AP

AP® Statistics
Sample Student Responses and Scoring Commentary

Inside:

Free-Response Question 2

☑ **Scoring Guidelines**

☑ **Student Samples**

☑ **Scoring Commentary**

Question 2: Focus on Exploring Data   4 points
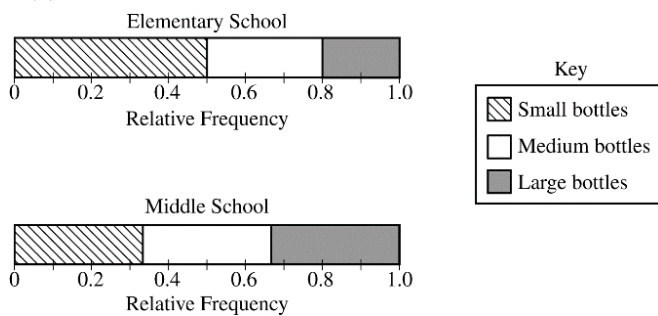
General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).

- The model solution represents an ideal response to each part of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

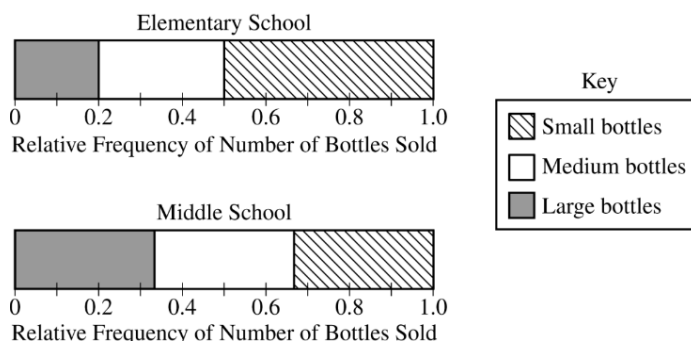| Model Solution	Scoring
(a)	**Essentially correct (E)** if the response satisfies the following two components: 1. The elementary school's segmented bar graph is correctly partitioned according to the given proportions 2. The middle school's segmented bar graph is correctly partitioned to create three equal areas **Partially correct (P)** if the response only satisfies one of the two components required for E. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- Responses that do not use the key given may still satisfy both components as long as a new key is provided that defines small, medium, and large bottles.
- Responses that do not use the key given and do not define a new key may receive credit for component 2 as long as the bar is vertically partitioned to create three equal areas, but do not satisfy component 1.
- Responses need to be within ±0.05 of the actual cutoff to be acceptable for demonstrating equal areas.
- Responses do not need to be drawn in the order from smallest bottles to largest bottles as shown in the model solution. For example, the graph could be drawn from largest bottles to smallest bottles.

Model Solution	Scoring
(b) No, the segment for small bottles for the elementary school is wider than the segment for small bottles for the middle school; however, the middle school students sold three times as many bottles as the elementary school students. So, if the elementary school sold x number of bottles, the middle school sold $3x$ number of bottles. For example, if the elementary students sold 100 bottles total, then they sold $0.5(100)$ or 50 small bottles. However, because the middle school students sold three times the total number of bottles as the elementary students, they would have sold 300 bottles total and $(0.\overline{3})(300) = 100$ small bottles. Because $100 > 50,$ the middle school sold more small bottles, and the elementary school's administrator is not correct.	**Essentially correct (E)** if the response satisfies the following three components: 1. States that the elementary school's administrator is incorrect 2. Provides correct mathematical support verifying the middle school sold more small bottles than the elementary school, consistent with the response to component 1 3. Includes context **Partially correct (P)** if the response satisfies only two of the three components required for E. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- A response that shows mathematical support by showing the inequalities $0.5x < 0.\overline{3}(3x),\ 0.5x < x,$ $0.5(1) < 0.\overline{3}(3),$ or $(0.5)\left(\dfrac{1}{4}\right) < (0.\overline{3})\left(\dfrac{3}{4}\right)$ may satisfy component 2.

- In order to satisfy component 3, a response must include "elementary," "middle," and "bottles."
- Part (b) should be scored consistent with the response to part (a).

