2024

AP® Physics C: Mechanics

Sample Student Responses and Scoring Commentary Set 2

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Free-Response Question 3

- ☑ Scoring Guidelines
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Question 3: Free-Response Question — 15 points

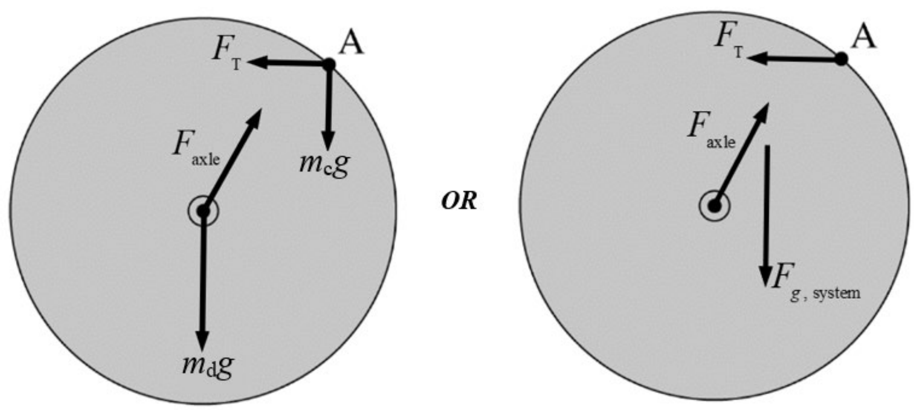
(a) For drawing and appropriately labeling separate gravitational forces that are exerted on the disk and the lump of clay and exerted at the correct locations — **1 point**

Scoring Note: Drawing a downward gravitational force exerted on the clay-disk system at the correct location can earn this point.

For drawing and appropriately labeling a leftward force exerted on the system at Point A — **1 point**

For drawing and appropriately labeling a force directed up and right that is exerted on the system at the axle, and no extraneous forces are present — **1 point**

Example Responses



Scoring Note: Examples of appropriate labels for the gravitational force include F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on disk," "F on disk by Earth," $F_{\text{Earth on Disk}}$, $F_{E,Disk}$, and $F_{Disk,E}$. The labels G or g are not appropriate labels for the gravitational force.

Scoring Note: Examples of appropriate labels for the normal force from the axle include F_N, F_{axle}, N, "normal force," and "axle force."

Scoring Note: Examples of appropriate labels for the tension force include F_t, F_T, T, F_{string}, and "Force from string."

Total for part (a) — 3 points

(b)	For a multi-step derivation that indicates the net torque is zero	**1 point**
	For indicating only the weight of the clay and the tension in the string exert torques on the system about the axle	**1 point**

Example Response

$\tau_{\text{net}} = \tau_{\text{clay}} - \tau_{\text{string}}$

For a correct expression for the torque exerted on the system by the weight of the clay **1 point**

Example Response

$\tau_{\text{clay}} = Rm_{\text{c}}g\cos\theta$

For a correct expression for the torque exerted on the system by the tension in the string **1 point**

Example Response

$\tau_{\text{string}} = RF_{\text{T}}\sin\theta$

Example Solution

$$\Sigma\tau = 0$$
$$\tau_{\text{clay}} - \tau_{\text{string}} = 0$$
$$F_{g,\text{ clay}}R\cos\theta - RF_{\text{T}}\sin\theta = 0$$
$$Rm_{\text{c}}g\cos\theta = RF_{\text{T}}\sin\theta$$
$$F_{\text{T}} = \frac{Rm_{\text{c}}g\cos\theta}{R\sin\theta}$$
$$F_{\text{T}} = m_{\text{c}}g\cot\theta$$

Scoring Note: A maximum of three points can be earned if the trigonometric functions (sin and cos) are reversed for both torque expressions.

