2024

# AP® Physics 2: Algebra-Based

## Sample Student Responses and Scoring Commentary

**Inside:**

Free-Response Question 3

- ☑ Scoring Guidelines
- ☑ Student Samples
- ☑ Scoring Commentary

## Question 3: Quantitative/Qualitative Translation — 12 points

**(a)(i)** For correctly determining the total resistance $R_{\text{total}}$ of the circuit **1 point**

**Example Response**

$$R_{\text{total}} = \frac{5R}{3}$$

For a multi-step derivation that includes correct substitutions of $\mathcal{E}$ and $R_{\text{total}}$ into the equation that describes Ohm's law, consistent with the first point of part (a)(i) **1 point**

**Example Response**

$$I_1 = \frac{3\mathcal{E}}{5R}$$

**Example Solution**

*Determine the total resistance of the circuit.*

*The resistance of the right-most branch containing resistors connected in series:*

$$R_s = \sum_i R_i$$

$$R_s = R + R$$

$$R_s = 2R$$

*The resistance of parallel branches that contain resistors:*

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

$$\frac{1}{R_p} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R}$$

$$R_p = \frac{2R}{3}$$

*The total resistance of the circuit:*

$$R_s = \sum_i R_i$$

$$R_{\text{total}} = R + \frac{2R}{3} = \frac{5R}{3}$$

*The total current in the circuit:*

$$I = \frac{\Delta V}{R} = \frac{3\mathcal{E}}{5R}$$

**(a)(ii)** For applying a result of Kirchhoff's law and/or Ohm's law that relates the current in $R_3$ to **one** of the following: **1 point**

- The current in or potential difference across $R_1$
- The current in or potential difference across $R_2$
- The potential difference across $R_3$

**Example Responses**

*One-third of the total current in the circuit is in* $R_3$*:* $I_3 = \frac{I_1}{3}$

**OR**

$I_1 = I_2 + I_3$

**OR**

$\Delta V_2 = \Delta V_3 + \Delta V_4$

For a correct expression from a multi-step derivation that is consistent with the final expression in part (a)(i) **1 point**

**Example Response**

$I = \frac{\mathcal{E}}{5R}$

**Example Solutions**

$I_1 = I_2 + I_3$

$\Delta V_2 = \Delta V_3 + \Delta V_4$

$I_2 R_2 = I_3 (R_3 + R_4)$

$I_2 R = I_3 (2R)$

$I_2 = 2I_3$

$I_1 = 2I_3 + I_3$

$I_3 = \frac{I_1}{3}$

$$I_3 = \frac{\left(\frac{3\mathcal{E}}{5R}\right)}{3}$$

$I_3 = \frac{\mathcal{E}}{5R}$

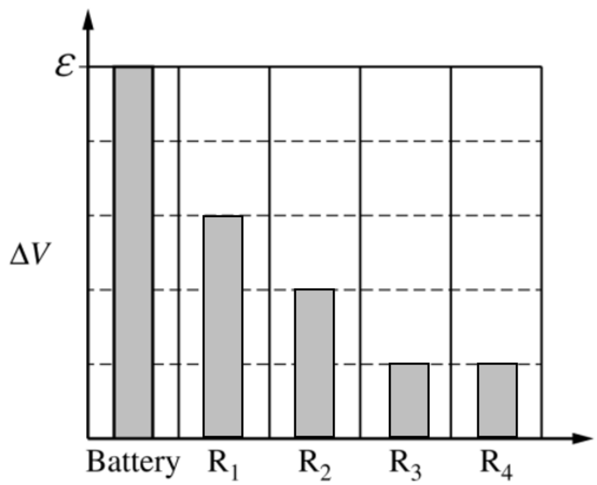
**OR**

*One-third of the total current in the circuit is in* $R_3$*. Therefore,* $I_3 = \frac{\mathcal{E}}{5R}$.