Model Solution	Scoring
(c) (i) The mosaic plot shows that the proportion of large bottles sold by High School A was 0.7 and the proportion of large bottles sold by High School B was 0.6. High School A sold a greater proportion of large bottles because 0.7 > 0.6. (ii) The number of bottles sold is represented by the area of the shaded region. The area of the rectangle representing large bottles sold by High School B is clearly larger than the area of the rectangle representing large bottles sold by High School A. Therefore, High School B sold more large bottles than High School A.	**Essentially correct (E)** if the response satisfies the following four components: 1. In part (c-i) the response indicates that High School A sold a greater proportion of large bottles 2. In part (c-i) the response bases reasoning on the height of the rectangles or the relative frequencies representing large bottles 3. In part (c-ii) the response indicates that High School B sold a greater number of large bottles 4. In part (c-ii) the response indicates that the area of the rectangle representing large bottles sold by High School B is larger than the area of the rectangle representing large bottles sold by High School A **Partially correct (P)** if the response satisfies two or three of the four components required for E. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:
- If a response refers to "proportions" rather than relative frequencies in part (c-i), the values 0.7 and 0.6 must be provided to satisfy component 2.
- If the response bases reasoning on the heights of rectangles in part (c-i), the response must clearly state that the rectangles representing large bottles are referenced.
- If the response bases reasoning on the areas of rectangles in part (c-ii), the response must clearly state that the areas representing large bottles are referenced.
- A response that provides reasonable estimates of the large bottle areas may satisfy component 4.

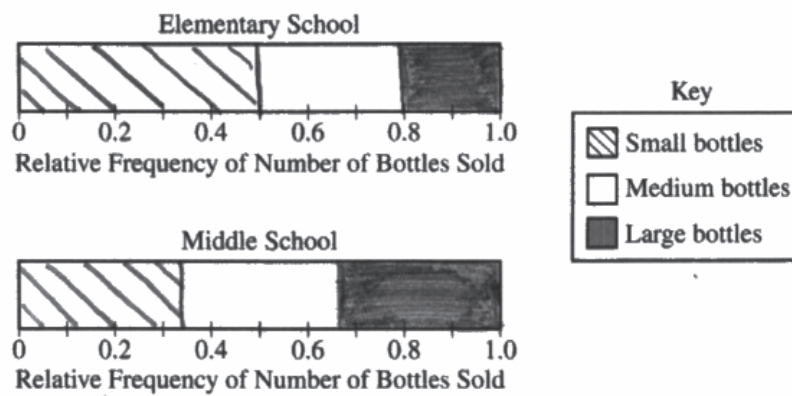| Scoring for Question 2	Score
Complete Response Three parts essentially correct	4
Substantial Response Two parts essentially correct and one part partially correct	3

Developing Response
Two parts essentially correct and no part partially correct
    OR
One part essentially correct and one or two parts partially correct
    OR
Three parts partially correct
	2

Minimal Response One part essentially correct and no part partially correct *OR* No part essentially correct and two parts partially correct	1

Question 2

Begin your response to **QUESTION 2** on this page.

2. A local elementary school decided to sell bottles printed with the school district's logo as a fund-raiser. The students in the elementary school were asked to sell bottles in three different sizes (small, medium, and large). The relative frequencies of the number of bottles sold for each size by the elementary school were 0.5 for small bottles, 0.3 for medium bottles, and 0.2 for large bottles.

A local middle school also decided to sell bottles as a fund-raiser, using the same three sizes (small, medium, and large). The middle school students sold three times the number of bottles that the elementary school students sold. For the middle school students, the proportion of bottles sold was equal for all three sizes.

(a) Complete the segmented bar graphs representing the relative frequencies of the number of bottles sold for each size by students at each school.

Elementary School

0   0.2   0.4   0.6   0.8   1.0
Relative Frequency of Number of Bottles Sold

Key

⬚ Small bottles
☐ Medium bottles
◼ Large bottles

Middle School

0   0.2   0.4   0.6   0.8   1.0
Relative Frequency of Number of Bottles Sold

(b) An administrator at the elementary school concluded that the elementary school students sold more small bottles than the middle school students did. Is the elementary school administrator's conclusion correct? Explain your response.

