2024

AP® Physics C: Mechanics

Sample Student Responses and Scoring Commentary Set 1

Inside:

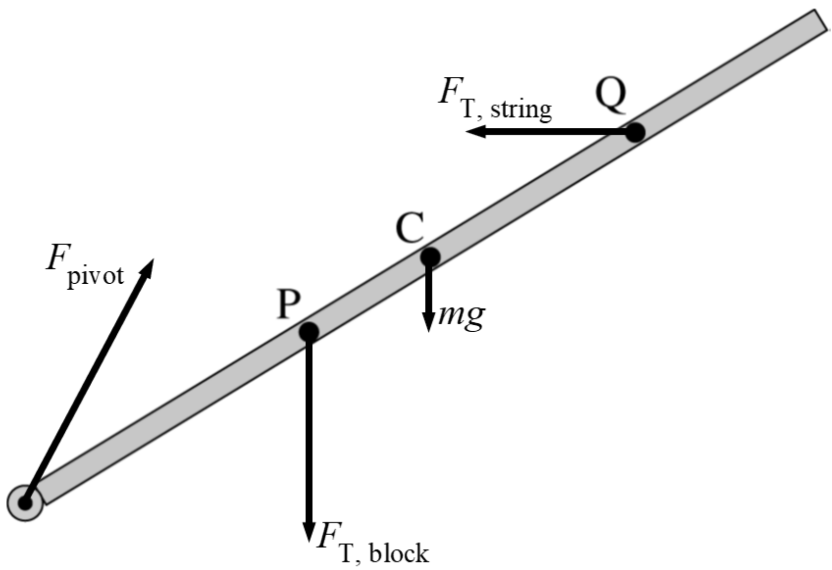
Free-Response Question 3

- ☑ Scoring Guidelines
- ☑ Student Samples
- ☑ Scoring Commentary

Question 3: Free-Response Question — 15 points

(a)	For drawing and appropriately labeling the downward forces that are exerted on the rod at points P and C **Scoring Note:** Labeling the downward force of tension as F_{block}, $3mg$, or similar, may earn this point.	**1 point**
	For drawing and appropriately labeling a leftward force that is exerted on the rod at Point Q	**1 point**
	For drawing and appropriately labeling a force that is directed up and to the right that is exerted on the rod at the pivot, and no extraneous forces are present	**1 point**

Example Response



Scoring Note: Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on block," "F on block by Earth," $F_{\text{Earth on block}}$, $F_{\text{E,Block}}$. The labels G and g are not appropriate labels for the force due to gravity.

Scoring Note: Examples of appropriate labels for the force from the pivot include: F_p, F_{pivot}, F_{n}, F_{N}, N, "normal force," "pivot force."

Scoring Note: Examples of appropriate labels for the tension force include F_t, F_{T}, T, F_{string}, and "Force from string."

Total for part (a) 3 points

(b) For indicating that the net torque exerted on the rod is equal to zero **1 point**

Example Response

$\Sigma\tau = 0$

For a correct expression for the torque exerted on the rod by the hanging mass **1 point**

Example Response

$3mg\sin\theta\left(\frac{3L}{8}\right)$

For a correct expression for the torque exerted on the rod by the gravitational force **1 point**

Example Response

$mg\sin\theta\left(\frac{4L}{8}\right)$

For a correct expression for the torque exerted on the rod by the string **1 point**

Example Responses

$F_T\sin(90° - \theta)\left(\frac{6L}{8}\right)$ **OR** $F_T\cos\theta\left(\frac{6L}{8}\right)$

Example Solution

$\Sigma\tau = 0$

$$3mg\sin\theta\left(\frac{3L}{8}\right) + mg\sin\theta\left(\frac{4L}{8}\right) - F_T\sin(90° - \theta)\left(\frac{6L}{8}\right) = 0$$

$$3mg\sin\theta\left(\frac{3L}{8}\right) + mg\sin\theta\left(\frac{4L}{8}\right) - F_T\cos\theta\left(\frac{6L}{8}\right) = 0$$

$$\left(\frac{9}{8}\right)mg\sin\theta + \left(\frac{4}{8}\right)mg\sin\theta = \left(\frac{6}{8}\right)F_T\cos\theta$$

$$13mg\sin\theta = 6F_T\cos\theta$$

$$F_T = \frac{13}{6}mg\tan\theta$$

Scoring Note: A maximum of three points may be earned if the trigonometric functions ($\sin$ and $\cos$) are reversed for all three torque terms.