**Total for part (a) 4 points**

| | | |
|---|---|---|
| **(b)** | For drawing a bar that indicates that the potential difference across $R_3$ is nonzero and is equal to the potential difference across $R_4$ | **1 point** |
| | For drawing a bar that indicates that the potential difference across $R_2$ is nonzero and is equal to the sum of the potential differences across $R_3$ and $R_4$ | **1 point** |
| | For drawing all bars correctly | **1 point** |

**Example Response**




**Total for part (b)** **3 points**

| | | |
|---|---|---|
| **(c)** | For indicating that the equation is correct or incorrect, consistent with the derivations in part (a) or the bar chart from part (b), with an attempt at a relevant justification | **1 point** |
| | For a justification that correctly relates $P$ to at least **one** of the following:<br>• The potential difference across the battery<br>• The current in the battery<br>• $R_1$, $R_2$, $R_3$, and $R_4$ | **1 point** |
| | For a justification that relates the equation $P = \frac{3\mathcal{E}^2}{5R}$ to at least **one** of the following:<br>• The bar chart from the response in part (b)<br>• The derivations in the responses to parts (a)(i) and (a)(ii)<br>**Scoring Note**: The justification must be consistent with the derivations in part (a) or the bar chart from part (b). | **1 point** |

**Example Solution**

*The equation is correct. The equation for $P$, which is power, can be written as $P = \frac{(\Delta V)^2}{R}$. According to the bar chart in part (b), the potential difference across the battery is $\mathcal{E}$. The total resistance of the circuit is $\frac{5R}{3}$, according to the derivation from part (a)(i). Therefore, $P = \frac{(\Delta V)^2}{R} = \frac{(\mathcal{E})^2}{\left(\frac{5R}{3}\right)} = \frac{3\mathcal{E}^2}{5R}$.*

**Total for part (c) 3 points**

| | | |
|---|---|---|
| **(d)** | For selecting that $P_{new} < P_{original}$ with an attempt at a relevant justification | **1 point** |
| | For a correct justification that indicates at least **one** of the following:<br>• The current in $R_1$ is less in the new circuit than in the original circuit<br>• The potential difference across $R_1$ is less in the new circuit than in the original circuit | **1 point** |

**Example Response**

$P_{new} < P_{original}$. *Since the* emf *of the battery is the same in the new circuit and the total resistance of the new circuit is greater, the current in $R_1$ is less in the new circuit. Therefore, $P_{new}$ is less than $P_{original}$.*

**Total for part (d) 2 points**

**Total for question 3 12 points**

## Question 3

**Begin your response to QUESTION 3 on this page.**

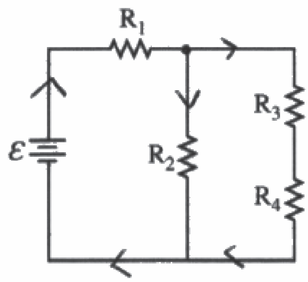



Figure 1

3. (12 points, suggested time 25 minutes)

A circuit consists of an ideal battery of emf $\mathcal{E}$ and four identical resistors $R_1$, $R_2$, $R_3$, and $R_4$, each of resistance $R$, as shown in Figure 1.

(a) For parts (a)(i) and (a)(ii), express your answers in terms of numerical values, $\mathcal{E}$, and $R$ only.

i. **Derive** an expression for the current $I_1$ in Resistor $R_1$.

$$R_{eq} = R + \left(\frac{1}{R} + \frac{1}{2R}\right)^{-1} = R + \frac{2R}{3} = \frac{5}{3}R$$

$$I_1 = \frac{V}{R_{eq}} = \frac{\mathcal{E}}{\frac{5}{3}R} = \boxed{\frac{3}{5}\left(\frac{\mathcal{E}}{R}\right)}$$

ii. **Derive** an expression for the current $I_3$ in Resistor $R_3$.

$$V_3 = I_3 R$$

$$V_{3\text{ and }4} = \mathcal{E} - I_1 R = \mathcal{E} - \frac{3}{5}\mathcal{E} = \frac{2}{5}\mathcal{E}$$

$$\frac{2}{5}\mathcal{E} = I_{3\text{ and }4}(2R)$$

$$I_{3\text{ and }4} = I_3 = \boxed{\frac{1}{5}\mathcal{E}/R}$$

## Question 3

Continue your response to **QUESTION 3** on this page.

(b) The partially completed bar chart in Figure 2 shows a bar that represents the absolute value $|\Delta V|$ of the potential difference across the ideal battery.

