
AP[®] Physics C: Electricity and Magnetism

Sample Student Responses and Scoring Commentary Set 2

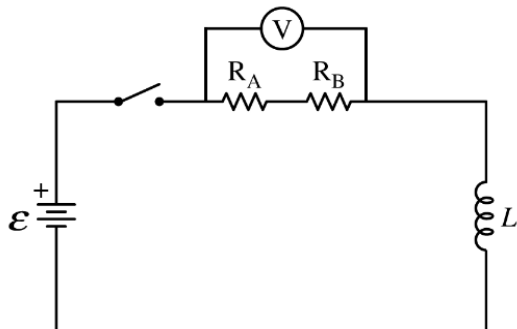
Inside:

Free-Response Question 2

- Scoring Guidelines**
- Student Samples**
- Scoring Commentary**

Question 2: Free-Response Question**15 points**

- (a)(i) For correctly placing the voltmeter in parallel with Resistor A , Resistor B , or the combination of resistors A and B **1 point**

Example Response

- (a)(ii) For a procedure that indicates that the voltmeter should be used to measure the potential difference for at least one time **1 point**

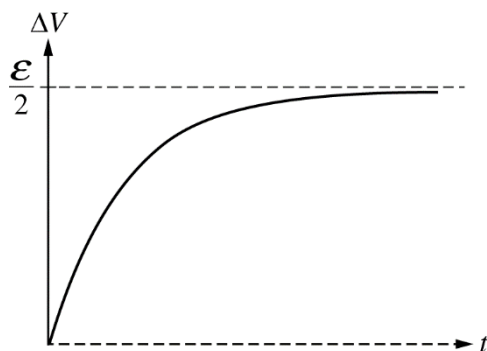
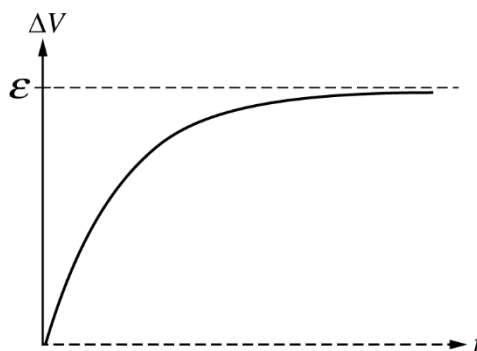
For measuring the potential difference from immediately after the switch is closed to when steady-state conditions have been established or until a time at which the time constant can be determined **1 point**

Example Response

Close the switch. Using the voltmeter, record the potential difference as a function of time until steady-state conditions are established.

Total for part (a) 3 points

- (b)(i) For correctly labeling potential difference on the vertical axis and time on the horizontal axis **1 point**
- For a concave-down and increasing curve **1 point**
- For a curve that starts at the origin and asymptotically approaches a nonzero potential difference value **1 point**
- For correctly labeling the horizontal asymptote that is consistent with the indication of the connection of the voltmeter in the response in part (a)(i) **1 point**

Example Responses*One Resistor***OR***Combination of Resistors*

(b)(ii) For indicating that a curve fit to the graph is that of an exponential function **1 point**

Alternate Solution

For indicating the time on the graph where the potential difference across Resistor A or Resistor B is approximately $\frac{0.63\mathcal{E}}{2}$ or that the potential difference across the combination of resistors A and B is approximately $0.63\mathcal{E}$

For indicating that the coefficient in front of the t of the curve-fit equation is equal to $\frac{2R}{L}$ **1 point**

Alternate Solution

For indicating that the time constant is equal to $\frac{L}{2R}$

Example Response

The data in the graph should be fit with an exponential function for the equation

$$V_R = \mathcal{E} \left(1 - e^{-t \frac{2R}{L}} \right). \text{ Because } \mathcal{E} \text{ and } L \text{ are known, } R \text{ can be calculated.}$$

Alternate Example Response

The potential difference at $0.63\mathcal{E}$ along the vertical axis corresponds to the time constant along the horizontal axis. Because the time constant is equal to $\frac{L}{2R}$, and L is known, R can be calculated.