If n represents the total number of bottles the elementary school students sold, then the elementary school students sold $0.5n$ small bottles, while the middle school students sold $0.33\overline{3}(3n) = n$ small bottles. Since $n > 1$, $0.5n < n$, and therefore the elementary school administrator's conclusion is incorrect.

Page 4   **GO ON TO THE NEXT PAGE.**

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.
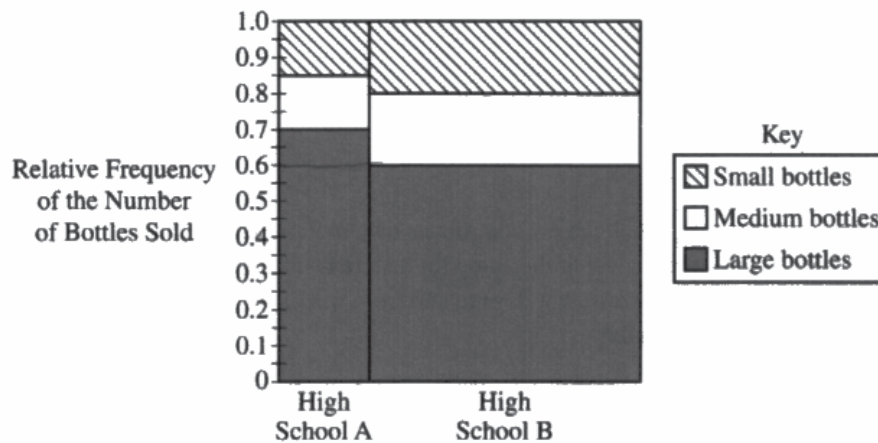
Q5261/4

Question 2

Continue your response to **QUESTION 2** on this page.

Two high schools are also selling the bottles and are competing to see which one sold more large bottles.

(c) A mosaic plot for the distribution of the number of bottles sold by each of the high schools is shown here.

Distribution of the Number of Bottles Sold by High School

Key
- ⊠ Small bottles
- ☐ Medium bottles
- ■ Large bottles

(i) Which of the two high schools sold a greater proportion of large bottles? Justify your answer.

High School A

High School A sold a proportion of 0.7 large bottles while High School B sold a proportion of 0.6 large bottles.

(ii) Which of the two high schools sold a greater number of large bottles? Justify your answer.

High School B sold a greater number of large bottles because the area in the mosaic plot for High School B's large bottle section is greater than the area in High School A's.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.
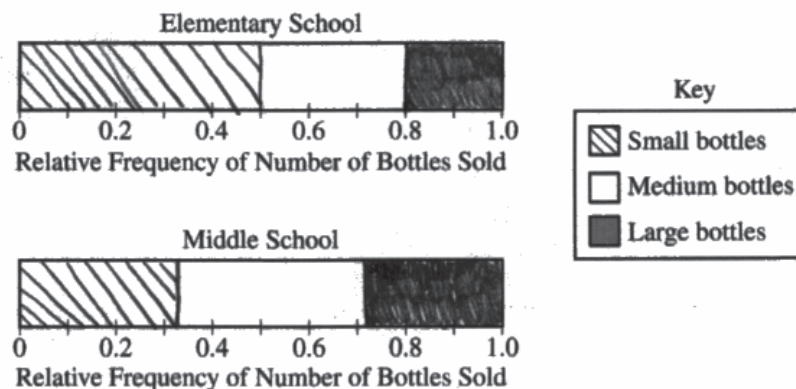
0086629

Q5261/5

Question 2

Begin your response to **QUESTION 2** on this page.

2. A local elementary school decided to sell bottles printed with the school district's logo as a fund-raiser. The students in the elementary school were asked to sell bottles in three different sizes (small, medium, and large). The relative frequencies of the number of bottles sold for each size by the elementary school were 0.5 for small bottles, 0.3 for medium bottles, and 0.2 for large bottles.

