2024

AP® Physics C: Electricity and Magnetism

Sample Student Responses and Scoring Commentary Set 2

Inside:

Free-Response Question 1

- ☑ Scoring Guidelines
- ☑ Student Samples
- ☑ Scoring Commentary

Question 1: Free-Response Question — 15 points

(a) For using a correct equation for electric flux — **1 point**

Example Response

$$\Phi_E = \frac{Q}{\varepsilon_0}$$

For the correct numerical answer — **1 point**

Scoring Note: Correct units are not required to earn this point.

Example Response

$$\Phi_E = 113\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C}}$$

Example Solution

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$\Phi_E = \frac{Q}{\varepsilon_0}$$

$$\Phi_E = \frac{1.0\times 10^{-9}\ \mathrm{C}}{8.85\times 10^{-12}\,\frac{\mathrm{C^2}}{\mathrm{N\cdot m^2}}}$$

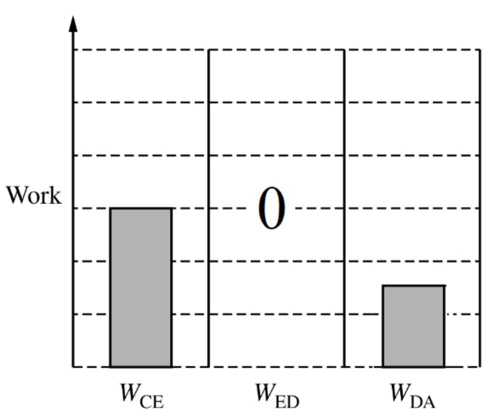
$$\Phi_E = 113\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C}}$$

Total for part (a) — 2 points

(b)(i) For indicating that $W_{\mathrm{ED}} = 0$ — **1 point**

For drawing a bar representing W_{DA} that has a height of approximately one and a half units — **1 point**

Example Response



(b)(ii) For using an equation that relates the electric field to potential difference **1 point**

Scoring Note: This point can be earned if the response begins with a correct relationship between electric field and potential difference in which numerical values are already substituted.

Example Responses

$$E_y = -\frac{dV}{dy} \quad \textbf{OR} \quad |E_y| = \left|-\frac{dV}{dy}\right| \quad \textbf{OR} \quad |E_y| = \left|-\frac{\Delta V}{\Delta y}\right| \quad \textbf{OR} \quad \Delta V = -\int \vec{E} \cdot d\vec{r}$$

For correct substitutions of values of electric potential and the distance between equipotential lines that can be used to calculate the approximate magnitude of the electric field at Position B **1 point**

Example Response

$$|E_y| = \left|-\frac{-20.0\text{ V} - (-10.0\text{ V})}{1.4\text{ m}}\right|$$

Example Solution

$$E_y = -\frac{dV}{dy}$$

$$|E_y| = \left|-\frac{dV}{dy}\right|$$

$$|E_y| = \left|-\frac{\Delta V}{\Delta y}\right|$$

$$|E_y| = \left|-\frac{-20.0\text{ V} - (-10.0\text{ V})}{1.4\text{ m}}\right|$$

$$|E_y| = 7.1\,\frac{\text{V}}{\text{m}}$$

Total for part (b) 4 points

(c) For selecting only $-y$ with an attempt at a relevant justification **1 point**

For indicating that the direction of the electric field vector is perpendicular to a line that is tangent to the equipotential line at Position C **1 point**

For indicating one of the following: **1 point**

- The test charge moves from a higher electric potential to a lower electric potential.
- The test charge and the rod have charges of the opposite sign.
- The test charge and the sphere have charges of the same sign.

Example Response

$-y$. The direction of an electric field vector is perpendicular to an equipotential line. Because the test charge has a positive charge, the test charge would move from a position of higher electric potential to a position of lower electric potential when an electric force is exerted on the test charge. Therefore, at Position C, the electric force is downward because that is the direction that is perpendicular to the equipotential line and in the direction of decreasing electric potential.