Total for part (b) 4 points

(c)	For indicating that the torque exerted on the rod by the string is always the same	**1 point**
	For stating that as the angle between the string and the rod increases, the force exerted on the rod by the string decreases	**1 point**

Example Response

Because the torque exerted on the rod by the string is always the same, as the angle between the string and the rod increases, the tension $F_{T,\text{new}}$ *must decrease.*

Total for part (c) 2 points

(d)(i)	For indicating the total mass is the sum of differentiable masses along the length of the rod	**1 point**

Example Response

$$M = \int dm$$

For correctly writing dm in terms of x **1 point**

Example Response

$$M = \int_0^{1.2} (6 + 10x)\, dx$$

For a correct numeric answer with correct units **1 point**

Example Response

$$M = 14.4 \text{ kg}$$

Example Solution

$$M = \int dm$$

$$M = \int \lambda \, dx$$

$$M = \int_0^{1.2} (6 + 10x)\, dx$$

$$M = \left(6x + \frac{10x^2}{2}\right)\Bigg|_0^{1.2\text{ m}}$$

$$M = 6 \text{ kg/m}(1.2 \text{ m}) + \frac{10 \text{ kg/m}^2 (1.2 \text{ m})^2}{2}$$

$$M = 14.4 \text{ kg}$$

(d)(ii)	For a correct substitution of λ into an integral expression of rotational inertia	**1 point**

Example Response

$$I = \int_0^{1.2\text{ m}} (A + Bx)x^2 dx$$

For a correct integration **1 point**

Example Response

$$I = \left(\frac{Ax^3}{3} + \frac{Bx^4}{4}\right)\Bigg|_0^{1.2\text{ m}}$$

For a correct numeric answer with correct units **1 point**

Example Response

$$I = 8.64 \text{ kg} \cdot \text{m}^2$$

Example Solution

$$I = \int r^2 dm \qquad dm = \lambda dr \text{ and } r = x$$

$$I = \int_0^{1.2\text{ m}} \lambda x^2 dx$$

$$I = \int_0^{1.2\text{ m}} (A + Bx)x^2 dx$$

$$I = \left(\frac{Ax^3}{3} + \frac{Bx^4}{4}\right)\Bigg|_0^{1.2\text{ m}}$$

$$I = \frac{(6.0\text{ kg/m})(1.2\text{ m})^3}{3} + \frac{\left(10.0\text{ kg/m}^2\right)(1.2\text{ m})^4}{4}$$

$$I = 8.64 \text{ kg} \cdot \text{m}^2$$

Total for part (d) 6 points

Total for question 3 15 points

Question 3

Begin your response to **QUESTION 3** on this page.