A local middle school also decided to sell bottles as a fund-raiser, using the same three sizes (small, medium, and large). The middle school students sold three times the number of bottles that the elementary school students sold. For the middle school students, the proportion of bottles sold was equal for all three sizes.

(a) Complete the segmented bar graphs representing the relative frequencies of the number of bottles sold for each size by students at each school.

Elementary School

Relative Frequency of Number of Bottles Sold

Key

Small bottles
Medium bottles
Large bottles

Middle School

Relative Frequency of Number of Bottles Sold

(b) An administrator at the elementary school concluded that the elementary school students sold more small bottles than the middle school students did. Is the elementary school administrator's conclusion correct? Explain your response.

The elementary school administrator's conclusion is not correct. While the elementary school sold a larger proportion of small bottles, the middle school students sold more bottles overall which impacts the proportion. For example, if the elementary school sold 20, they would have sold 10 small bottles (p=0.5). The middle school would have then sold 60 total and 20 small bottles (p = ⅓).

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.
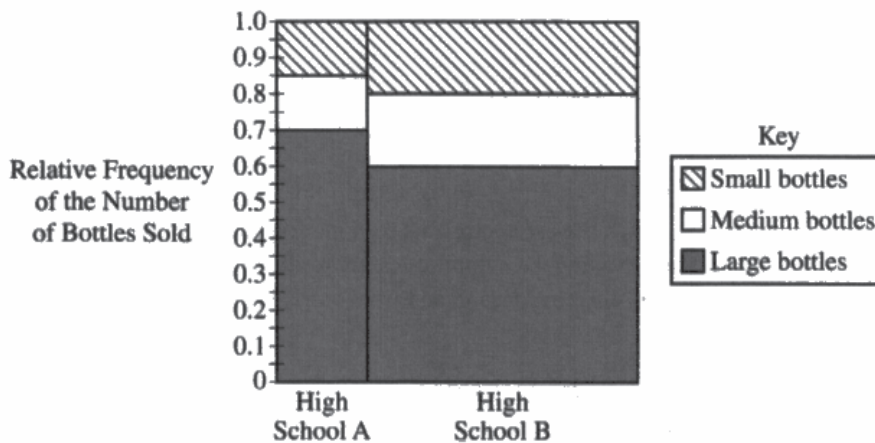
Q5261/4

Question 2

Continue your response to **QUESTION 2** on this page.

Two high schools are also selling the bottles and are competing to see which one sold more large bottles.

(c) A mosaic plot for the distribution of the number of bottles sold by each of the high schools is shown here.

Distribution of the Number of Bottles Sold by High School

(i) Which of the two high schools sold a greater proportion of large bottles? Justify your answer.

High School A sold a greater proportion (p=0.7) of large bottles than High School B (p=0.6).

(ii) Which of the two high schools sold a greater number of large bottles? Justify your answer.

High School B sold a greater number of large bottles because it has the greater shaded area.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.
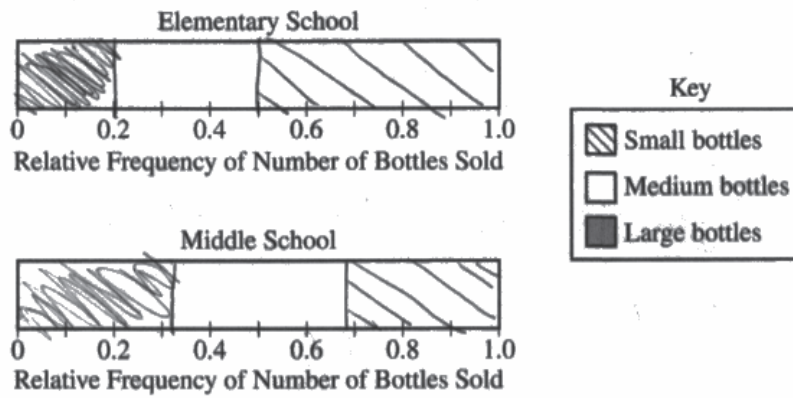
0152986

Q5261/5

Question 2

Begin your response to **QUESTION 2** on this page.