Total for part (c) 3 points

(d)(i) For using an appropriate equation for determining the electric potential from a line of uniform charge **1 point**

Example Responses

$$V = \frac{1}{4\pi\varepsilon_0}\sum\frac{q_i}{r_i} \quad \textbf{OR} \quad V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} \quad \textbf{OR} \quad \Delta V = -\int\vec{E}\cdot d\vec{r}$$

For a correct determination of r, the distance between Point P and a point on the line of uniform charge **1 point**

Example Responses

$$V_\mathrm{P} = k\sum\frac{Q}{y_\mathrm{P} - y} \quad \textbf{OR} \quad V_\mathrm{P} = k\int\left(\frac{1}{y_\mathrm{P} - y}\right)dq$$

For a correct integral with $\lambda\, dy$ substituted for dq **1 point**

Example Response

$$V_\mathrm{P} = -k\lambda\int\left(\frac{1}{y_\mathrm{P} - y}\right)dy$$

For the correct limits of integration **1 point**

Example Response

$$V_\mathrm{P} = -k\lambda\int_0^{2L}\left(\frac{1}{y_\mathrm{P} - y}\right)dy$$

Example Solutions

$$V = \frac{1}{4\pi\varepsilon_0}\sum\frac{q_i}{r_i}$$

$$V_P = k\int\left(\frac{1}{y_P - y}\right)dq$$

$$dq = -\lambda\, dy$$

$$V_P = -k\lambda\int\left(\frac{1}{y_P - y}\right)dy$$

$$V_P = -k\lambda\int_0^{2L}\left(\frac{1}{y_P - y}\right)dy$$

$$V_P = k\lambda\ln(y_P - y)\,\Big|_0^{2L} = -k\lambda\ln(y_P - y)\,\Big|_{2L}^{0}$$

$$V_P = -k\lambda\ln\left(\frac{y_P}{y_P - 2L}\right)$$

OR

$$\Delta V = -\int\vec{E}\cdot d\vec{r}$$

$$E(y) = \int\frac{k}{r^2}\,dq$$

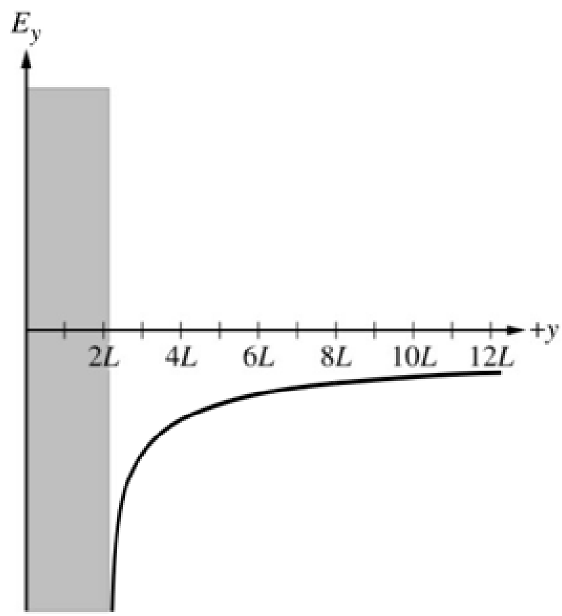
$$E(y) = \int_0^{2L}\frac{k\lambda}{(y - y')^2}\,dy' = k\lambda\left(\frac{1}{y - 2L} - \frac{1}{y}\right)$$

$$V_P = -\int_\infty^{y_P}E(y)\,dy = -k\lambda\int_\infty^{y_P}\left(\frac{1}{y - 2L} - \frac{1}{y}\right)dy$$

$$V_P = k\lambda\ln\left(\frac{y_P}{y_P - 2L}\right)$$

(d)(ii)	For sketching a curve or line that continually approaches the horizontal axis as position increases	**1 point**
	For sketching a concave down curve that is always negative	**1 point**

Example Response



Total for part (d) **6 points**

Total for question 1 **15 points**

Question 1

Begin your response to **QUESTION 1** on this page.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