Total for part (b) 4 points

(c)	For indicating a direct relationship between the torque exerted by the clay and the tension in the string	**1 point**
	For correctly relating a greater torque exerted by the clay from at least **one** of the following: • An increase in the angle between the radial direction and the weight of the clay • A greater perpendicular component of the weight of the clay • A greater lever arm	**1 point**

Example Response

The torque caused by the weight of the clay at Point B *is greater than when the clay is at Point* A *because the component of the weight that is perpendicular to the lever arm is larger. To maintain equilibrium, the net torque on the system is still zero, therefore the tension* $F_{\mathrm{T,\,new}}$ *must be greater.*

Total for part (c) 2 points

(d)(i)	For using integration to calculate rotational inertia	**1 point**
	For **one** of the following: • Substituting $\rho(2\pi r)dr$ for dm • Indicating the correct limits of integration	**1 point**

Example Response

$$I = \int_0^{0.3\text{ m}} r^2 \rho(2\pi r)\,dr$$

For a correct answer of $I = 0.012\text{ kg}\cdot\text{m}^2$, including units	**1 point**

Example Response

$$I = 0.012\text{ kg}\cdot\text{m}^2$$

Example Solution

$$I = \int r^2 dm \qquad dm = \rho dA \quad \text{and} \quad dA = 2\pi r\,dr$$

$$I = \int_0^R r^2 \rho dA$$

$$I = \int_0^{0.3\text{ m}} 2\pi\beta r^4 dr$$

$$I = \left.\frac{2\pi\beta R^5}{5}\right|_0^{0.3\text{ m}}$$

$$I = \frac{2\pi(4.0\text{ kg/m}^3)(0.3\text{ m})^5}{5}$$

$$I = 0.012\text{ kg}\cdot\text{m}^2$$

(d)(ii) For using the rotational form of Newton's second law **1 point**

For a correct expression for the net torque exerted on the clay-disk system **1 point**

Example Response

$\tau_{net} = Rm_c g$

For indicating that the rotational inertia of the clay-disk system is the sum of the rotational inertia of the disk and the rotational inertia of the lump of clay **1 point**

Example Response

$I_{system} = I_{disk} + I_{clay}$

Example Solution

$\Sigma\tau = I\alpha$

$\alpha = \dfrac{\tau_{net}}{I}$

$\alpha = \dfrac{Rm_c g}{I_{disk} + I_{clay}}$

$\alpha = \dfrac{(0.3\text{ m})(0.60\text{ kg})\left(9.8\text{ m/s}^2\right)}{\left(\left(0.012\text{ kg}\cdot\text{m}^2\right) + (0.60\text{ kg})(0.3\text{m})^2\right)}$

$\alpha = 26.7\ \text{rad/s}^2$

Total for part (d) 6 points

Total for question 3 15 points

Question 3

Begin your response to QUESTION 3 on this page.

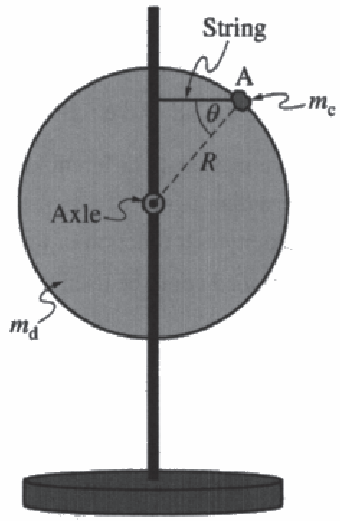
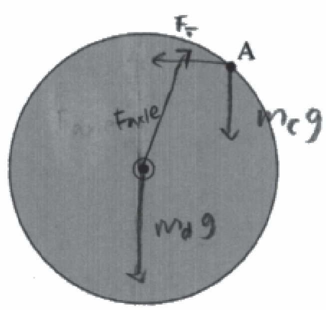


Figure 1

<u>Note:</u> Figure not drawn to scale.

