
AP[®] Statistics

Sample Student Responses and Scoring Commentary

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Free-Response Question 4

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Question 4: Focus on Probability and Sampling Distributions**4 points****General Scoring Notes**

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

Model Solution	Scoring
<p>(a) Let G represent the number of geodes a player opens until finding a red crystal. G follows a geometric distribution with $p = 0.08$.</p> <p>(i) $\mu = E(G) = \frac{1}{0.08} \approx 12.5$ geodes</p> <p>(ii) $\sigma_G = \frac{\sqrt{1 - 0.08}}{0.08} \approx 11.99$ geodes</p>	<p>Essentially correct (E) if the response satisfies at least three of the following four components:</p> <ol style="list-style-type: none"> In part (a-i) the response correctly calculates the mean In part (a-i) the response provides supporting work for the calculation of the correct mean In part (a-ii) the response correctly calculates the standard deviation In part (a-ii) the response provides supporting work for the calculation of the correct standard deviation <p>Partially correct (P) if the response satisfies only two of the four components required for E.</p> <p>Incorrect (I) if the response does not meet the criteria for E or P.</p>

Additional Notes:

- An arithmetic or transcription error in a response can be ignored if correct work is shown.
- A response with the mean rounded to 12 or 13 does not satisfy component 1.
- A response with a standard deviation of 12 geodes satisfies component 3.

Model Solution	Scoring
<p>(b) (i) $P(Y = 3) = (0.92)^2(0.08) \approx 0.067712$</p> <p>(ii) $P(Y = 4) = 1 - P(Y = 1 \text{ or } 2 \text{ or } 3)$ $\approx 1 - (0.08 + 0.0736 + 0.067712)$ ≈ 0.778688</p> <p><i>OR</i></p> <p>If Conrad opens 4 geodes, then he either finds no red geodes or he finds a red geode on the fourth one he opens; therefore,</p> $P(Y = 4) = (0.92)^4 + (0.92)^3(0.08)$ $\approx 0.778688.$	<p>Essentially correct (E) if the response satisfies at least three of the following four components:</p> <ol style="list-style-type: none"> In part (b-i) the response correctly calculates the probability of opening 3 geodes In part (b-i) the response provides supporting work for the calculation of the correct probability In part (b-ii) the response correctly calculates the probability of opening 4 geodes, consistent with the response to part (b-i) In part (b-ii) the response provides supporting work for the calculation of the correct probability <p>Partially correct (P) if the response satisfies only two of the four components required for E.</p> <p>Incorrect (I) if the response does not meet the criteria for E or P.</p>

Additional Notes:

- A response may satisfy component 2 or component 4 by the following or a combination of the following:
 - Probability formula:** Displaying a correct formula for computing the geometric probability, such as:
 - $(1 - 0.08)^2(0.08)$ or $(0.92)^2(0.08)$ for part (b-i)
 - Calculator function syntax: Using calculator function notation with the correct value of the parameter identified, such as:
 - $\text{geompdf}(p = 0.08, x = 3)$ for part (b-i)
 - $1 - \text{geomcdf}(p = 0.08, x = 3)$ for part (b-ii)
 - $\text{binompdf}(n = 4, p = 0.08, x = 0) + \text{geompdf}(p = 0.08, x = 4)$ for part (b-ii)
 - $\text{binompdf}(n = 4, p = 0.92, x = 4) + \text{geompdf}(p = 0.08, x = 4)$ for part (b-ii)
- An arithmetic or transcription error in a response can be ignored if correct work is shown.

Model Solution	Scoring
<p>(c) (i) $\mu = E(Y)$ $\approx (1)(0.08) + (2)(0.0736)$ $+ (3)(0.0677) + (4)(0.778688)$ $\approx 0.08 + 0.1472 + 0.2031 + 3.1148$ ≈ 3.545 geodes.</p> <p>(ii) The mean of 3.545 geodes is the average number of geodes that result from a long run of many, many trials of opening randomly selected geodes and counting the number opened until either a red geode is found or the fourth geode is opened.</p>	<p>Essentially correct (E) if the response satisfies both components 1 and 2 and at least two of components 3–5:</p> <ol style="list-style-type: none"> In part (c-i) the response states the correct mean of the distribution consistent with values calculated in part (b) In part (c-i) the response shows appropriate work to calculate the mean using the values calculated in part (b) In part (c-ii) the interpretation includes the concept of repeating the selection process over a long period of time In part (c-ii) the interpretation includes the concept of an average or mean In part (c-ii) the interpretation includes the context of number of geodes opened <p>Partially correct (P) if the response does not meet the criteria for E but satisfies two or three of components 1–4.</p> <p>Incorrect (I) if the response does not meet the criteria for E or P.</p>