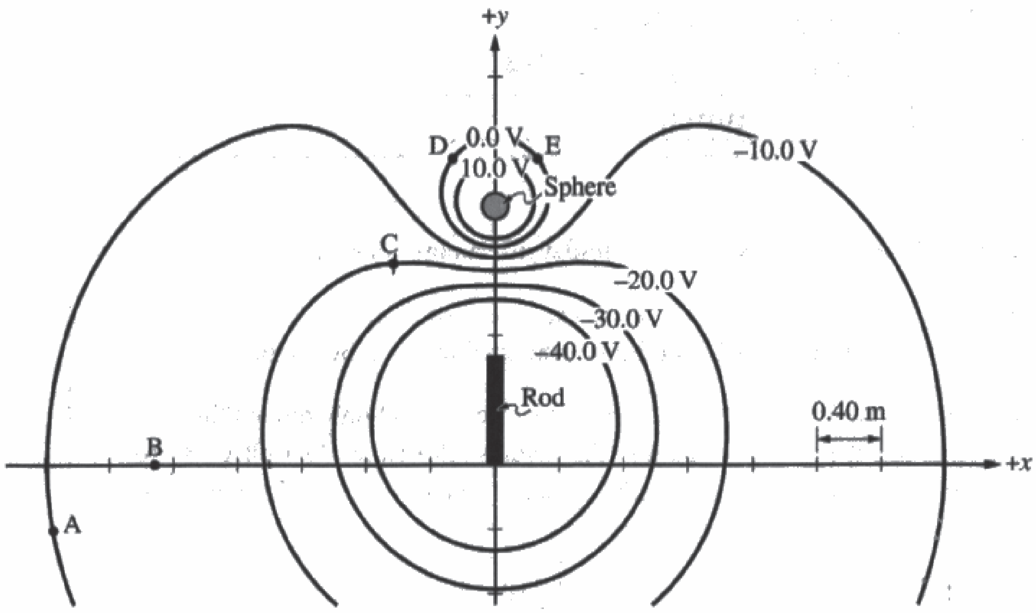


Figure 1

1. A nonconducting rod of uniform negative linear charge density is near a sphere with charge +1.0 nC. The rod and sphere are held at rest on the y-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and on the y-axis are 0.40 m apart.

(a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the 0.0 V equipotential line.

$\phi_E = \frac{Q_{enc}}{\varepsilon_0} = \frac{1 \cdot 10^{-9}}{8.85 \cdot 10^{-12}} = 112.994 \frac{N \cdot m^2}{C}$

GO ON TO THE NEXT PAGE.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 1

Continue your response to QUESTION 1 on this page.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C→E→D→A. The test charge is held momentarily at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work W_{ED} done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of W_{CE}. If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a "0" in the corresponding columns, as appropriate.

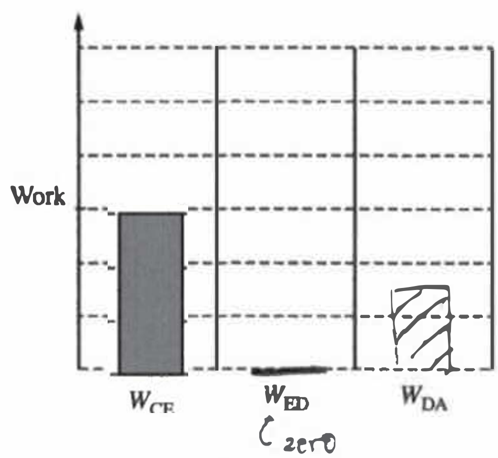


Figure 2

ii. **Calculate** the approximate magnitude of the x-component of the electric field at Position B.

$$E = \frac{-dV}{dr} \approx -\left(\frac{-20-(-10)}{3\cdot 0.4\cdot 0.2}\right)$$

$$= -\left(\frac{-10}{1.4}\right)$$

$$= -7.143\ \frac{V}{m}$$

GO ON TO THE NEXT PAGE.

Question 1

Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position C. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

____ $+x$ ____ $+y$ ____ Directly away from the sphere

____ $-x$ _✓_ $-y$ ____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

First, the electric field is always perpindicular to valtage lines so the electric field is along the y-axis at C. By definition E moves in the direction of decreasing electric potential, therefore we know E points in the -y direction. Since the charge is positive the force due to the electric field is in the same direction as the field. therefore the force is in the -y direction.