- In Figure 2, **draw** a bar to represent $|\Delta V|$ across each resistor, relative to the emf $\mathcal{E}$ of the ideal battery.
- The height of each bar should be proportional to the value of $|\Delta V|$ represented by that bar. If $|\Delta V|$ is zero, write a "0" in that column.

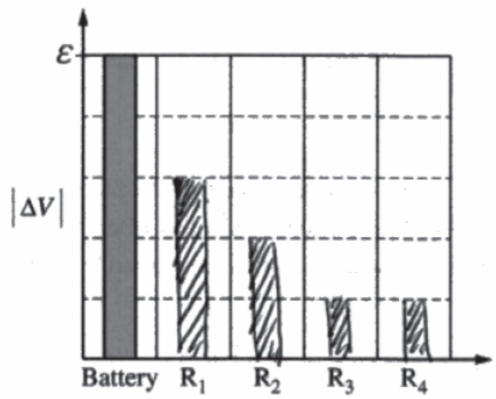



Figure 2

A student claims that the rate at which energy is dissipated (power) by the circuit can be expressed as $P = \frac{3\mathcal{E}^2}{5R}$.

(c) **State** whether the expression for $P$ is correct or incorrect. **Justify** your answer by referring to the derivations from part (a) or the bar chart from part (b).

This expression is correct. P is known to be equal to IV, and I was calculated to be $\frac{3}{5}\left(\frac{\mathcal{E}}{R}\right)$ in part (a). Given that $V = \mathcal{E}$, a quick substitution yields $P = \left[\frac{3}{5}\left(\frac{\mathcal{E}}{R}\right)\right] \cdot \mathcal{E}$ or $P = \frac{3}{5}\left(\frac{\mathcal{E}^2}{R}\right)$.

## Question 3

Continue your response to **QUESTION 3** on this page.

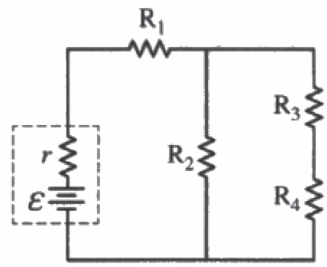



Figure 3

When the ideal battery is connected in the original circuit, the rate at which energy is dissipated by Resistor $R_1$ is $P_{original}$. The ideal battery is now replaced with a nonideal battery of emf $\mathcal{E}$ and internal resistance $r$ to form the new circuit shown in Figure 3. The rate at which energy is dissipated by Resistor $R_1$ in the new circuit is $P_{new}$.

(d) **Indicate** whether $P_{new}$ is greater than, less than, or equal to $P_{original}$.

____ $P_{new} > P_{original}$ ✓ $P_{new} < P_{original}$ ____ $P_{new} = P_{original}$

Briefly **justify** your answer.

The nonideal battery's internal resistance contributes to the overall resistance of the circuit and ~~decreases~~ increases it, so the current through the first resistor would decrease. Its known that $P \propto I^2$, so a lower current would reduce the energy dissipated by $R_1$.







Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

## Question 3

Begin your response to **QUESTION 3** on this page.

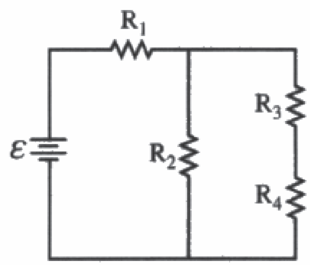



Figure 1

3. (12 points, suggested time 25 minutes)

A circuit consists of an ideal battery of emf $\mathcal{E}$ and four identical resistors $R_1$, $R_2$, $R_3$, and $R_4$, each of resistance $R$, as shown in Figure 1.

(a) For parts (a)(i) and (a)(ii), express your answers in terms of numerical values, $\mathcal{E}$, and $R$ only.

i. **Derive** an expression for the current $I_1$ in Resistor $R_1$.

$$\frac{\mathcal{E}}{R_1} = I_1$$

ii. **Derive** an expression for the current $I_3$ in Resistor $R_3$.

$$\frac{\mathcal{E}}{R_T} = I_T$$

$$I_T R_1 = \mathcal{E}_1$$

$$\mathcal{E} - \mathcal{E}_1 = \mathcal{E}_{234}$$

$$R_T = \frac{2R}{3} + R_1$$

$$\frac{\mathcal{E}_{234}}{2R} = I_{34}$$

$$I_{34} = I_3$$

$$R_T = \frac{5R}{3}$$





GO ON TO THE NEXT PAGE.