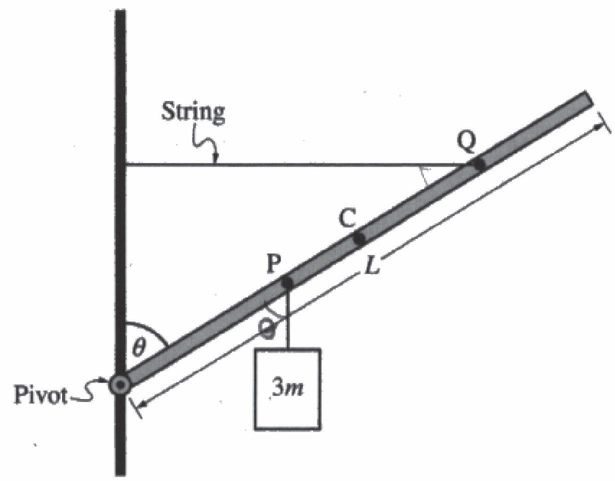
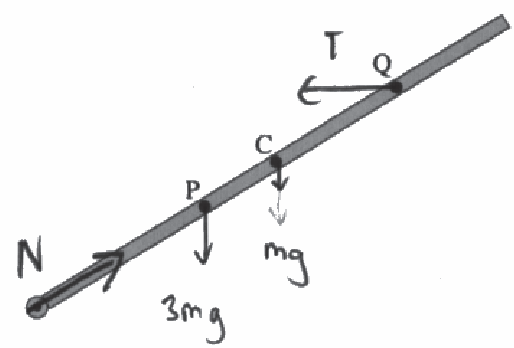


Figure 1

Note: Figure not drawn to scale.

3. A uniform rod of length L and mass m is attached to a pivot on a vertical pole, as shown in Figure 1. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

   (a) On the following representation of the rod, **draw** and **label** the forces (not components) that are exerted on the rod. Each force must be represented by a distinct arrow that starts on and points away from the point at which the force is exerted on the rod.



[redacted]

Question 3

Continue your response to QUESTION 3 on this page.

(b) In Figure 1, Point P is located $\frac{3}{8}L$ from the pivot and Point Q is located $\frac{6}{8}L$ from the pivot. **Derive** an equation for the tension F_T in the horizontal string in terms of L, m, θ, and physical constants, as appropriate.

$\Sigma\tau = 0$

$\frac{3}{8}L(3mg)\sin\theta + \frac{1}{2}Lmg\sin\theta - \frac{6}{8}LF_T\sin(90-\theta) = 0$

$\frac{9}{8}Lmg\sin\theta + \frac{L}{2}mg\sin\theta - \frac{6}{8}LF_T\cos\theta = 0$

$\frac{13}{8}mg\sin\theta + \frac{6}{8}F_T\cos\theta = 0$

$\frac{6}{8}F_T\cos\theta = \frac{13}{8}mg\sin\theta$

$F_T\cos\theta = \frac{13}{6}mg\sin\theta$

$F_T = \boxed{\frac{13}{6}mg\tan\theta}$

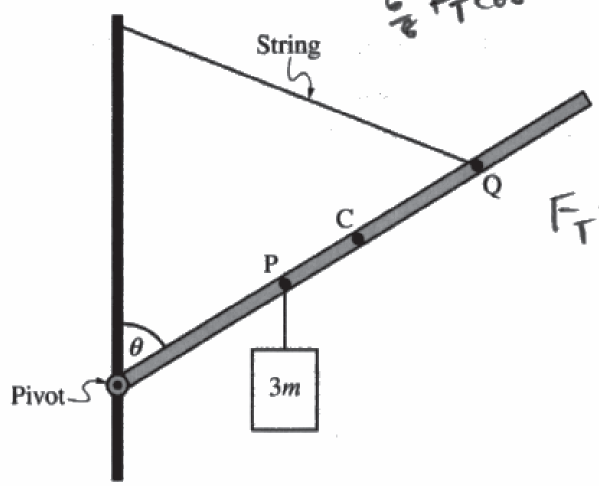


Figure 2

Note: Figure not drawn to scale.

(c) The original string is replaced with a longer string that connects Point Q to a higher location on the vertical pole, as shown in Figure 2. The angle θ remains the same. How does the new tension $F_{T,\,new}$ compare with the original tension F_T from part (b) ? **Justify** your reasoning.

As the angle between F_T and ℓ increases, $\tau = rF\sin\theta$ the torque must stay the same to keep the system in equilibrium. Therefore, F_T must decrease.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 3

Continue your response to **QUESTION 3** on this page.