Question 1

Continue your response to **QUESTION 1** on this page.

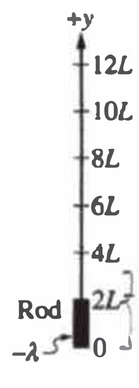


Figure 3

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the y-axis in the orientation shown in Figure 3. Position P (not shown) is located on the y-axis a distance y_P from the origin, where $y_P > 2L$.

(d) The electric potential V_P at y_P is $V_P = -k\lambda \ln\left(\frac{y_P}{y_P - 2L}\right)$.

i. Using integral calculus, derive the expression for V_P provided.

$dV = \frac{k\,dQ}{r}$ $\quad -\lambda = \frac{dQ}{dr}$

$\Rightarrow V = \int \frac{k\,dQ}{r}$

$= -\int \frac{k\lambda\,dr}{r}$

$= -k\lambda \ln|r|$

$V_P = -k\lambda \ln|r| \Big|_{y_P - 2L}^{y_P}$

difference in start and end potential

$= -k\lambda \ln|y_P| + k\lambda \ln|y_P - 2L|$

$= -k\lambda\left[\ln|y_P| - \ln|y_P - 2L|\right]$

$= -k\lambda \ln\left(\frac{y_P}{y_P - 2L}\right)$

Question 1

Continue your response to **QUESTION 1** on this page.

ii. On Figure 4, **sketch** a graph of the y-component E_y of the electric field resulting from the rod as a function of y in the region $2L < y < 12L$.

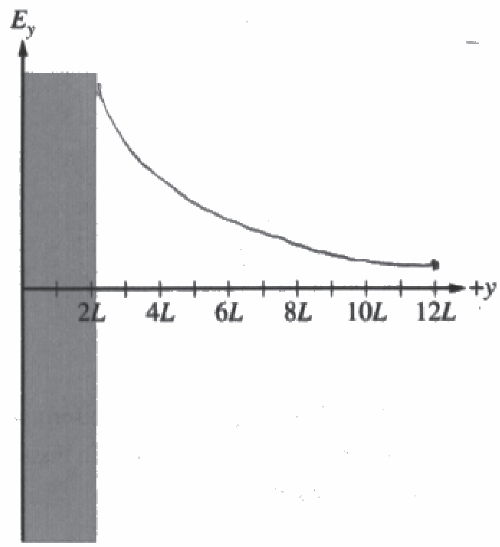


Figure 4

GO ON TO THE NEXT PAGE.

Question 1

Begin your response to **QUESTION 1** on this page.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

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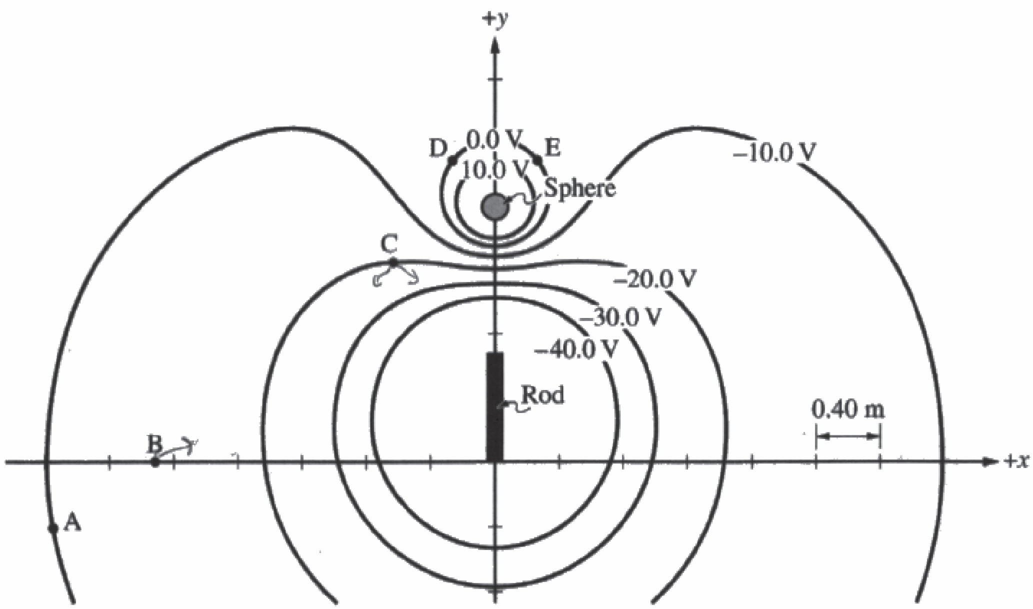