## Question 3

Continue your response to **QUESTION 3** on this page.

(b) The partially completed bar chart in Figure 2 shows a bar that represents the absolute value $|\Delta V|$ of the potential difference across the ideal battery.

- In Figure 2, **draw** a bar to represent $|\Delta V|$ across each resistor, relative to the emf $\mathcal{E}$ of the ideal battery.
- The height of each bar should be proportional to the value of $|\Delta V|$ represented by that bar. If $|\Delta V|$ is zero, write a "0" in that column.

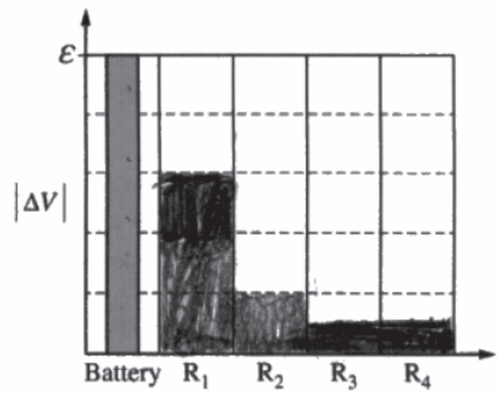



Figure 2

A student claims that the rate at which energy is dissipated (power) by the circuit can be expressed as $P = \dfrac{3\mathcal{E}^2}{5R}$.

(c) **State** whether the expression for $P$ is correct or incorrect. **Justify** your answer by referring to the derivations from part (a) or the bar chart from part (b).

$$P = \frac{\mathcal{E}^2}{\frac{5}{3}R} = \frac{3\mathcal{E}^2}{5R}$$

The expression is correct because from Part a) $R_T = \frac{5}{3}R$

## Question 3

Continue your response to **QUESTION 3** on this page.

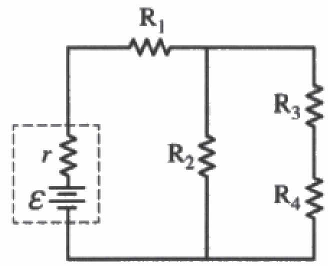



Figure 3

When the ideal battery is connected in the original circuit, the rate at which energy is dissipated by Resistor $R_1$ is $P_{original}$. The ideal battery is now replaced with a nonideal battery of emf $\mathcal{E}$ and internal resistance $r$ to form the new circuit shown in Figure 3. The rate at which energy is dissipated by Resistor $R_1$ in the new circuit is $P_{new}$.

(d) **Indicate** whether $P_{new}$ is greater than, less than, or equal to $P_{original}$.

____ $P_{new} > P_{original}$ ✓ $P_{new} < P_{original}$ ____ $P_{new} = P_{original}$

Briefly **justify** your answer.

With the internal resistance now added to the circuit, ε will decrease and the overall resistance of the circuit will increase. Since resistance is inverseley poportional to Power. $P_{new}$ is going to be less than $P_{original}$

$$P = \frac{V^2}{R}$$

## Question 3

**Begin your response to QUESTION 3 on this page.**

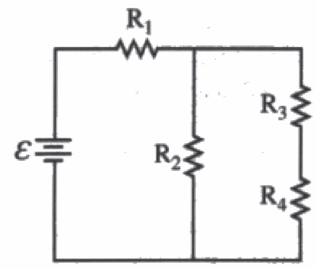



Figure 1

$R_{eq} = R_1 + \frac{1}{\frac{1}{R_3+R_4} + \frac{1}{R_2}}$

$= R_1 + \frac{R_2 + R_3 + R_4}{R_2(R_3+R_4)}$

$= R_1 + \frac{R_2(R_3+R_4)}{R_2+R_3+R_4}$

3. (12 points, suggested time 25 minutes)

A circuit consists of an ideal battery of emf $\mathcal{E}$ and four identical resistors $R_1$, $R_2$, $R_3$, and $R_4$, each of resistance $R$, as shown in Figure 1.

(a) For parts (a)(i) and (a)(ii), express your answers in terms of numerical values, $\mathcal{E}$, and $R$ only.

i. **Derive** an expression for the current $I_1$ in Resistor $R_1$.