3. A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal string is connected from the pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

   (a) On the following representation of the clay-disk system, **draw** and **label** the external forces (not components) exerted on the system. Each force must be represented by a distinct arrow that starts on, and points away from, the point at which the force is exerted on the system.



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Question 3

Continue your response to **QUESTION 3** on this page.

(b) **Derive** an expression for the tension F_T in the string when the clay is at Point A, as shown in Figure 1, in terms of R, m_d, m_c, θ, and physical constants, as appropriate.

τ_{net} (through center of disk) $= m_c g \cdot R\cos\theta - F_T \cdot R\sin\theta = 0$

$F_T R\sin\theta = m_c g R\cos\theta$

$F_T = \dfrac{\cos\theta}{\sin\theta} m_c g = \boxed{\dfrac{m_c g}{\tan\theta}}$

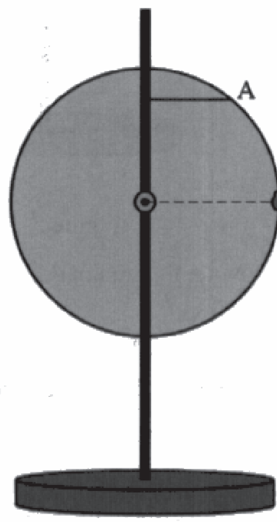

Figure 2

Note: Figure not drawn to scale.

(c) The string remains connected to the edge of the disk at Point A. The clay is moved to Point B, which is horizontally in line with the axle, as shown in Figure 2. How does the new tension $F_{T,\,new}$ compare with tension F_T from part (b)? **Justify** your reasoning.

It increases. The lever arm of gravity on the clay is greater ($R\cos\theta$ increases as θ decreases), so the torque exerted by gravity on the clay is greater, so the torque exerted by tension in the opposite direction must be greater to balance it, so F_T is greater (because the lever arm of F_T does not change).

Question 3

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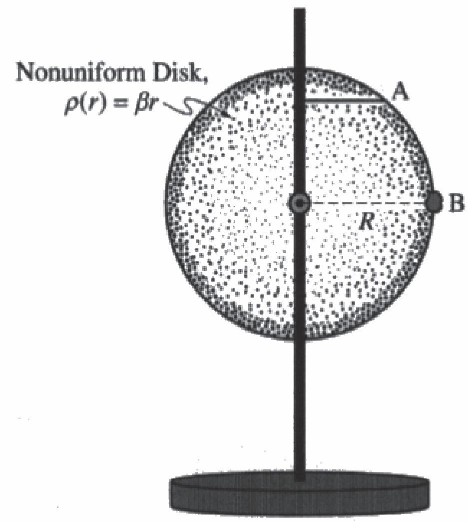

Figure 3

Note: Figure not drawn to scale.

(d) A nonuniform disk is now attached to the axle. The lump of clay is attached to the disk at Point B, as shown in Figure 3. The clay has mass $m_c = 0.60$ kg and the disk has a radius $R = 0.30$ m. The mass density of the disk varies radially and can be modeled by $\rho(r) = \beta r$, where r is the radial distance from the axle and $\beta = 4.0\,\text{kg/m}^3$.

i. **Calculate** the rotational inertia of the disk about the axle.

$I = \int x^2 dm \quad \frac{dm}{dx} = 2\pi x \rho = 2\pi \beta x^2 \rightarrow dm = 2\pi\beta x^2 dx$

$I = \int_0^R x^4 \cdot 2\pi \beta \, dx = \boxed{\frac{2\pi\beta R^5}{5}} = \boxed{0.012215 \text{ kg}\cdot\text{m}^2}$

ii. The string connecting the disk to the pole is cut. **Calculate** the magnitude of the initial angular acceleration of the clay-disk system.