Figure 1

1. A nonconducting rod of uniform negative linear charge density is near a sphere with charge +1.0 nC. The rod and sphere are held at rest on the y-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and on the y-axis are 0.40 m apart.

   (a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the 0.0 V equipotential line.

$\frac{C \cdot N \cdot m^2}{C^2} \rightarrow \frac{N \cdot m^2}{C}$

$|\Phi_E| = \frac{Q_{enclosed}}{\varepsilon_0} = \frac{10^{-9}\,C}{\varepsilon_0} = \boxed{112.99\ \frac{N \cdot m^2}{C}}$

Question 1

Continue your response to QUESTION 1 on this page.

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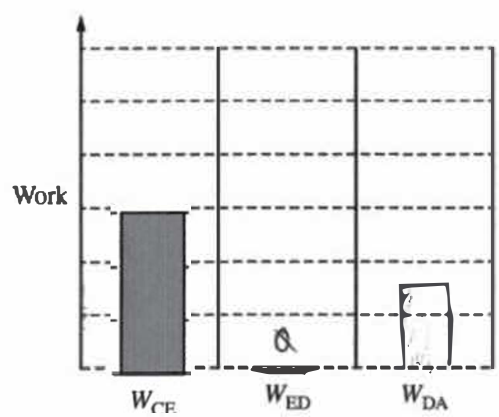

Figure 2

ii. **Calculate** the approximate magnitude of the x-component of the electric field at Position B.

$x \approx -2.1\,m$

$V \approx$ ~~-20 V~~ $-5V$

$E = \frac{-20-(-10)}{1.4-2.8} = $ **7.14 N/C**

Question 1

Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position C. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

____ $+x$ ____ $+y$ ____ Directly away from the sphere

____ $-x$ _X_ $-y$ ____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

The charge is repelled by the sphere and attracted by the rod, so it goes ~~dow~~ in the -y direction.

Question 1

Continue your response to **QUESTION 1** on this page.

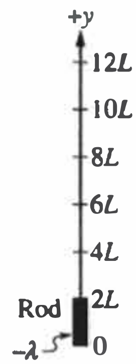

Figure 3

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the y-axis in the orientation shown in Figure 3. Position P (not shown) is located on the y-axis a distance y_P from the origin, where $y_P > 2L$.

$-k\lambda$ ·

(d) The electric potential V_P at y_P is $V_P = -k\lambda \ln\left(\dfrac{y_P}{y_P - 2L}\right)$.

i. Using integral calculus, **derive** the expression for V_P provided.

$\Delta V = -\int \vec{E} \cdot d\vec{r}$

Question 1

Continue your response to **QUESTION 1** on this page.

ii. On Figure 4, **sketch** a graph of the y-component E_y of the electric field resulting from the rod as a function of y in the region $2L < y < 12L$.

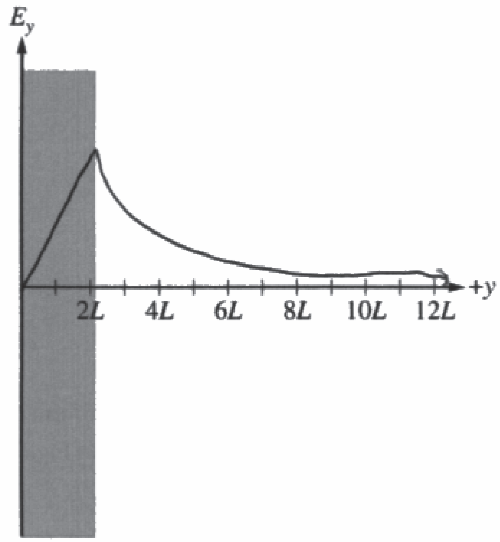

Figure 4

Question 1

Begin your response to **QUESTION 1** on this page.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