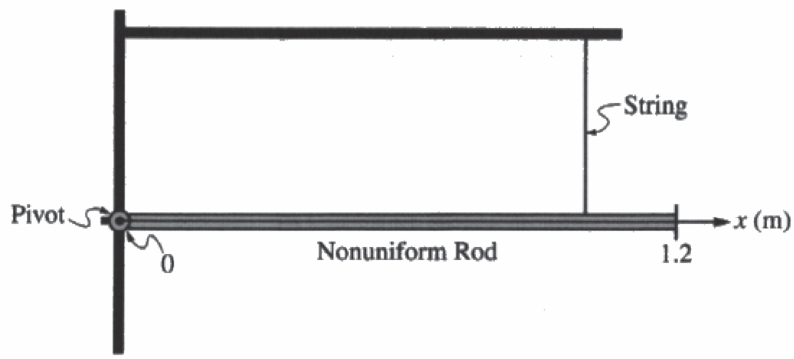


Figure 3

Note: Figure not drawn to scale.

(d) A nonuniform rod is now attached to the pivot, as shown in Figure 3. There is negligible friction between the nonuniform rod and the pivot. The rod has a length of 1.2 m and a linear mass density $\lambda(x) = A + Bx$, where x is the distance from the pivot, $A = 6.0$ kg/m, and $B = 10.0$ kg/m^2.

i. **Calculate** the mass of the rod.

$\lambda = \frac{M}{x} = \frac{dM}{dx}$ $\quad dM = \lambda dx \quad M = \int \lambda dx = \int_0^{1.2} 6.0 + 10.0x\,dx$

$M = \boxed{14.4 \text{ kg}}$

ii. **Calculate** the rotational inertia of the rod about the pivot.

$I = \int r^2 dm = \int x^2 (A + Bx) dx = \int_0^{1.2} 10x^3 + 6x^2 dx$

$dm = \lambda dx$

$I = \boxed{8.64 \text{ kg} \cdot \text{m}^2}$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

[redacted]

Question 3

Begin your response to QUESTION 3 on this page.

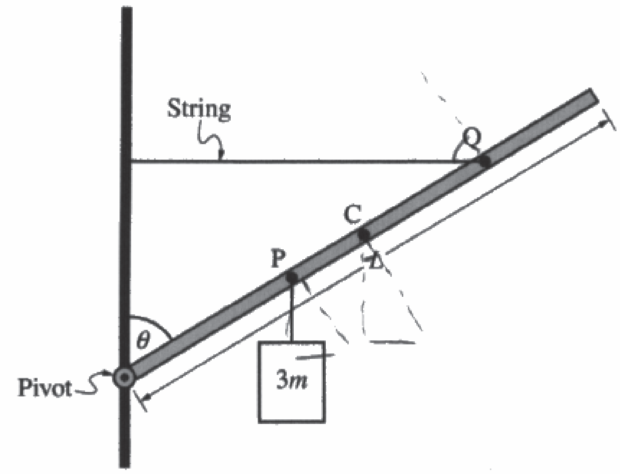
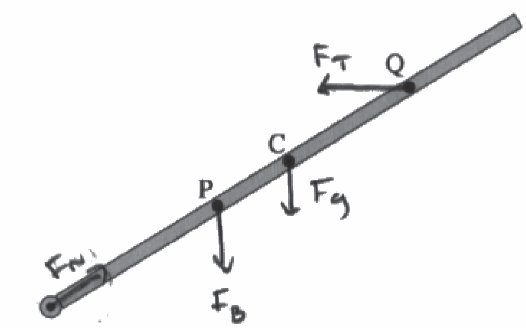


Figure 1

<u>Note:</u> Figure not drawn to scale.

3. A uniform rod of length L and mass m is attached to a pivot on a vertical pole, as shown in Figure 1. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

   (a) On the following representation of the rod, **draw** and **label** the forces (not components) that are exerted on the rod. Each force must be represented by a distinct arrow that starts on and points away from the point at which the force is exerted on the rod.