Total for part (b) 6 points

(c)	For a multi-step derivation that begins with an attempt at using Kirchhoff's loop rule	1 point
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Example Response

$$\mathcal{E} - \Delta V_R - \Delta V_L = 0$$

	For indicating that the potential difference across the series combination of resistors is $I(2R)$	1 point
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	For indicating that the absolute value of the potential difference across the inductor is $L \frac{dI}{dt}$	1 point
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Example Solution

$$\mathcal{E} - \Delta V_R - \Delta V_L = 0$$

$$\mathcal{E} - I(2R) - L \frac{dI}{dt} = 0$$

$$\frac{\mathcal{E}}{L} - \frac{2IR}{L} = \frac{dI}{dt}$$

$$-\frac{R}{L} \left(2I - \frac{\mathcal{E}}{R} \right) = \frac{dI}{dt}$$

	Total for part (c)	3 points
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(d)	For selecting $ \Delta V_2 < \Delta V_1 $ with an attempt at a relevant justification	1 point
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	For indicating that the total resistance has increased	1 point
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	For indicating that the current decreases due to the increased resistance	1 point
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OR

For indicating that the potential difference across the inductor will increase, and, therefore, the potential difference across R_A must decrease

Example Response

$|\Delta V_2| < |\Delta V_1|$, because the inductor has nonnegligible resistance, and the total resistance of the new circuit increases as compared to the original circuit. Therefore, the current in the new circuit is reduced compared to that of the original circuit when steady-state conditions are established.

OR

$|\Delta V_2| < |\Delta V_1|$, because the inductor has nonnegligible resistance, and the total resistance of the new circuit increases as compared to the original circuit. Therefore, the potential difference across the inductor increases, which decreases the potential difference across R_A compared to that of the original circuit when steady-state conditions are established.

	Total for part (d)	3 points
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	Total for question 2	15 points
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Question 2

Begin your response to **QUESTION 2** on this page.

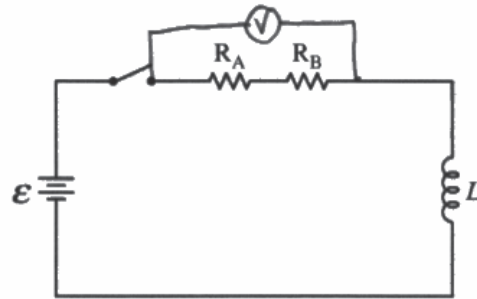


Figure 1

2. Students are asked to determine the resistance R of identical resistors R_A and R_B . The resistors are connected in series with each other, a battery of known emf \mathcal{E} , an inductor of known inductance L , and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that increases with time to determine R .

(a)

- i. On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that increases with time.



Voltmeter Symbol

- ii. **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for R using a measured quantity that increases with time. Provide enough detail so that another student could replicate the experiment.

The students should measure the voltage at numerous different times. Since the current at any time t is given by $I(t)$ the resistance $R = \frac{V(t)}{2I(t)}$ for all times t .

Question 2

Continue your response to **QUESTION 2** on this page.

(b)

i. On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

- Label the quantities graphed on the vertical and horizontal axes.
- Sketch a line or curve that represents the expected trend of the collected data.
- Label any appropriate intercepts and/or asymptotes in terms of the quantities provided.

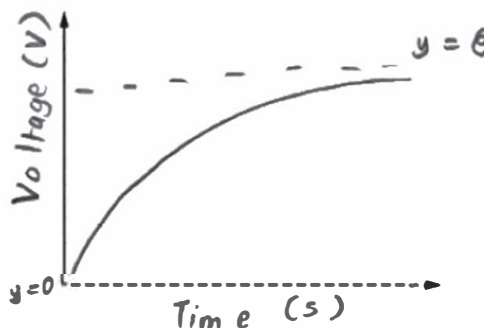


Figure 2

ii. Describe how the information from the graph in part (b)(i) would be used to determine the experimental value for R .

Calculate $V(t)$, $R = \frac{V(t)}{I(t)}$



Question 2

Continue your response to **QUESTION 2** on this page.

- (c) Starting with an appropriate application of Kirchhoff's loop rule, **derive**, but do NOT solve, a differential equation that can be used to determine the current I in the inductor at time t after the switch is closed. Express your answer in terms of R , \mathcal{E} , L , t , and physical constants, as appropriate.

$$\text{Loop Rule: } \mathcal{E} - 2IR - L \frac{di}{dt} = 0$$

$$\Rightarrow \mathcal{E} - 2IR = L \frac{di}{dt}$$

After reaching steady state, the absolute value of the potential difference across R_A is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across R_A is $|\Delta V_2|$.

- (d) **Indicate** whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.