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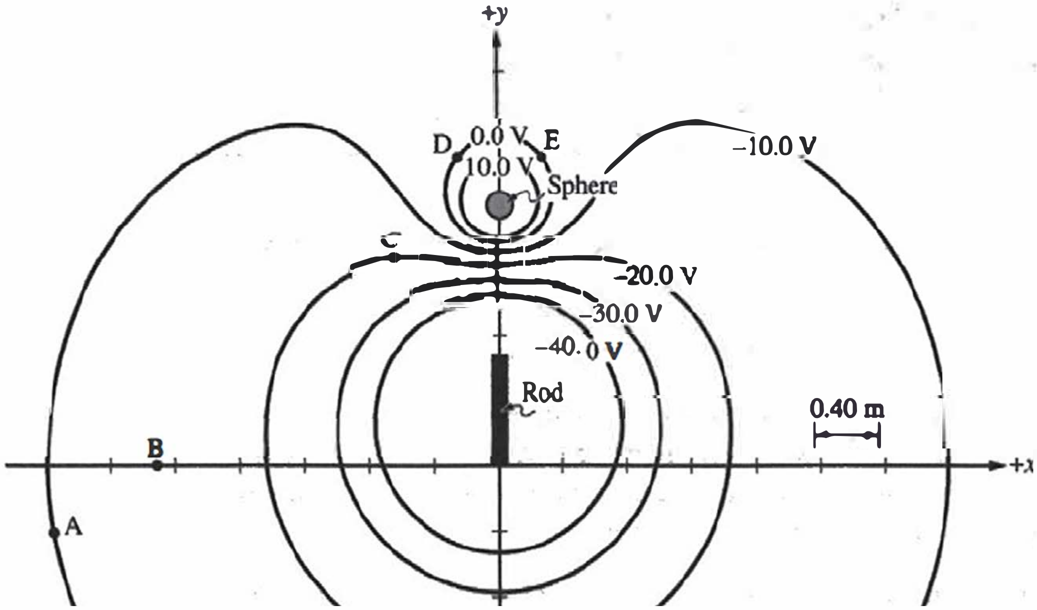

Figure 1

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   (a) Calculate the absolute value of the electric flux through the Gaussian surface whose cross section is the 0.0 V equipotential line.

$\Phi_E = \oint E \cdot dA = \frac{q_{en}}{\varepsilon_0}$ $q_{en} = +1\,nC$

$EA\cos\theta = \frac{1\,nC}{\varepsilon_0}$ $\boxed{\Phi_E = \frac{+1\,nC}{\varepsilon_0}}$

  GO ON TO THE NEXT PAGE.

Question 1

Continue your response to QUESTION 1 on this page.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C→E→D→A. The test charge is held momentarily at rest at each position.

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- **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of W_{CE}. If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a "0" in the corresponding columns, as appropriate.

$W = \int qE\,dl$

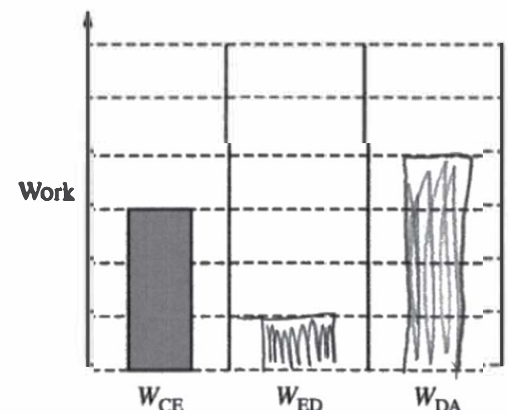

Figure 2

ii. **Calculate** the approximate magnitude of the x-component of the electric field at Position B.

$E_x = \frac{KQ}{2.2^2}$ $V = \frac{KQ}{r}$ $E = \frac{KQ}{r^2} \leftarrow E = \frac{F}{q}$

$E = \frac{10}{2.2} \approx 4.545\,\frac{N}{C}$ $E = -\frac{dV}{dx}$ distance = 2.2 m

Question 1

Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position C. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

__✓__ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ __✓__ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

Due to positive charges being attracted to negative charges, the positive test charge will want to goto the most heavily induced negative zone being the rod, Thus giving it a +x and -y direction

Question 1

Continue your response to **QUESTION 1** on this page.