GO ON TO THE NEXT PAGE.

[redacted]

Question 3

Continue your response to **QUESTION 3** on this page.

(b) In Figure 1, Point P is located $\frac{3}{8}L$ from the pivot and Point Q is located $\frac{6}{8}L$ from the pivot. **Derive** an equation for the tension F_T in the horizontal string in terms of L, m, θ, and physical constants, as appropriate.

$\tau = Fr$

$\tau_T - \tau_G - \tau_B = 0$

$F_T(\frac{6}{8}L)\cos\theta - mg(\frac{1}{2}L)\cos\theta - 3mg(\frac{3}{8}L)\cos\theta = 0$

$\frac{6}{8}LF_T - \frac{1}{2}Lmg - \frac{9}{8}Lmg = 0$

$\frac{6}{8}LF_T = \frac{13}{8}Lmg$

$6F_T = 13mg$

$F_T = \frac{13mg}{6}$

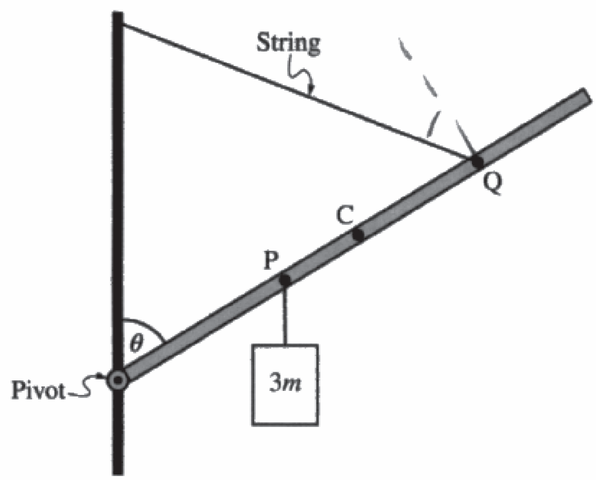


Figure 2

Note: Figure not drawn to scale.

(c) The original string is replaced with a longer string that connects Point Q to a higher location on the vertical pole, as shown in Figure 2. The angle θ remains the same. How does the new tension $F_{T,new}$ compare with the original tension F_T from part (b) ? **Justify** your reasoning.

$F_{T\,new}$ is greater than F_T because $\tau = F\cdot r = Fr\cos\theta$, since the angle between the force and the lever arm decreased, the force has increased

  GO ON TO THE NEXT PAGE.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 3

Continue your response to **QUESTION 3** on this page.

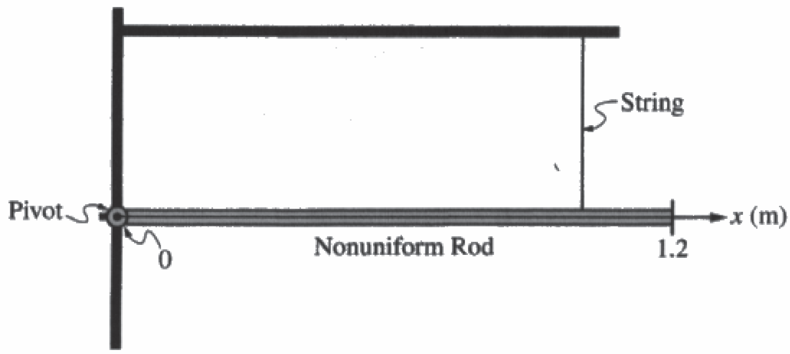


Figure 3

Note: Figure not drawn to scale.

(d) A nonuniform rod is now attached to the pivot, as shown in Figure 3. There is negligible friction between the nonuniform rod and the pivot. The rod has a length of 1.2 m and a linear mass density $\lambda(x) = A + Bx$, where x is the distance from the pivot, $A = 6.0$ kg/m, and $B = 10.0$ kg/m^2.

i. **Calculate** the mass of the rod.