τ_{net} about center of disk : $m_c g R = I_{tot}\alpha$

$I_{tot} = I_{disk} + I_{clay} = 0.012215 + 0.6 \cdot 0.3^2 = 0.066215$

$\alpha = \frac{m_c g R}{I_{tot}} = \frac{0.6 \cdot 10 \cdot 0.3}{0.066215} = \boxed{27.1844 \text{ rad/s}^2}$

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Question 3

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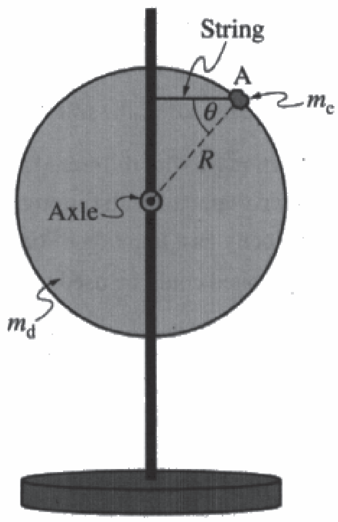
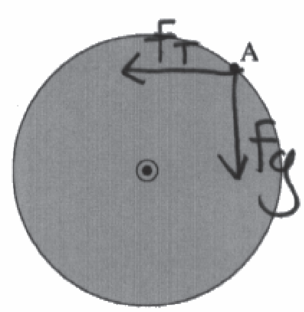


Figure 1

Note: Figure not drawn to scale.

3. A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal string is connected from the pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

   (a) On the following representation of the clay-disk system, **draw** and **label** the external forces (not components) exerted on the system. Each force must be represented by a distinct arrow that starts on, and points away from, the point at which the force is exerted on the system.



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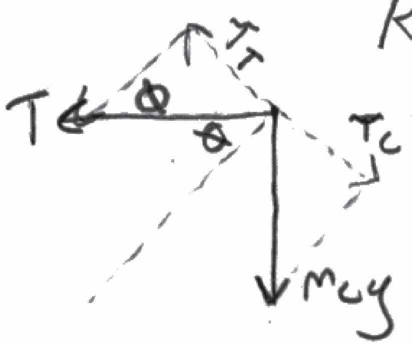
Question 3

Continue your response to **QUESTION 3** on this page.

(b) **Derive** an expression for the tension F_T in the string when the clay is at Point A, as shown in Figure 1, in terms of R, m_d, m_c, θ, and physical constants, as appropriate.

$\alpha = 0 \Rightarrow$

$\Sigma T = 0$



$R m_c g \cos\alpha = T \sin\alpha R$

$m_c g \cot(\alpha) = T$

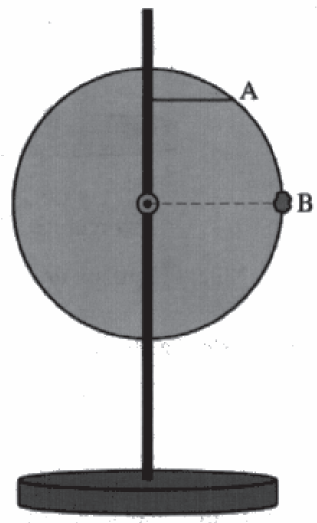


Figure 2

Note: Figure not drawn to scale.

(c) The string remains connected to the edge of the disk at Point A. The clay is moved to Point B, which is horizontally in line with the axle, as shown in Figure 2. How does the new tension $F_{T,\text{new}}$ compare with tension F_T from part (b)? **Justify** your reasoning.

AT Point B, ~~a~~ greater component of the clay weight exerts a greater torque on the cylinder than at Point A so $F_{T}\text{new} > F_T$ to keep $\Sigma T = 0$

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Question 3

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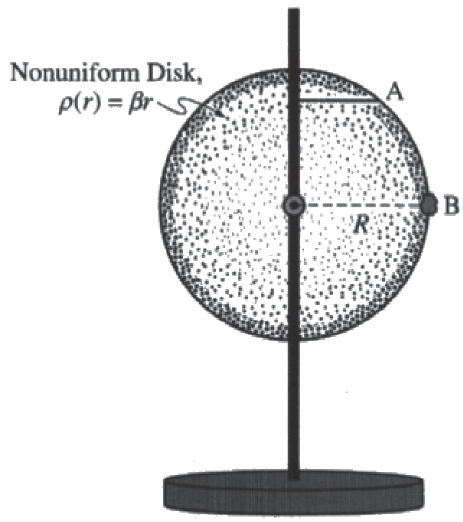

Figure 3

Note: Figure not drawn to scale.