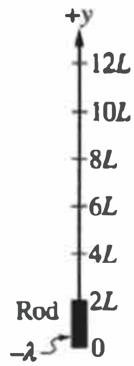

Figure 3

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the y-axis in the orientation shown in Figure 3. Position P (not shown) is located on the y-axis a distance y_P from the origin, where $y_P > 2L$.

(d) The electric potential V_P at y_P is $V_P = -k\lambda \ln\left(\dfrac{y_P}{y_P - 2L}\right)$.

i. Using integral calculus, derive the expression for V_P provided.

$$V_P = \int_0^{2L} -k\lambda \ln\left(\frac{y_P}{y_P - 2L}\right)$$

$$\ln\left(y_P\left(\frac{1}{1-2L}\right)\right)$$

$$= -k\lambda \int_0^{2L} \ln\left(\frac{y_P}{y_P - 2L}\right)$$

$$V_P = -k\lambda \frac{2L^2}{e\, y_P}$$

Question 1

Continue your response to **QUESTION 1** on this page.

ii. On Figure 4, **sketch** a graph of the y-component E_y of the electric field resulting from the rod as a function of y in the region $2L < y < 12L$.

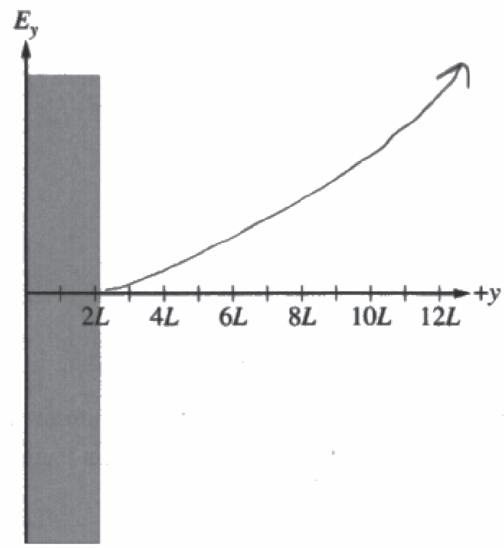

Figure 4

Question 1

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses were expected to demonstrate the ability to:

- Determine the absolute value of the electric flux due to a symmetrical charge distribution by using Gauss's law.
- Use an energy bar chart to qualitatively represent the absolute value of the work done by an external force when moving a test charge between equipotential lines.
- Calculate the magnitude of a component of an electric field by using electric potential values.
- Indicate the direction of the motion of a positive test charge after the test charge is released from rest in an electric field, requiring the application of both Coulomb's law and the relationship between electric field vectors and equipotential lines.
- Derive the relationship between electric potential and the location of a point that is located on an axis that is extended from the end of a rod of uniform negative linear charge density by using integral calculus.
- Sketch a graph representing the value of the electric field along an axis that is produced by a rod with a uniform negative charge density, resulting in a negative inverse square relationship between the field and the distance from the rod.

Sample: 1A
Score: 14

Part (a) earned 2 points. The first point was earned for using a correct expression for electric flux. The second point was earned for providing a correct numerical value for the absolute value of the electric flux through the Gaussian surface that is coincident with the zero equipotential line. Part (b) earned 4 points. The first point was earned for correctly indicating that the work W_{ED} done on the test charge by an external force is zero. A line clearly drawn at zero is sufficient to earn this point. The second point was earned for correctly indicating that the work W_{DA} done on the test charge by an external force is half of W_{CE} by drawing a bar with a height of approximately 1.5 units. The third point was earned for correctly using the equation $E = -\frac{dV}{dx}$ to relate electric field to electric potential difference. The fourth point was earned for correctly substituting values for electric potential and the distance between equipotential lines that are consistent with Figure 1 that can be used to approximate the magnitude of the electric field at Position B. Part (c) earned 3 points. The first point was earned for correctly indicating the $-y$ -direction only and providing an attempt at a relevant justification. The second point was earned for correctly indicating that the direction of the electric field is perpendicular to the equipotential surface. The third point was earned for correctly indicating that the electric field, and, therefore, the electric force, is directed such that the test charge moves from a higher electric potential to a lower electric potential. Part (d) earned 5 points. The first point was earned for beginning the derivation with an appropriate equation for determining the electric potential from a line of uniform charge. The second point was earned for correctly defining the distance from Position P to a point on the line of uniform charge. The third point was earned for correctly providing an integral with the substitution for $dq = \lambda\, dr$. The fourth point was earned for using the correct limits of integration. The fifth point was earned for showing a curve that is always approaching the horizontal axis. The sixth point was not earned because the response shows a curve that is neither always negative nor concave down.