$\lambda = A + Bx = 6 + 10x$

$\lambda = \frac{M}{L}$

$\int \lambda dL = \int dM$

$M = \int_0^{1.2} 6 + 10x\,dx = \boxed{14.4\text{ kg}}$

ii. **Calculate** the rotational inertia of the rod about the pivot.

$I = \int r^2 dm$ $\lambda dL = dm$

$I = \int r^2 \lambda dr$

$I = \int x^2 \lambda dx$ ← x so it's in same terms as $\lambda(x)$

$I = \int_0^{1.2} x^2 (6 + 10x)\,dx = \boxed{8.641\text{ kg}\cdot\text{m}^2}$

GO ON TO THE NEXT PAGE.

[redacted]

Question 3

Begin your response to **QUESTION 3** on this page.

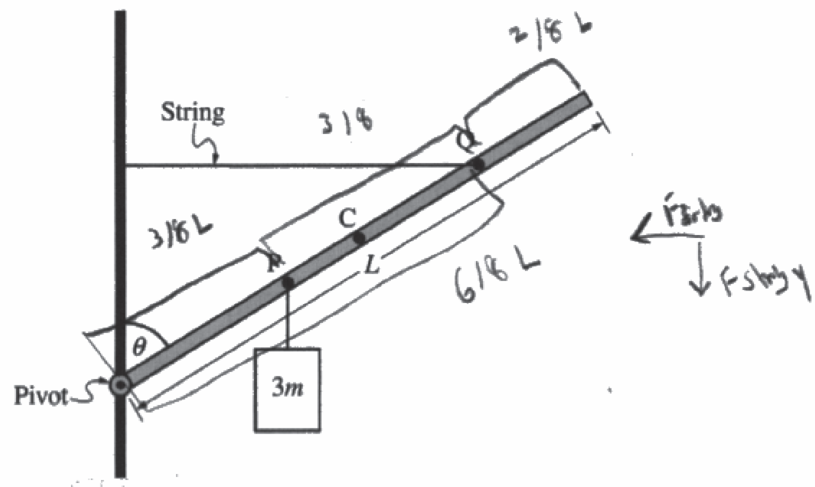
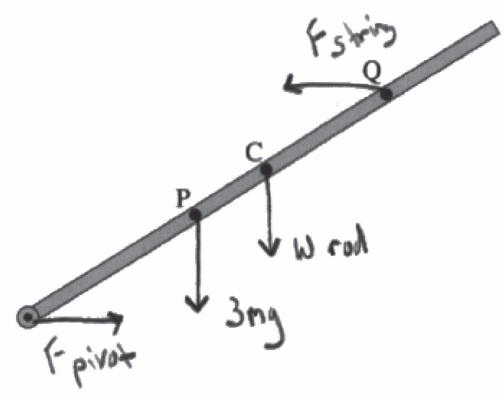


Figure 1

Note: Figure not drawn to scale.

3. A uniform rod of length L and mass m is attached to a pivot on a vertical pole, as shown in Figure 1. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

   (a) On the following representation of the rod, **draw** and **label** the forces (not components) that are exerted on the rod. Each force must be represented by a distinct arrow that starts on and points away from the point at which the force is exerted on the rod.



Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

[redacted]

Q5266/10

Question 3

Continue your response to **QUESTION 3** on this page.

(b) In Figure 1, Point P is located $\frac{3}{8}L$ from the pivot and Point Q is located $\frac{6}{8}L$ from the pivot. **Derive** an equation for the tension F_T in the horizontal string in terms of L, m, θ, and physical constants, as appropriate.

