# AP' Physics l: Algebra-Based Scoring Guidelines 

## Question 1: Short Answer

(a) For an explanation that indicates that the maximum kinetic energy and maximum potential 1 point energy are the same due to energy conservation

Scoring Note: This point may be earned for only stating "conservation of energy."

## Example Response

The maximum kinetic energy and maximum potential energy of the car-spring system are both 4 J , because energy is conserved in this system.
(b) For using the equation for frequency or period in a ratio

1 point

## Example Responses

$\frac{\frac{1}{2 \pi} \sqrt{\frac{k}{m_{2}}}}{\frac{1}{2 \pi} \sqrt{\frac{k}{m_{1}}}}$ OR $\frac{\frac{1}{2 \pi} \sqrt{\frac{k}{m_{1}}}}{\frac{1}{2 \pi} \sqrt{\frac{k}{m_{2}}}} \boldsymbol{O R} \frac{2 \pi \sqrt{\frac{m_{2}}{k}}}{2 \pi \sqrt{\frac{m_{1}}{k}}}$ OR $\frac{2 \pi \sqrt{\frac{m_{1}}{k}}}{2 \pi \sqrt{\frac{m_{2}}{k}}}$

Scoring Note: Simplified versions of the above ratios also earn this point.

For substituting the total mass $4 m_{0}$ into the correct ratio: $\frac{f_{2}}{f_{1}}$ or $\frac{T_{1}}{T_{2}}$

## Example Response

$T=2 \pi \sqrt{\frac{m}{k}}$
$f=\frac{1}{2 \pi} \sqrt{\frac{k}{m}}$
$\frac{f_{2}}{f_{1}}=\frac{\frac{1}{2 \pi} \sqrt{\frac{k}{4 m_{0}}}}{\frac{1}{2 \pi} \sqrt{\frac{k}{m_{0}}}}$
$\frac{f_{2}}{f_{1}}=\frac{1}{2}$
(c)(i) For a valid explanation in terms of work or energy for why the systems' energies should be the same

Accept one of the following:

- No work is done on the system
- The maximum spring potential energy is the same
- The force exerted on the system is perpendicular to the direction of motion


## Example Response

The maximum potential energy of the system does not depend upon the mass of the system, therefore there will be no change when the block is added.
(c)(ii) For drawing a single straight line with a horizontal intercept that is the same as the horizontal intercept of the original graph of 4 J

For drawing a line with a vertical intercept that is less than the vertical intercept in the original graph

For drawing a line with the correct vertical intercept of 1 J

## Example Response



Total for part (c) 4 points
Total for question $1 \quad 7$ points
(a)(i) For indicating two quantities that, when graphed together, produce a straight line whose $\mathbf{1}$ point slope can be used to determine the acceleration $a$

## Example Response

Vertical Axis : __Position Horizontal Axis: Time squared

| Position $x$ <br> $(\mathrm{~m})$ | Time $t$ <br> $(\mathrm{~s})$ | Time squared $t^{2}$ <br> $\left(\mathrm{~s}^{2}\right)$ |
| :---: | :---: | :---: |
| 0.06 | 0.39 | 0.15 |
| 0.14 | 0.59 | 0.35 |
| 0.24 | 0.77 | 0.59 |
| 0.37 | 0.96 | 0.92 |
| 0.55 | 1.20 | 1.44 |

(a)(ii) The axes have a linear scale and are identified (labels OR units) so that when graphed correctly, the data will span more than half of the horizontal and vertical axes

| For plotting at least 4 of the data points correctly | $\mathbf{1}$ point |
| :--- | :---: |
| For drawing a best-fit line that approximates the trend of the data | $\mathbf{1}$ point |

## Example Response



## Alternate Example Response



Scoring Note: The following tables represent the most common linearized graphs with the data that were used to determine the acceleration.