$|\Delta V_2| > |\Delta V_1|$ $|\Delta V_2| < |\Delta V_1|$ $|\Delta V_2| = |\Delta V_1|$

Justify your answer.

After a long time, the inductor will act as a resistor, reducing the current in the circuit and thus reducing the voltage across R_A as $V = iR$.

Question 2

Begin your response to **QUESTION 2** on this page.

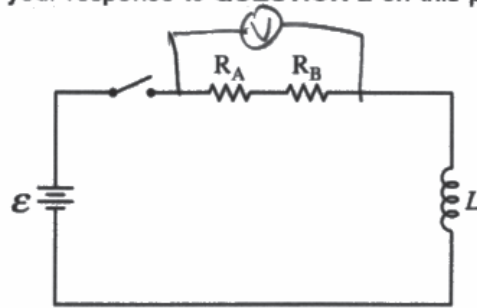


Figure 1

2. Students are asked to determine the resistance R of identical resistors R_A and R_B . The resistors are connected in series with each other, a battery of known emf \mathcal{E} , an inductor of known inductance L , and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that increases with time to determine R .

(a)

- i. On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that increases with time.



Voltmeter Symbol

- ii. **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for R using a measured quantity that increases with time. Provide enough detail so that another student could replicate the experiment.

measure V as a function of time of resistors. $(\frac{dV}{dt})$
wait until steady $\rightarrow V = \mathcal{E}$

Question 2

Continue your response to QUESTION 2 on this page.

(b)

i. On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

- Label the quantities graphed on the vertical and horizontal axes.
- Sketch a line or curve that represents the expected trend of the collected data.
- Label any appropriate intercepts and/or asymptotes in terms of the quantities provided.

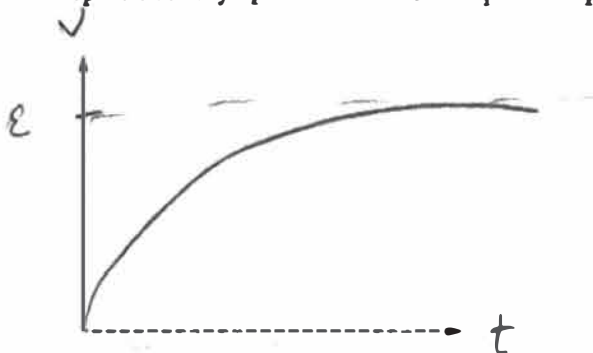


Figure 2

ii. Describe how the information from the graph in part (b)(i) would be used to determine the experimental value for R .

$$\epsilon = IR$$

$$\frac{\epsilon}{I} = 2R$$

$$R = \frac{\epsilon}{2I}$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.



Question 2

Continue your response to QUESTION 2 on this page.

- (c) Starting with an appropriate application of Kirchhoff's loop rule, derive, but do NOT solve, a differential equation that can be used to determine the current I in the inductor at time t after the switch is closed. Express your answer in terms of R , \mathcal{E} , L , t , and physical constants, as appropriate.

$$\mathcal{E} - IR - IR - L \frac{dI}{dt} = 0$$

After reaching steady state, the absolute value of the potential difference across R_A is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across R_A is $|\Delta V_2|$.

- (d) Indicate whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.

$|\Delta V_2| > |\Delta V_1|$ $|\Delta V_2| < |\Delta V_1|$ $|\Delta V_2| = |\Delta V_1|$

Justify your answer.

$$V = IR$$

voltage of inductor does not depend on R . If the inductance is the same, then

$$|\Delta V_2| = |\Delta V_1|$$

Question 2

Begin your response to **QUESTION 2** on this page.

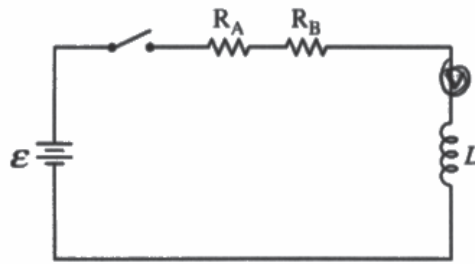


Figure 1

2. Students are asked to determine the resistance R of identical resistors R_A and R_B . The resistors are connected in series with each other, a battery of known emf \mathcal{E} , an inductor of known inductance L , and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that increases with time to determine R .

(a)

- i. On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that increases with time.



Voltmeter Symbol

- ii. **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for R using a measured quantity that increases with time. Provide enough detail so that another student could replicate the experiment.