$\Delta V = IR$

$\mathcal{E} = IR$

$I = \frac{\mathcal{E}}{R}$

ii. **Derive** an expression for the current $I_3$ in Resistor $R_3$.

$\Delta V = IR$

## Question 3

Continue your response to **QUESTION 3** on this page.

(b) The partially completed bar chart in Figure 2 shows a bar that represents the absolute value $|\Delta V|$ of the potential difference across the ideal battery.

- In Figure 2, **draw** a bar to represent $|\Delta V|$ across each resistor, relative to the emf $\mathcal{E}$ of the ideal battery.
- The height of each bar should be proportional to the value of $|\Delta V|$ represented by that bar. If $|\Delta V|$ is zero, write a "0" in that column.

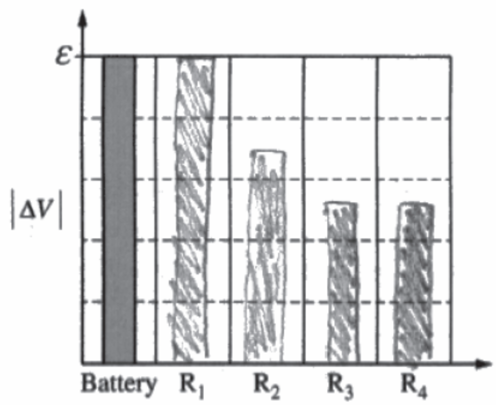



Figure 2

A student claims that the rate at which energy is dissipated (power) by the circuit can be expressed as $P = \frac{3\mathcal{E}^2}{5R}$.

(c) **State** whether the expression for $P$ is correct or incorrect. **Justify** your answer by referring to the derivations from part (a) or the bar chart from part (b).

the expression for P is correct

## Question 3

Continue your response to **QUESTION 3** on this page.

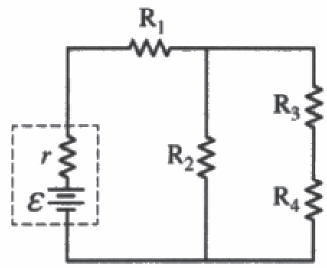



Figure 3

When the ideal battery is connected in the original circuit, the rate at which energy is dissipated by Resistor $R_1$ is $P_{original}$. The ideal battery is now replaced with a nonideal battery of emf $\mathcal{E}$ and internal resistance $r$ to form the new circuit shown in Figure 3. The rate at which energy is dissipated by Resistor $R_1$ in the new circuit is $P_{new}$.

(d) **Indicate** whether $P_{new}$ is greater than, less than, or equal to $P_{original}$.

____ $P_{new} > P_{original}$    _✓_ $P_{new} < P_{original}$    ____ $P_{new} = P_{original}$

Briefly **justify** your answer.

battery changed from ideal to nonideal with internal resistance. more resistance slower rate of which energy dissipates

## Question 3

**Note:** Student samples are quoted verbatim and may contain spelling and grammatical errors.

### Overview

The responses were expected to demonstrate the ability to:
- Determine the equivalent resistance of a circuit containing resistors in both series and parallel.
- Derive mathematical expressions for the current in a resistor in series with the battery and the current in a resistor on a parallel branch of a circuit using Ohm's law and Kirchhoff's laws.
- Demonstrate understanding of the relationships among potential differences across each resistor in a circuit through the use of a bar chart.
- Analyze a given expression for power to determine if it is correct based on previous circuit analysis.
- Apply functional dependence to determine how the addition of a nonideal battery to a circuit affects the rate at which energy is dissipated across a resistor in the circuit.