String on 6/8 L of rod   Fnet = 0 , τnet = 0

$F_T - F_P = 0$

$F_T = F_P$

$F_T = (6/8L)(4mg\cos\theta)$

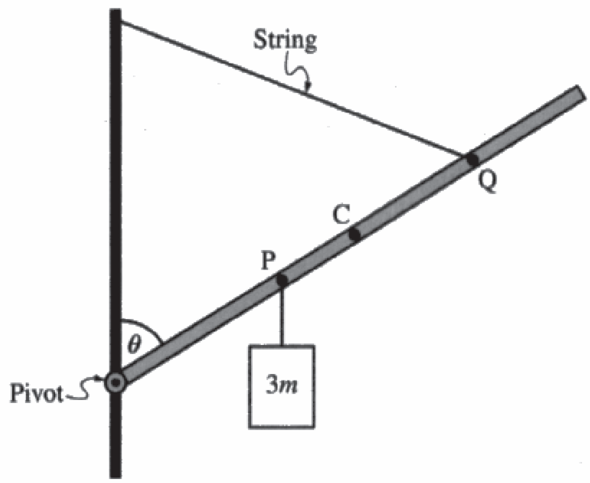


Figure 2

Note: Figure not drawn to scale.

(c) The original string is replaced with a longer string that connects Point Q to a higher location on the vertical pole, as shown in Figure 2. The angle θ remains the same. How does the new tension $F_{T,\,new}$ compare with the original tension F_T from part (b) ? **Justify** your reasoning.

If the L of the String increased the new tension will be greater than the original tension b/c $F_T \propto L$.

GO ON TO THE NEXT PAGE.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 3

Continue your response to **QUESTION 3** on this page.

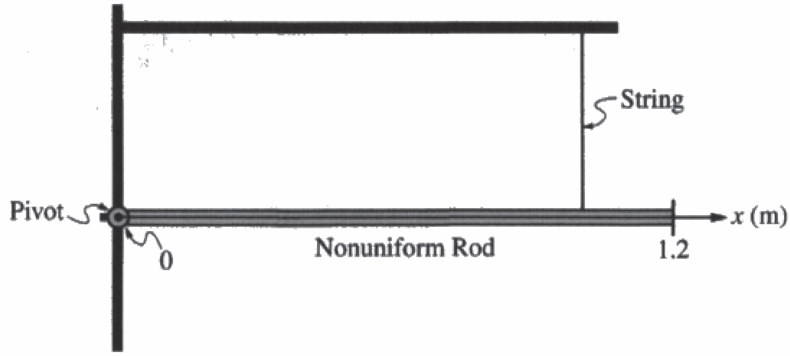


Figure 3

Note: Figure not drawn to scale.

(d) A nonuniform rod is now attached to the pivot, as shown in Figure 3. There is negligible friction between the nonuniform rod and the pivot. The rod has a length of 1.2 m and a linear mass density $\lambda(x) = A + Bx$, where x is the distance from the pivot, $A = 6.0$ kg/m, and $B = 10.0$ kg/m^2.

$\lambda x = 6 + 10x$

i. **Calculate** the mass of the rod.

$\int \lambda = \int 6 + 10x$

$= \int 6x + 5x^2 \quad x = Kg + Kg/m$

$= 3x^2 + 5/3 x^3 = * Kg$

$5/3(1.2)^3 =$ **2.88 Kg**

ii. **Calculate** the rotational inertia of the rod about the pivot.

$I = \int r^2 dm \qquad dm = 6 + 10x$

$I \; (6+10x) \int_0^{1.2} x^2 \, dx$

$(10) \quad \frac{1}{3}x^3 \Big|_0^{1.2} =$ **0.576** $\times 10 =$ **5.76**

[redacted]

Question 3

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses were expected to demonstrate the ability to:

- Draw and label force vectors on a rigid body diagram.
- Determine the magnitude of a torque associated with a force exerted on a rigid body.
- Determine the magnitude of torque exerted on a rigid body due to the gravitational force.
- Derive an unknown force exerted on a rigid body that is in a state of translational and rotational equilibrium.
- Demonstrate a conceptual understanding of the relationship between the torque provided by a force and the angle at which the force is exerted.
- Calculate the mass of a thin rod of nonuniform density based on the linear mass density of the rod.
- Calculate the moment of inertia of a thin rod of nonuniform density.
- Determine the differential mass dm in terms of x from the linear mass density of the rod.