The current could be measured using an ammeter, and this would produce a graph that allows the time constant to be determined and therefore R , since L is known.

Question 2

Continue your response to QUESTION 2 on this page.

(b)

i. On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

- Label the quantities graphed on the vertical and horizontal axes.
- Sketch a line or curve that represents the expected trend of the collected data.
- Label any appropriate intercepts and/or asymptotes in terms of the quantities provided.

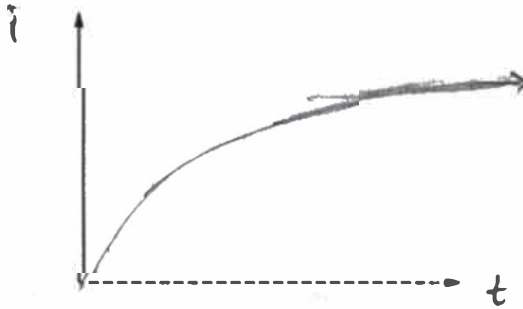


Figure 2

ii. Describe how the information from the graph in part (b)(i) would be used to determine the experimental value for R .

$V = IR$, so the resistance can be calculated from I and V at different places in time.

Question 2

Continue your response to **QUESTION 2** on this page.

- (c) Starting with an appropriate application of Kirchhoff's loop rule, **derive**, but do NOT solve, a differential equation that can be used to determine the current I in the inductor at time t after the switch is closed. Express your answer in terms of R , \mathcal{E} , L , t , and physical constants, as appropriate.

$$\mathcal{E} - IR_A - IR_B - L \frac{dI}{dt} = 0$$

$$\mathcal{E} - L \frac{dI}{dt} = I(R_A + R_B) - \mathcal{E}$$

After reaching steady state, the absolute value of the potential difference across R_A is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across R_A is $|\Delta V_2|$.

- (d) Indicate whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.

$|\Delta V_2| > |\Delta V_1|$ $|\Delta V_2| < |\Delta V_1|$ $|\Delta V_2| = |\Delta V_1|$

Justify your answer.

R_A is unaffected by the resistance of the inductor since $V = IR$ and the current remains the same along with R_A .

Question 2

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses were expected to demonstrate the ability to:

- Determine how to correctly connect a voltmeter to measure an increasing electric potential difference over time in an RL circuit that contains two identical resistors, an inductor, and a battery that are in series.
- Describe a procedure for using a voltmeter to collect data that allow for a graphically determined experimental value of the resistance of one resistor, based on a quantity that increases over time.
- Create a graph that represents the data collected.
- Explain how the information from the graph would be utilized to determine the experimental value of the resistance of one resistor.
- Using Kirchhoff's loop rule, derive, but do not solve, a differential equation to determine the current in the inductor at a given time after the switch is closed.
- Determine how changing the physical attributes of the inductor (i.e., adding resistance to an ideal inductor) would affect the electric potential difference across one of the resistors.

Sample: 2A

Score: 13

Part (a) earned 3 points. The first point was earned for correctly placing the voltmeter in parallel with the combination of resistors A and B. The second point was earned for correctly indicating that the voltmeter should be used to measure the potential difference for at least one time. The third point was earned for correctly indicating that the student should measure the potential difference from immediately after the switch is closed to when steady-state conditions have been established. Part (b) earned 6 points. The first point was earned for correctly labeling potential difference on the vertical axis and time on the horizontal axis. The second point was earned for correctly drawing a concave-down and increasing curve. The third point was earned for correctly drawing a curve that starts at the origin and asymptotically approaches a nonzero value. The fourth point was earned for correctly labeling the horizontal asymptote that is consistent with the indication of the connection of the voltmeter in part (a)(i). The fifth point was not earned because the response does not indicate that a curve fit to the graph is that of an exponential function, or that the time on the graph where the potential difference across Resistor A or Resistor B is approximately $\frac{0.63}{2}\mathcal{E}$, or that the potential difference across the combination of resistors A and B is approximately $0.63\mathcal{E}$. The sixth point was not earned because the response does not indicate that the coefficient of t in the curve-fit equation is equal to $\frac{2R}{L}$ or that the time constant is equal to $\frac{2L}{R}$.

Part (c) earned 3 points. The first point was earned for correctly attempting a multi-step derivation that begins with an attempt at using Kirchhoff's loop rule. The second point was earned for correctly indicating that the potential difference across the series combination of resistors is $I(2R)$. The third point was earned for correctly indicating that the absolute value of the potential difference across the inductor is $L\frac{dI}{dt}$. Part (d) earned 3 points.