(d) A nonuniform disk is now attached to the axle. The lump of clay is attached to the disk at Point B, as shown in Figure 3. The clay has mass $m_c = 0.60$ kg and the disk has a radius $R = 0.30$ m. The mass density of the disk varies radially and can be modeled by $\rho(r) = \beta r$, where r is the radial distance from the axle and $\beta = 4.0\ \text{kg/m}^3$.

i. **Calculate** the rotational inertia of the disk about the axle.

$\delta I = \int r^2 \delta m = \int r^2 B r\, \delta r =$ $\quad B = 4$

$B\int_0^R r^3 = I = \frac{BR^4}{4} = R^4 = .0081\ \text{kgm}^2$

ii. The string connecting the disk to the pole is cut. **Calculate** the magnitude of the initial angular acceleration of the clay-disk system.

$\alpha = \frac{\Sigma \tau}{I} = \frac{m_c g R}{R^4} = \frac{m_c g}{R^3} = \frac{.60 \cdot 10}{.3^3}$

$g = 10\ \text{m/s}^2$ $\quad = 222\ \text{rad/s}^2$

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Question 3

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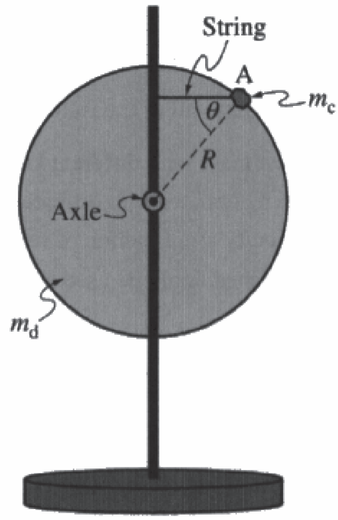
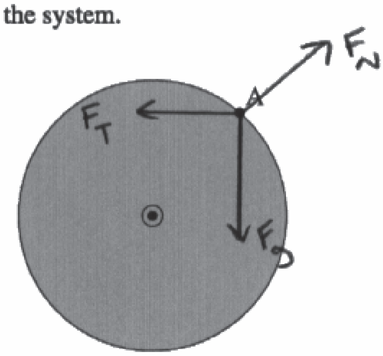


Figure 1

Note: Figure not drawn to scale.

3. A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal string is connected from the pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

   (a) On the following representation of the clay-disk system, **draw** and **label** the external forces (not components) exerted on the system. Each force must be represented by a distinct arrow that starts on, and points away from, the point at which the force is exerted on the system.



  GO ON TO THE NEXT PAGE.

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Question 3

Continue your response to **QUESTION 3** on this page.

(b) **Derive** an expression for the tension F_T in the string when the clay is at Point A, as shown in Figure 1, in terms of R, m_d, m_c, θ, and physical constants, as appropriate.

$$\Sigma F_y = m_c g - F_N \cos\theta = m_c(0)$$

$$F_N = \frac{m_c g}{\cos\theta}$$

$$\Sigma F_x = \frac{m_c g}{\cos\theta}\sin\theta - F_T = m_c(0)$$

$$\boxed{F_T = m_c g \tan\theta}$$

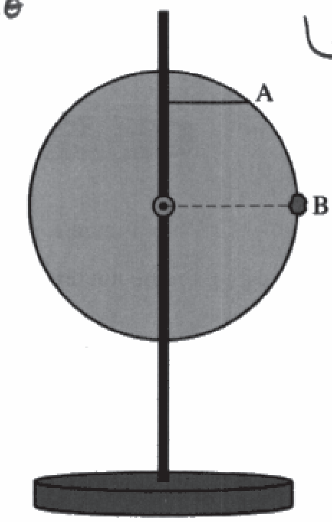

Figure 2

Note: Figure not drawn to scale.