Additional Notes:

- Supporting work for finding the expected value must include at least two of the terms in the equation to show the pattern, such as $1(0.08) + 2(0.0736) + \dots$
 - Calculator notation does not satisfy component 2, such as $1\text{-VAR STATS}(L1, L2)$.
 - An arithmetic or transcription error in a response can be ignored if correct work is shown.
 - The response $\frac{1}{(.08 + .0736 + .0677 + .0623)} \approx 3.526$ does not satisfy component 1 or 2.
 - In part (c-i) if the response has incorrect values in part (b), but uses the values from the correct probability distribution, the response may satisfy components 1 and 2.
 - The numerical value of the mean is not required to satisfy components 3–5.
-

Scoring for Question 4	Score
Complete Response Three parts essentially correct	4
Substantial Response Two parts essentially correct and one part partially correct	3
Developing Response Two parts essentially correct and no part partially correct <i>OR</i> One part essentially correct and one or two parts partially correct <i>OR</i> Three parts partially correct	2
Minimal Response One part essentially correct and no part partially correct <i>OR</i> No part essentially correct and two parts partially correct	1

Question 4

Begin your response to **QUESTION 4** on this page.

4. In an online game, players move through a virtual world collecting geodes, a type of hollow rock. When broken open, these geodes contain crystals of different colors that are useful in the game. A red crystal is the most useful crystal in the game. The color of the crystal in each geode is independent and the probability that a geode contains a red crystal is 0.08.

(a) Sarah, a player, will collect and open geodes until a red crystal is found.

- (i) Calculate the mean of the distribution of the number of geodes Sarah will open until a red crystal is found. Show your work.

The distribution is geometric with $p = .08$ because Sarah is opening geodes until a red crystal is found. The mean of the distribution is $\frac{1}{p} = \frac{1}{.08} = 12.5$ geodes

- (ii) Calculate the standard deviation of the distribution of the number of geodes Sarah will open until a red crystal is found. Show your work.

Standard deviation of number of geodes Sarah opens until a red crystal is found is $\frac{\sqrt{1-p}}{p} = \frac{\sqrt{1-.08}}{.08} = 11.99$ geodes. Distribution is geometric with $p = .08$

- (b) Another player, Conrad, decides to play the game and will stop opening geodes after finding a red crystal or when 4 geodes have been opened, whichever comes first. Let Y = the number of geodes Conrad will open. The table shows the partially completed probability distribution for the random variable Y .

Number of geodes Conrad will open, y	1	2	3	4
Probability, $P(Y = y)$	0.08	0.0736	0.06712	0.77868

Question 4

Continue your response to QUESTION 4 on this page.

(i) Calculate $P(Y = 3)$. Show your work.

$$P(Y = 3) = (1-p)^{x-1} p = (1-.08)^{3-1} (.08)$$

$Y =$ number of geodes Conrad will open $= 0.067712$

There is a 0.067712 probability that Conrad will open three geodes

(ii) Calculate $P(Y = 4)$. Show your work.

$$P(Y = 4) = 1 - P(Y \leq 3) = 1 - .08 - .0736 - .067712$$

$Y =$ number of geodes Conrad will open $= 0.778688$

There is a 0.778688 probability that Conrad will open four geodes

(c) Consider the table and your results from part (b).

(i) Calculate the mean of the distribution of the number of geodes Conrad will open. Show your work.

$Y =$ number of geodes Conrad will open

$$M_y = (1)(.08) + (2)(.0736) + 3(.067712) + 4(.778688)$$

$$= 3.545088 \text{ geodes}$$

(ii) Interpret the mean of the distribution of the number of geodes Conrad will open, which was calculated in part (c-i).

On average, Conrad will open 3.545088 geodes if he stops opening geodes after finding a red crystal or when 4 geodes have been opened. If Conrad plays the game this way many, many times, the average number of geodes he will open will approximate 3.545088 geodes.

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Question 4

Begin your response to **QUESTION 4** on this page.

4. In an online game, players move through a virtual world collecting geodes, a type of hollow rock. When broken open, these geodes contain crystals of different colors that are useful in the game. A red crystal is the most useful crystal in the game. The color of the crystal in each geode is independent and the probability that a geode contains a red crystal is 0.08.

(a) Sarah, a player, will collect and open geodes until a red crystal is found.

