6, 8.5, and 8.6, and learning objectives DAT-3.L, VAR-4.D, VAR-4.E, VAR-8.1, and VAR-8.J.

Sample: 5A
Score: 4

The response earned the following: Part (a) – E; Part (b) – E; Part (c) – E; Part (d) – E.

In part (a), the response provides the correct probability as a decimal, satisfying component 1. The response also shows work for the correct calculation by including the correct denominator and showing how the numerator is computed, satisfying component 2. Part (a) was scored essentially correct (E). Note that the response makes a clear probability statement, which is considered a positive element in holistic scoring.

In part (b) the response provides the correct probability, satisfying component 1. The response also shows work for the correct calculation using the rule for conditional probability, satisfying component 2. Part (b) was scored essentially correct (E). Again, this response makes a clear probability statement, which is considered a positive element in holistic scoring.

In part (c-i) the response correctly names the test, satisfying component 1. In part (c-ii) the response has the correct null hypothesis, "there is no relationship" and the correct alternative hypothesis "there is a relationship," satisfying component 2. In part (c-ii) the response has sufficient context by including reference to "months spent collecting baseball cards" and "which type of card is majority in collection," satisfying component 3. Part (c) was scored essentially correct (E).

In part (d) the response states that the p-value is less than alpha and makes the correct decision of rejecting the null hypothesis, satisfying component 1. The response then states a correct conclusion in terms of the alternative hypothesis, with context "months spent collecting" and "which type of card," and non-definitive language "significant evidence," satisfying component 2. Part (d) was scored essentially correct (E).

Question 5 (continued)

Sample: 5B
Score: 3

The response earned the following: Part (a) – I; Part (b) – E; Part (c) – E; Part (d) – E.

In part (a) the response provides the incorrect probability; therefore, component 1 is not satisfied. Although the response provides work, it is not work for the correct calculation; therefore, component 2 is not satisfied. Part (a) was scored incorrect (I).

In part (b), the presentation of the probability as a fraction "$\frac{80}{91}$" provides the correct probability with work shown, satisfying both components 1 and 2. Part (b) was scored essentially correct (E). Note that this response makes clear probability statements in both parts (a) and (b) which would be considered positive in holistic scoring.

In part (c-i) the response correctly names the test, satisfying component 1. In part (c-ii) the response has the correct null hypothesis, "There is no association" and the correct alternative hypothesis, "There is an association," satisfying component 2. In part (c-ii) the response has sufficient context by including reference to "the number of months spent collecting" and "the type of card," satisfying component 3. Part (c) was scored essentially correct (E).

In part (d) the response states that the *p*-value is less than alpha and makes a correct decision of rejecting the null hypothesis, satisfying component 1. The response then states a correct conclusion in terms of the alternative hypothesis, with context "number of months spent collecting" and "the type of card is the majority" and non-definitive language "significant evidence," satisfying component 2. Part (d) was scored essentially correct (E).

Sample: 5C
Score: 2

The response earned the following: part (a) - I; part (b) - E; part (c) - P; part (d) - P.

In part (a) the response provides the incorrect probability; therefore, component 1 is not satisfied. Although the response provides work, it is not work for the correct calculation; therefore, component 2 is not satisfied. Part (a) was scored incorrect (I).

In part (b) the presentation of the probability as the fraction "$\frac{80}{91}$" satisfies both components 1 and 2. Part (b) was scored essentially correct (E).

In part (c-i) the response does not identify the correct test identifying "homogenity" instead of independence. Therefore, component 1 is not satisfied. In part (c-ii) the response has the correct null hypothesis, "There is no relationship" and the correct alternative hypothesis, "There is a relationship," satisfying component 2. In part (c-ii) the response has sufficient context "number of months spent collecting" and "which type of card is the majority," satisfying component 3. Part (c) was scored partially correct (P).

In part (d) the response provides a correct decision of rejecting the null hypothesis and states that the *p*-value is less than alpha, satisfying component 1. Although the response provides a conclusion, in context, consistent with the alternative hypothesis, definitive language is used "the alternate hypothesis would be accepted." Therefore, component 2 is not satisfied. Part (d) was scored partially correct (P).