(c) The string remains connected to the edge of the disk at Point A. The clay is moved to Point B, which is horizontally in line with the axle, as shown in Figure 2. How does the new tension $F_{T,\,new}$ compare with tension F_T from part (b)? **Justify** your reasoning.

New tension is increased because

Question 3

Continue your response to QUESTION 3 on this page.

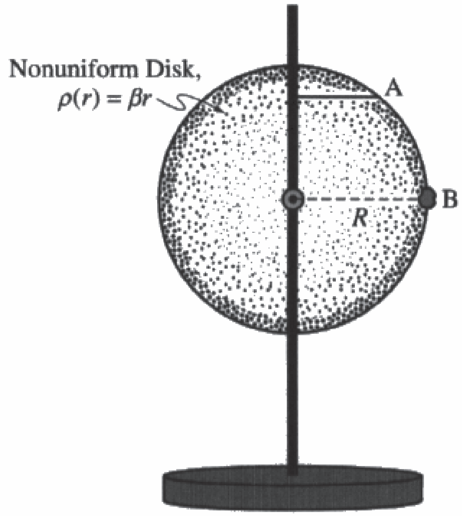

Figure 3

Note: Figure not drawn to scale.

(d) A nonuniform disk is now attached to the axle. The lump of clay is attached to the disk at Point B, as shown in Figure 3. The clay has mass $m_c = 0.60$ kg and the disk has a radius $R = 0.30$ m. The mass density of the disk varies radially and can be modeled by $\rho(r) = \beta r$, where r is the radial distance from the axle and $\beta = 4.0\ \text{kg/m}^3$.

i. **Calculate** the rotational inertia of the disk about the axle.

$\rho = \frac{dm}{dr}$

$\beta r\, dr = dm$

$dm = 4.0 r \cdot dr$

$I = \int r^2 \cdot dm$

$I = \int_0^{0.30} r^2 (4.0 r)\, dr$

$I = \left[r^4 \right]_0^{0.30} \rightarrow \boxed{I = 0.0081\ \text{kg}\cdot\text{m}^2}$

ii. The string connecting the disk to the pole is cut. **Calculate** the magnitude of the initial angular acceleration of the clay-disk system.

$\Sigma \tau = \Sigma r \times F = I\alpha$

$\Sigma \tau = 0.30\ (0.60(9.8)) \sin 90° = 0.0081\,\alpha$

$\boxed{\alpha = 220\ \text{rad/sec}^2}$

Question 3

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses were expected to demonstrate the ability to:

- Draw and label force vectors on a rigid body diagram.
- Calculate the magnitude of a torque associated with a force exerted on a rigid body.
- Calculate unknown forces exerted on an extended rigid body in translational or rotational equilibrium.
- Derive an expression using Newton's second law in rotational form.
- Explain how a quantity changes for different scenarios using physical reasoning.
- Calculate the magnitude and direction of the torque associated with a given force exerted on a rigid body.
- Calculate unknown quantities such as net torque, angular acceleration, or rotational inertia of a rigid body undergoing angular acceleration.