### Sample: 3A
### Score: 12

Part (a) earned 4 points. The first point was earned for correctly determining the total resistance of the circuit in terms of $R$. The second point was earned for showing a multi-step derivation that correctly substitutes the emf and the determined total resistance into Ohm's law. (Note: A correct response with no supporting work can earn the first point of part (a)(i) but not the second point.) The third point was earned for correctly relating the current $I_3$ in $R_3$ to "$V_3$" using Ohm's law and then using Kirchhoff's loop rule to determine the potential difference across the outer parallel branch of the circuit to determine $I_3$. The fourth point was earned for correctly including a multi-step derivation that is consistent with the final expression from part (a)(i) and is written only in terms of the required variables. Part (b) earned 3 points. The first point was earned for correctly indicating that the potential difference across $R_3$ is nonzero and is equal to the potential difference across $R_4$. The second point was earned for correctly indicating a potential difference across $R_2$ that is nonzero and is equal to the sum of the potential differences across $R_3$ and $R_4$. The third point was earned for correctly showing all potential differences with the correct values. Part (c) earned 3 points. The first point was earned for correctly indicating that the expression is correct, consistent with the derivations in part (a), and including a relevant justification. The second point was earned for correctly relating $P$ to both the battery emf and the current in the battery in the equation $P = IV$, though only one of these substitutions is necessary to earn this point. The third point was earned for correctly using the battery emf and the current in the battery, derived in part (a)(i), in an appropriate expression for $P$ that shows the expected expression for $P$ matches the given expression for $P$. Part (d) earned 2 points. The first point was earned for correctly selecting $P_{\text{new}} < P_{\text{original}}$ with an attempt at a relevant justification. The second point was earned for correctly indicating that the current in resistor $R_1$ is less in the new circuit which leads to a lower $P$.

### Sample: 3B
### Score: 7

Part (a) earned 1 point. The first point was not earned because the response does not correctly determine the total resistance of the circuit in terms of $R$. The second point was not earned because the response is not multi-step and does not correctly substitute a value for total resistance into Ohm's law. The third point was earned for attempting to apply a result of Kirchhoff's laws and Ohm's law that relates the current in $R_3$ to the potential difference across $R_1$. The fourth point was not earned because the response does not contain a correct expression

## Question 3 (continued)

that is consistent with the final expression from part (a)(i). Part (b) earned 2 points. The first point was earned for correctly indicating that the potential difference across $R_3$ is nonzero and is equal to the potential difference across $R_4$. The second point was earned for correctly indicating a potential difference across $R_2$ that is nonzero and is equal to the sum of the potential differences across $R_3$ and $R_4$. The third point was not earned because the response does not correctly show all potential differences with the correct values. Part (c) earned 3 points. The first point was earned for correctly indicating that the expression is correct, consistent with the derivation of total resistance from part (a)(ii) and including a relevant justification. The second point was earned for correctly relating $P$ to both the battery emf and the total resistance in the equation "$P = \frac{V^2}{R}$." The third point was earned for using the potential difference across the battery and the total resistance of the circuit, derived in part (a)(ii), in an appropriate expression for $P$ that matches the given expression for $P$. Part (d) earned 1 point for correctly selecting $P_{new} < P_{original}$ with an attempt at a relevant justification. The second point was not earned because the response does not directly indicate that either the current in, or potential difference across, $R_1$ is less in the new circuit than in the original circuit. The response includes a correct justification that the inclusion of the nonideal battery will lower the total power dissipation of the circuit but does not address what happens to $R_1$.

**Sample: 3C**
**Score: 2**

Part (a) did not earn any points. The first point was not earned because the response does not correctly determine the total resistance of the circuit in terms of $R$. The second point was not earned because the response does not correctly substitute the determined value for total resistance into Ohm's law. The third point was not earned because the response does not attempt to apply a result of Kirchhoff's laws or Ohm's law that relates the current in $R_3$ to values in the rest of the circuit. The fourth point was not earned because the response does not contain a correct expression that is consistent with the final expression in part (a)(i). Part (b) earned 1 point for correctly indicating that the potential difference across $R_3$ is nonzero and is equal to the potential difference across $R_4$. The second point was not earned because the response does not correctly indicate a potential difference across $R_2$ that is nonzero and is equal to the sum of the potential differences across $R_3$ and $R_4$. The third point was not earned because the response does not correctly show all potential differences with the correct values. Part (c) did not earn any points. The first point was not earned because the response does not include an attempt at a relevant justification. The second point was not earned because the response does not include a justification. The third point was not earned because the response does not include a justification. Part (d) earned 1 point for correctly selecting $P_{new} < P_{original}$ with an attempt at a relevant justification. The second point was not earned because the response does not correctly indicate that either the current in, or potential difference across, $R_1$ is less in the new circuit than in the original circuit.