Question 1 (continued)

Sample: 1B
Score: 9

Part (a) earned 2 points. The first point was earned for using a correct expression for electric flux. The second point was earned for providing a correct numerical value for the absolute value of the electric flux through the Gaussian surface that is coincident with the zero equipotential line. Part (b) earned 4 points. The first point was earned for correctly indicating that the work W_{ED} done on the test charge by an external force is zero. The second point was earned for correctly indicating that the work W_{DA} done on the test charge by an external force is half of W_{CE} by drawing a bar with a height of approximately 1.5 units. The third point was earned for correctly using the equation $E = -\frac{dV}{dx}$ to relate electric field to potential difference. The fourth point was earned for correctly substituting values for electric potential and the distance between equipotential lines that are consistent with Figure 1 that can be used to approximate the magnitude of the electric field at Position B. Part (c) earned 2 points. The first point was earned for correctly indicating the $-y$ -direction only and providing an attempt at a relevant justification. The second point was not earned because the response does not indicate that the direction of the electric field is perpendicular to the equipotential surface. The third point was earned for correctly addressing the relative signs of the test charge, rod, and/or sphere. Part (d) earned 1 point. The first point was not earned because although the response does indicate a possible starting point for the derivation, the equation is not used to answer the question. The second point was not earned because the response does not correctly define the distance from Position P to a point on the line of uniform charge. The third point was not earned because the response does not provide an integral with the substitution for $dq = \lambda\, dy$. The fourth point was not earned because the response does not use the correct limits of integration. The fifth point was earned for correctly sketching a curve that is always approaching the horizontal axis. The sixth point was not earned because the response shows a sketch that is neither always negative nor concave down.

Question 1 (continued)

Sample: 1C
Score: 3

Part (a) earned 1 point. The first point was earned for correctly using a correct expression for electric flux. The second point was not earned because the response does not provide a correct numerical value for the absolute value of the electric flux through the Gaussian surface that is created by the zero equipotential line. Part (b) earned 1 point. The first point was not earned because the response does not indicate that the work W_{ED} done on the test charge by an external force is zero. The second point was not earned because the response does not indicate that the work W_{DA} done on the test charge by an external force is half of W_{CE}. The third point was earned for correctly using the equation $E = -\frac{dV}{dx}$ to relate electric field to potential difference. The fourth point was not earned because the response does not correctly substitute values for the electric potential and the distance between equipotential lines that are consistent with Figure 1 that can be used to approximate the magnitude of the electric field at Position B. Part (c) earned 1 point. The first point was not earned because the response does not select that the net electric force exerted on the test charge is in the $-y$-direction only. The second point was not earned because the response does not indicate that the direction of the electric field is perpendicular to the equipotential surface. The third point was earned for correctly addressing the relative charges of the test charge, the sphere, and/or the rod. Part (d) did not earn any points. The first point was not earned because although the response does indicate a possible starting point for the derivation, the equation is not used to answer the question. The second point was not earned because the response does not correctly define the distance from Position P to a point on the line of uniform charge. The third point was not earned because the response does not provide an integral with the substitution for $dq = \lambda dy$. The fourth point was not earned because the response does not use the correct limits of integration. The fifth point was not earned because the response sketch does not approach the horizontal axis. The sixth point was not earned because the response sketch is neither always negative nor concave down.