- (i) Calculate the mean of the distribution of the number of geodes Sarah will open until a red crystal is found. Show your work.

$$\mu = E(X) = \frac{1}{p} = \frac{1}{0.08} = 12.5$$

- (ii) Calculate the standard deviation of the distribution of the number of geodes Sarah will open until a red crystal is found. Show your work.

$$\sigma = \sqrt{\frac{q}{p}} = \sqrt{\frac{(1-0.08)}{0.08}} = \sqrt{\frac{0.92}{0.08}} = 3.391$$

- (b) Another player, Conrad, decides to play the game and will stop opening geodes after finding a red crystal or when 4 geodes have been opened, whichever comes first. Let Y = the number of geodes Conrad will open. The table shows the partially completed probability distribution for the random variable Y .

Number of geodes Conrad will open, y	1	2	3	4
Probability, $P(Y = y)$	0.08	0.0736	0.06712	0.77848

Question 4

Continue your response to QUESTION 4 on this page.

- (i) Calculate
- $P(Y = 3)$
- . Show your work.

$$P(Y=3) = q^{r-1} \cdot p = 0.92^{3-1} \cdot 0.08 = 0.06712$$

- (ii) Calculate
- $P(Y = 4)$
- . Show your work.

$$P(Y=4) = 0.08 + 0.0736 + 0.068 = 0.221$$

$$1 - 0.221 = 0.778688$$

- (c) Consider the table and your results from part (b).

- (i) Calculate the mean of the distribution of the number of geodes Conrad will open. Show your work.

$$1(0.08) + 2(0.0736) + 3(0.06712) + 4(0.778688)$$

$$0.08 + 0.1472 + 0.203136 + 3.114752$$

$$\mu = 3.545$$

- (ii) Interpret the mean of the distribution of the number of geodes Conrad will open, which was calculated in part (c-i).

The mean is the expected value of the number of geodes Conrad will open.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

0147170

Question 4

Begin your response to **QUESTION 4** on this page.

4. In an online game, players move through a virtual world collecting geodes, a type of hollow rock. When broken open, these geodes contain crystals of different colors that are useful in the game. A red crystal is the most useful crystal in the game. The color of the crystal in each geode is independent and the probability that a geode contains a red crystal is 0.08.

(a) Sarah, a player, will collect and open geodes until a red crystal is found.

- (i) Calculate the mean of the distribution of the number of geodes Sarah will open until a red crystal is found. Show your work.

$$\mu_x = \frac{1}{p} = \frac{1}{0.08} = \boxed{12.5}$$

- (ii) Calculate the standard deviation of the distribution of the number of geodes Sarah will open until a red crystal is found. Show your work.

$$\sigma_x = \frac{\sqrt{1-p}}{p} = \frac{\sqrt{1-0.08}}{0.08} = \boxed{11.989}$$

- (b) Another player, Conrad, decides to play the game and will stop opening geodes after finding a red crystal or when 4 geodes have been opened, whichever comes first. Let Y = the number of geodes Conrad will open. The table shows the partially completed probability distribution for the random variable Y .

Number of geodes Conrad will open, y	1	2	3	4
Probability, $P(Y = y)$	0.08	0.0736		

Question 4

Continue your response to QUESTION 4 on this page.

(i) Calculate $P(Y = 3)$. Show your work.

$$P(Y=3) = (1-p)^{x-1} p = (1-0.08)^{3-1} (0.08)$$

$$= \boxed{0.0677}$$

(ii) Calculate $P(Y = 4)$. Show your work.

$$P(Y=4) = (1-0.08)^{4-1} (0.08)$$

$$= \boxed{0.0623}$$

(c) Consider the table and your results from part (b).

(i) Calculate the mean of the distribution of the number of geodes Conrad will open. Show your work.

$$\mu_X = \frac{1}{p}$$

$$0.08 + 0.0736 + 0.0677 + 0.0623 = 0.2836$$

$$\frac{1}{0.2836} = \boxed{3.526}$$

(ii) Interpret the mean of the distribution of the number of geodes Conrad will open, which was calculated in part (c-i).

Conrad will open about 3.526 geodes on average if he decides to play the game and will stop opening geodes after finding a red crystal or when 4 geodes have been opened.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

0222655



Question 4

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The primary goals of the question were to assess a student’s ability to (1) calculate the mean of a geometric distribution; (2) calculate the standard deviation of a geometric distribution; (3) calculate a probability for a geometric distribution; (4) calculate a probability for a geometric distribution using a previously calculated probability; (5) calculate the mean of a geometric distribution using previously calculated probabilities; and (6) interpret the mean of a geometric distribution using previously calculated probabilities.

This question primarily assesses skills in skill category 3: Using Probability and Simulation, and skill category 4: Statistical Argumentation. Skills required for responding to this question include (3.A) Determine relative frequencies, proportions, or probabilities using simulation or calculations, (3.B) Determine parameters for probability distributions, and (4.B) Interpret statistical calculations and findings to assign meaning or assess a claim.