2. A local elementary school decided to sell bottles printed with the school district's logo as a fund-raiser. The students in the elementary school were asked to sell bottles in three different sizes (small, medium, and large). The relative frequencies of the number of bottles sold for each size by the elementary school were 0.5 for small bottles, 0.3 for medium bottles, and 0.2 for large bottles.

A local middle school also decided to sell bottles as a fund-raiser, using the same three sizes (small, medium, and large). The middle school students sold three times the number of bottles that the elementary school students sold. For the middle school students, the proportion of bottles sold was equal for all three sizes.

(a) Complete the segmented bar graphs representing the relative frequencies of the number of bottles sold for each size by students at each school.

Elementary School

Relative Frequency of Number of Bottles Sold

Key

▨	Small bottles
☐	Medium bottles
■	Large bottles

Middle School

Relative Frequency of Number of Bottles Sold

(b) An administrator at the elementary school concluded that the elementary school students sold more small bottles than the middle school students did. Is the elementary school administrator's conclusion correct? Explain your response.

No because the middle school sold 3x more bottles meaning the number of bottles was more even if the proportion was not.
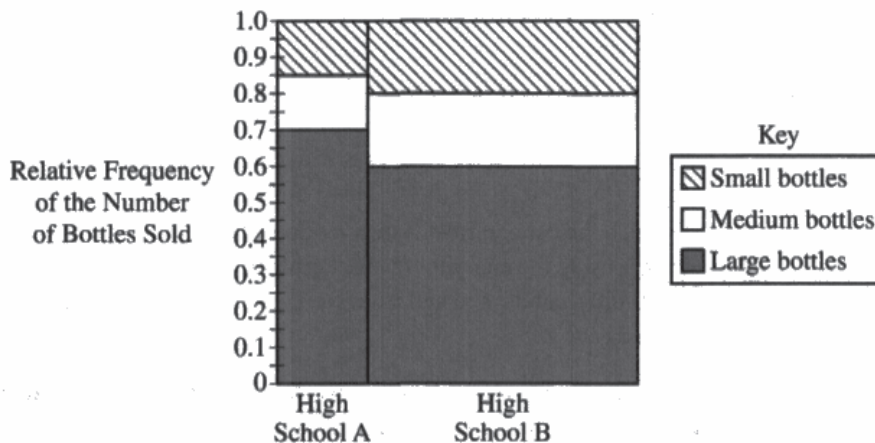
Page 4 **GO ON TO THE NEXT PAGE.**

QS261/4

Question 2

Continue your response to **QUESTION 2** on this page.

Two high schools are also selling the bottles and are competing to see which one sold more large bottles.

(c) A mosaic plot for the distribution of the number of bottles sold by each of the high schools is shown here.

Distribution of the Number of Bottles Sold by High School

(i) Which of the two high schools sold a greater proportion of large bottles? Justify your answer.

High school A because it had a proportion of 0.7 large bottles while High school B had a proportion of 0.6

(ii) Which of the two high schools sold a greater number of large bottles? Justify your answer.

High School B because the area shown on the graph is ~~that~~ larger.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

0002705

QS261/5

Question 2

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The primary goals of this question were to assess a student's ability to (1) use summarized data from a description to complete two segmented bar graphs for two different schools; (2) determine whether an administrator's conclusion is correct based on the previously constructed two segmented bar graphs and description; and (3) use information presented in a mosaic plot to determine the larger proportion and the larger number of large bottles for two different high schools.

This question primarily assesses skills in skill category 2: Data Analysis, skill category 3: Using Probability and Simulation, and skill category 4: Statistical Argumentation. Skills required for responding to this question include (2.B) Construct numerical or graphical representations of distributions, (2.D) Compare distributions or relative positions of points within a distribution, (3.A) Determine relative frequencies, proportions, or probabilities using simulation, and (4.B) Interpret statistical calculations and findings to assign meaning or assess a claim or calculations.

This question covers content from Unit 1: Exploring One-Variable Data, Unit 2: Exploring Two-Variable Data, and Unit 4: Probability, Random Variables, and Probability Distributions of the course framework in the AP Statistics Course and Exam Description. Refer to topics, 1.4, 2.2 and 4.3, and learning objectives UNC-1.C, UNC-1.E, UNC-1.P, VAR-4.A, and VAR-4.B.