The first point was earned for correctly selecting $|\Delta V_2| < |\Delta V_1|$ with an attempt at a relevant justification. The second point was earned for correctly indicating that the total resistance has increased. The third point was earned for correctly indicating that the current decreases due to the increased resistance.

Question 2 (continued)**Sample: 2B****Score: 9**

Part (a) earned 3 points. The first point was earned for correctly placing the voltmeter in parallel with the combination of resistors A and B. The second point was earned for correctly indicating that the voltmeter should be used to measure the potential difference for at least one time. The third point was earned for correctly indicating that the student should measure the potential difference from immediately after the switch is closed to when steady-state conditions have been established. Part (b) earned 4 points. The first point was earned for correctly labeling potential difference on the vertical axis and time on the horizontal axis. The second point was earned for correctly drawing a concave-down and increasing curve. The third point was earned for correctly drawing a curve that starts at the origin and asymptotically approaches a nonzero value. The fourth point was earned for correctly labeling the horizontal asymptote that is consistent with the indication of the connection of the voltmeter in part (a)(i). The fifth point was not earned because the response does not indicate that a curve fit to the graph is that of an exponential function, or that the time on the graph where the potential difference across Resistor A or Resistor B is approximately $\frac{0.63}{2}\mathcal{E}$, or that the potential difference across the combination of resistors A and B is approximately $0.63\mathcal{E}$. The sixth point was not earned because the response does not correctly indicate that the time constant is equal to $\frac{L}{R_{\text{eq}}}$ and that R_{eq} is equal to $2R$. Part (c) earned 2 points.

The first point was not earned because the response begins with an attempt at using Kirchhoff's loop rule but does not contain a multi-step derivation. The second point was earned for correctly indicating that the potential difference across the series combination of resistors is $I(2R)$ because the response shows that the potential difference across each resistor is IR . The third point was earned for correctly indicating that the absolute value of the potential difference across the inductor is $L\frac{dI}{dt}$. Part (d) did not earn any points. The first point was not earned because the response does not correctly select $|\Delta V_2| < |\Delta V_1|$. The second point was not earned because the response does not indicate that the total resistance has increased. The third point was not earned because the response does not indicate that the current decreases due to the increased resistance or that, because the potential difference across the inductor increases, the potential difference across Resistor A will decrease.

Question 2 (continued)**Sample: 2C****Score: 4**

Part (a) did not earn any points. The first point was not earned because the response incorrectly places the voltmeter in series in the circuit. The second point was not earned because the response incorrectly indicates that an ammeter should be used to measure current in the circuit; a voltmeter was the only equipment provided in the prompt. The third point was not earned because the response does not indicate that the student should measure the potential difference for more than one time. Part (b) earned 2 points. The first point was not earned because the response incorrectly labels current on the vertical axis. The second point was earned for correctly drawing a concave-down and increasing curve. The third point was earned for correctly drawing a curve that starts at the origin and asymptotically approaches a nonzero value. The fourth point was not earned because the response does not label the horizontal asymptote. The fifth point was not earned because the response does not indicate that a curve fit to the graph is that of an exponential function, or that the time on the graph where the potential difference across Resistor A or Resistor B is approximately $\frac{0.63}{2}\mathcal{E}$, or that the potential difference across the combination of resistors A and B is approximately $0.63\mathcal{E}$. The sixth point was not earned because the response does not indicate that the coefficient of t in the curve-fit equation is equal to $\frac{2R}{L}$ or that the time constant is equal to $\frac{2L}{R}$. Part (c) earned 2 points. The first point was earned for correctly attempting a multi-step derivation that begins with an attempt at using Kirchhoff's loop rule. The second point was not earned because while the response correctly indicates that the potential difference across the series combination of resistors is $I(2R)$; however, the response does not include the provided value of R for resistors A and B. The third point was earned for correctly indicating that the absolute value of the potential difference across the inductor is $L\frac{dI}{dt}$. Part (d) did not earn any points. The first point was not earned because the response does not correctly select $|\Delta V_2| < |\Delta V_1|$. The second point was not earned because the response does not indicate that the total resistance has increased. The third point was not earned because the response does not indicate that the current decreases due to the increased resistance or that, because the potential difference across the inductor increases, the potential difference across Resistor A will decrease.