This question covers content from Unit 4: Probability, Random Variables, and Probability Distributions in the AP Statistics Course and Exam Description. Refer to topics 4.5, 4.6, 4.8, and 4.12, and learning objectives UNC-3.E, UNC-3.F, VAR-4.A, VAR-4.E, VAR-5.C, and VAR-5.D.

Sample: 4A

Score: 4

The response earned the following: Part (a) – E; Part (b) – E; Part (c) – E.

In part (a), the response correctly calculates the mean, satisfying component 1. The response provides appropriate supporting work for the correct mean, satisfying component 2. The response correctly calculates the standard deviation, satisfying component 3. The response provides appropriate supporting work for the correct standard deviation, satisfying component 4. Part (a) was scored essentially correct (E).

In part (b), the response correctly calculates the probability of opening three geodes, satisfying component 1. The response provides appropriate supporting work for the correct probability of opening three geodes, satisfying component 2. The response correctly calculates the probability of opening four geodes, satisfying component 3. The response provides appropriate supporting work for the correct probability of opening four geodes, satisfying component 4. Part (b) was scored essentially correct (E).

In part (c), the response states the correct mean of the distribution consistent with values calculated in part (b), satisfying component 1. The response provides appropriate supporting work for the correct mean using values calculated in part (b), satisfying component 2. The response provides an interpretation of the mean that includes the concept of repeating the selection process over a long period of time, “many, many times,” satisfying component 3. The response includes the concept of an average or mean, “the average,” satisfying component 4. The response includes the context of the number of geodes opened, “number ... he will open,” satisfying component 5. Part (c) was scored essentially correct (E).

Question 4 (continued)**Sample: 4B****Score: 3**

The response earned the following: Part (a) – P; Part (b) – E; Part (c) – E.

In part (a) the response correctly calculates the mean, satisfying component 1. The response provides appropriate supporting work for the correct mean, satisfying component 2. The response does not correctly calculate the standard deviation. The response fails to satisfy component 3. The response does not provide appropriate supporting work for the correct standard deviation. The response fails to satisfy component 4. Part (a) was scored partially correct (P).

In part (b) the response correctly calculates the probability of opening three geodes, satisfying component 1. The response provides appropriate supporting work for the correct probability of opening three geodes, satisfying component 2. The response correctly calculates the probability of opening four geodes, satisfying component 3. The response provides appropriate supporting work for the correct probability of opening four geodes, satisfying component 4. Part (b) was scored essentially correct (E).

In part (c) the response states the correct mean of the distribution consistent with values calculated in part (b), satisfying component 1. The response provides appropriate supporting work for the correct mean using values calculated in part (b), satisfying component 2. The response does not provide an interpretation of the mean that includes the concept of repeating the selection process over a long period of time. The response fails to satisfy component 3. The response includes the concept of an average or mean, “expected value,” satisfying component 4. The response includes the context of the number of geodes opened, “number of geodes Conrad will open,” satisfying component 5. Part (c) was scored essentially correct (E).

Sample: 4C**Score: 2**

The response earned the following: Part (a) – E; Part (b) – P; Part (c) – I.

In part (a) the response correctly calculates the mean, satisfying component 1. The response provides appropriate supporting work for the correct mean, satisfying component 2. The response correctly calculates the standard deviation, satisfying component 3. The response provides appropriate supporting work for the correct standard deviation, satisfying component 4. Part (a) was scored essentially correct (E).

In part (b) the response correctly calculates the probability of opening three geodes, satisfying component 1. The response provides appropriate supporting work for the correct probability of opening three geodes, satisfying component 2. The response does not correctly calculate the probability of opening four geodes. The response calculates only the probability that the fourth geode is red, not the probability that the fourth geode is red or that no red geode is found in the first four selections. The response fails to satisfy component 3. The response does not provide appropriate supporting work for the correct probability of opening four geodes. The response fails to satisfy component 4. Part (b) was scored partially correct (P).

In part (c) the response does not state the correct mean of the distribution consistent with values calculated in part (b). The response fails to satisfy component 1. The response does not provide appropriate supporting work for the correct mean using values calculated in part (b). The response fails to satisfy component 2. The response does not provide an interpretation of the mean that includes the concept of repeating the selection process over a long period of time. The response fails to satisfy component 3. The response includes the concept of an average or mean, “on average,” satisfying component 4. The response includes the context of the number of geodes opened, “geodes have been opened,” satisfying component 5. The response does not satisfy two or three components from components 1, 2, 3, or 4. Part (c) was scored incorrect (I).