Sample: 3A
Score: 15

Part (a) earned 3 points. The first point was earned for including appropriately labeled downward forces at points P and C. The second point was earned for including an appropriately labeled leftward force at Point Q. The third point was earned for including an appropriately labeled force at the pivot that is directed up and to the right. Part (b) earned 4 points. The first point was earned for indicating that net torque is zero. The second point was earned for including a correct expression for torque due to the hanging mass. The third point was earned for including a correct expression for the torque due to the force of gravity exerted on the rod. The fourth point was earned for including a correct expression for torque due to the string. Part (c) earned 2 points. The first point was earned for indicating that the torque provided by the string is always the same. The second point was earned for stating that the larger angle between the string and rod leads to a decreased tension force. Part (d) earned 6 points. The first point was earned for attempting to integrate, indicating that total mass is the sum of differential masses. The second point was earned for correctly writing dm in terms of x. The third point was earned for including a correct numeric answer with correct units. The fourth point was earned for correctly substituting λ into an integral expression for moment of inertia. The fifth point was earned for correctly integrating the expression. The sixth point was earned for including a correct numeric answer with correct units.

Question 3 (continued)

Sample: 3B
Score: 11

Part (a) earned 3 points. The first point was earned for including appropriately labeled downward forces at points P and C, where "F_B" is acceptable for the force due to the block. The second point was earned for including an appropriately labeled leftward force at Point Q. The third point was earned for including an appropriately labeled force at the pivot that is directed up and to the right. Part (b) earned 2 points. The first point was earned for indicating that net torque is zero. The second point was not earned because the response's expression for torque due to the hanging mass is incorrect, due to "$\cos\theta$." The third point was not earned because the response's expression for torque due to gravity on the rod is incorrect, due to "$\cos\theta$." The fourth point was earned for including a correct expression for the torque due to the string. Notice that trig functions are not reversed on ALL torque terms. Part (c) did not earn any points. The first point was not earned because the response does not indicate that the torque provided by the string is always the same. The second point was not earned because the response does not state that a larger angle between the string and rod leads to a decreased tension force. Part (d) earned 6 points. The first point was earned for attempting to integrate, indicating that the total mass is the sum of differential masses. The second point was earned for correctly writing dm in terms of x. The third point was earned for including a correct numeric answer with correct units. The fourth point was earned for correctly substituting λ into an integral expression for moment of inertia. The fifth point was earned for correctly integrating the expression. The sixth point was earned for including a correct numeric answer with correct units.

Sample: 3C
Score: 4

Part (a) earned 2 points. The first point was earned for including appropriately labeled downward forces at points P and C. The second point was earned for including an appropriately labeled leftward force at Point Q. The third point was not earned because the response does not include an appropriately labeled force at the pivot that is directed up and to the right. Part (b) earned 1 point for indicating that net torque is zero. The second point was not earned because the response does not include a correct expression for torque due to the hanging mass. The third point was not earned because the response does not include a correct expression for torque due to the force of gravity exerted on the rod. The fourth point was not earned because the response does not include a correct expression for torque due to the string. Part (c) did not earn any points. The first point was not earned because the response does not indicate that the torque provided by the string is always the same. The second point was not earned because the response does not discuss the angle between the string and rod. Part (d) earned 1 point for attempting to integrate, indicating that total mass is the sum of differential masses. The second point was not earned because the response does not write dm in terms of x. The third point was not earned because the response does not include a correct numeric answer with correct units. The fourth point was not earned because the response has no correct substitution into an integral expression. The fifth point was not earned because the response does not integrate the expression. The sixth point was not earned because the response does not include a correct numeric answer with correct units.