Sample: 3A
Score: 15

Part (a) earned 3 points. The first point was earned for correctly indicating the gravitational forces exerted on the clay and the disk with appropriately labeled arrows starting at the correct locations and pointing downwards. The second point was earned for correctly indicating the force due to tension with an appropriately labeled arrow starting at the correct location and pointing leftwards. The third point was earned for correctly indicating an axle force with an appropriately labeled arrow starting at the axle and pointing at an angle which is up and to the right. Part (b) earned 4 points. The first point was earned for a multi-step derivation that indicates the net torque is zero. The second point was earned for correctly indicating that only the torques applied by the weight of the clay and the tension in the string are exerted about the axle. The third point was earned for including a correct expression for the torque from the weight of the clay. The fourth point was earned for including a correct expression for the torque from the tension in the string. Part (c) earned 2 points. The first point was earned for indicating the direct relationship between the torque due to the weight of the clay and tension in the string. The second point was earned for indicating the greater torque on the system due to an increase in the lever arm between the axle and the gravitational force exerted on the clay. Part (d) earned 6 points. The first point was earned for using an integration to calculate the rotational inertia. The second point was earned for indicating the correct limits of integration. The third point was earned for a correct answer including units. The fourth point was earned for using the rotational form of Newton's second law. The fifth point was earned for indicating the correct expression for the torque exerted on the clay-disk system due to the gravitational force exerted on the clay. The sixth point was earned for indicating that the rotational inertia of the clay-disk system is the sum of the rotational inertia of the disk and the rotational inertia of the lump of clay.

Question 3 (continued)

Sample: 3B
Score: 11

Part (a) earned 1 point. The first point was not earned because the response indicates only the gravitational force exerted on the clay, with an appropriately labeled arrow starting at the correct location and pointing downwards. The second point was earned for correctly indicating a leftward force exerted on the system at Point A. The third point was not earned because the response does not indicate an axle or a normal force with an appropriately labeled arrow starting on and pointing away from the axle. Part (b) earned 4 points. The first point was earned for showing a multi-step derivation that indicates the net torque is zero. The second point was earned for correctly indicating only the weight of the clay and the tension in the string exert torques on the system about the axle. The third point was earned for including a correct expression for the torque by the weight of the clay. The fourth point was earned for including a correct expression for the torque by the tension in the string. Part (c) earned 2 points. The first point was earned for indicating the direct relationship between the torque due to the weight of the clay and the tension in the string. The second point was earned for correctly relating a greater torque by the clay to a greater perpendicular component of the weight of the clay. Part (d) earned 4 points. The first point was earned for using an integration to calculate rotational inertia. The second point was earned for correctly indicating the limits of integration. The third point was not earned because the response does not provide a correct answer. The fourth point was earned for using the rotational form of Newton's second law. The fifth point was earned for indicating the correct expression for the net torque exerted on the clay-disk system. The sixth point was not earned because the response does not indicate that the rotational inertia of the clay-disk system is the sum of the rotational inertia of the disk and the rotational inertia of the lump of clay.

Sample: 3C
Score: 5

Part (a) earned 1 point. The first point was not earned because the response does not indicate that there are separate gravitational forces exerted on the disk and the lump of clay at the correct locations. The second point was earned for correctly indicating and appropriately labeling a leftward force exerted on the system at Point A. The third point was not earned because the response does not indicate a force exerted on the system at the axle. Part (b) did not earn any points. The first point was not earned because the response does not use a multi-step derivation that indicates the net torque is zero. The response only uses the sum of forces. The second point was not earned because the response does not include two torques from the weight of the clay and tension in the string on the system about the axle. The third point was not earned because the response does not include a correct expression for the torque exerted on the system by the weight of the clay. The fourth point was not earned because the response does not include a correct expression for the torque exerted on the system by the tension in the string. Part (c) did not earn any points. The first point was not earned because the response does not show a direct relationship between the torque by the clay and the tension in the string. The second point was not earned because the response does not correctly relate a greater torque by the clay to a greater perpendicular component of the weight of the clay. Part (d) earned 4 points. The first point was earned for using an integration to calculate rotational inertia. The second point was earned for correctly indicating the limits of integration. The third point was not earned because the response does not provide a correct answer. The fourth point was earned for using the rotational form of Newton's second law. The fifth point was earned for indicating the correct expression for the torque exerted on the clay-disk system. The sixth point was not earned because the response neglects the rotational inertia of the lump of clay.