Sample: 2A
Score: 4

The response earned the following: Part (a) - E; Part (b) - E; Part (c) - E.

In part (a) the response partitions the elementary school's segmented bar graph correctly according to the given proportions, satisfying component 1. The response correctly partitions the middle school's segmented bar graph to create three equal areas, satisfying component 2. Part (a) was scored essentially correct (E).

In part (b) the response states, "the elementary school administrator's conclusion is incorrect," satisfying component 1. The response provides correct mathematical support verifying the middle school sold more small bottles than the elementary school by including "$0.5n$," "$0.33\overline{3}(3n) = n$," and "$0.5n < n$," satisfying component 2. By including the words "bottles," "elementary," and "middle," the response includes context, satisfying component 3. Part (b) was scored essentially correct (E).

In part (c-i) the response indicates that High School A sold a greater proportion of large bottles and provides the values 0.7 and $0.7 > 0.6$. Note that no explicit comparison (e.g., $0.7 > 0.6$) is required. Components 1 and 2 have been satisfied. In part (c-ii) the response indicates that High School B sold a greater number of large bottles and bases the response on the area of the rectangle representing large bottles sold by High School B in the mosaic plot being greater than the area of the rectangle representing large bottles sold by High School A, satisfying components 3 and 4. Part (c) was scored essentially correct (E).

Question 2 (continued)

Sample: 2B
Score: 3

The response earned the following: Part (a) - E; Part (b) - E; Part (c) - P.

In part (a) the response partitions the elementary school's segmented bar graph correctly according to the given proportions, satisfying component 1. The response correctly partitions the middle school's segmented bar graph to create three approximately equal areas. Note that the medium/large boundary is located at 0.72. This is within the ± 0.05 limit (i.e., $0.67 + 0.05 = 0.72$). Component 2 is satisfied. Part (a) was scored essentially correct (E).

In part (b) the response states, "The elementary school administrator's conclusion is not correct," satisfying component 1. The response provides correct mathematical support verifying the middle school sold more small bottles than the elementary school with a specific example including "20 ... sold 10 small ... ($p = 0.5$) ... 60 total ... and 20 small" and "($p = 1 / 3$)," satisfying component 2. By including the words "elementary," "bottles," and "middle," the response includes context, satisfying component 3. Part (b) was scored essentially correct (E).

In part (c-i) the response indicates that High School A sold a greater proportion of large bottles and provides the values 0.7 and $0.6,$ satisfying components 1 and 2. In part (c-ii) the response indicates that High School B sold a greater number of large bottles, satisfying component 3. The response states, "it has the greater shaded area," but does not clearly state that the area mentioned is that representing large bottles, so component 4 is not satisfied. If the response had said "the greater solidly shaded area," this would have satisfied component 4, as it refers specifically to the large bottle area. Only three of the four components are satisfied, so part (c) was scored partially correct (P).

Sample: 2C
Score: 2

The response earned the following: Part (a) - E; Part (b) - I; Part (c) - P.

In part (a) the response partitions the elementary school's segmented bar graph correctly according to the given proportions, satisfying component 1. The response correctly partitions the middle school's segmented bar graph to create three equal areas, satisfying component 2. Responses need not be drawn from smallest to largest. Part (a) was scored essentially correct (E).

In part (b) the response states "No," satisfying component 1. The response does not provide correct mathematical support verifying the middle school sold more small bottles than the elementary school, so component 2 is not satisfied. The response does not include all required context, as the word "elementary" is not included. Component 3 is not satisfied. Only one of the three components are satisfied, so part (b) was scored incorrect (I).

In part (c-i) the response indicates that High School A sold a greater proportion of large bottles and provides the values 0.7 and $0.6,$ satisfying components 1 and 2. In part (c-ii) the response indicates that High School B sold a greater number of large bottles, satisfying component 3. The response states "the area shown on the graph is larger" but does not clearly state that the area mentioned is that representing large bottles, so component 4 is not satisfied. Only three of the four components are satisfied, so part (c) was scored partially correct (P).