| Graph: $v$ vs. $t$ |  |
| :---: | :---: |
| $v\left(\frac{\mathrm{~m}}{\mathrm{~s}}\right)$ | $t(\mathrm{~s})$ |
| 0.15 | 0.20 |
| 0.40 | 0.49 |
| 0.56 | 0.68 |
| 0.68 | 0.87 |
| 0.75 | 1.08 |


| Graph: $2 x$ vs. $t^{2}$ |  |
| :---: | :---: |
| $2 x(\mathrm{~m})$ | $t^{2}\left(\mathrm{~s}^{2}\right)$ |
| 0.12 | 0.15 |
| 0.28 | 0.35 |
| 0.48 | 0.59 |
| 0.74 | 0.92 |
| 1.10 | 1.44 |


| Graph: $2 v_{\text {avg }}$ vs. $t$ |  |
| :---: | :---: |
| $2 v_{\text {avg }}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)$ | $t(\mathrm{~s})$ |
| 0.31 | 0.39 |
| 0.47 | 0.59 |
| 0.62 | 0.77 |
| 0.77 | 0.96 |
| 0.92 | 1.20 |


| Graph: $x$ vs. $\frac{1}{2} t^{2}$ |  |
| :---: | :---: |
| $x(\mathrm{~m})$ | $\frac{1}{2} t^{2}\left(\mathrm{~s}^{2}\right)$ |
| 0.06 | 0.08 |
| 0.14 | 0.17 |
| 0.24 | 0.30 |
| 0.37 | 0.46 |
| 0.55 | 0.72 |


| Graph: $v_{\text {avg }}{ }^{2}$ vs. $x$ |  |
| :---: | :---: |
| $v_{\text {avg }}{ }^{2}\left(\frac{\mathrm{~m}^{2}}{\mathrm{~s}^{2}}\right)$ | $x(\mathrm{~m})$ |
| 0.02 | 0.06 |
| 0.06 | 0.14 |
| 0.10 | 0.24 |
| 0.15 | 0.37 |
| 0.21 | 0.55 |


| Graph: $\sqrt{x}$ vs. $t$ |  |
| :---: | :---: |
| $\sqrt{x}(\sqrt{\mathrm{~m}})$ | $t(\mathrm{~s})$ |
| 0.24 | 0.39 |
| 0.37 | 0.59 |
| 0.49 | 0.77 |
| 0.61 | 0.96 |
| 0.74 | 1.20 |

(a)(iii) For attempting to find the slope, $\left(\frac{\text { rise }}{\text { run }}\right)$ or $\left(\frac{\Delta y}{\Delta x}\right)$, of the best-fit line drawn in part (a)(ii) 1 point

Scoring Note: An indication that a calculator was used for linear regression to determine the value of the slope may earn this point.

For using the slope in a valid kinematic equation to calculate the acceleration $\quad 1$ point
Scoring Note: This point can be earned if evidence of a kinematic equation exists in graphed quantities (e.g., a graph of position as a function of $\frac{1}{2} t^{2}$ ).

## Example Response

slope $=\frac{\Delta y}{\Delta x}=\frac{\Delta \text { position }}{\Delta \text { time }^{2}}=\frac{0.48 \mathrm{~m}-0.18 \mathrm{~m}}{1.2 \mathrm{~s}^{2}-0.4 \mathrm{~s}^{2}}=0.375 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$
$\Delta x=v_{0} t+\frac{1}{2} a t^{2}$
$\frac{\Delta x}{t^{2}}=\frac{1}{2} a$
slope $\times 2=a$
$a=0.75 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$
(b)(i) For indicating a quantity to be measured

Accept one of the following:

- The angle $\theta$ with the horizontal
- The height $h$ and length $L$ of the ramp

Scoring Note: Stating only the height needs to be measured can earn this point if an energy approach is used.
(b)(ii) For providing a correct expression relating the acceleration of gravity to the acceleration $\mathbf{1}$ point measured

Scoring Note: If $\cos \theta$ is used, the response must specify that $\theta$ was measured from the vertical.

## Example Response

$m g_{\exp } \sin \theta=m a$
$g_{\exp }=\frac{a}{\sin \theta}$

OR
$\sin \theta=\frac{h}{L}$
$g_{\exp }=\left(\frac{L}{h}\right) a$

OR
$m g_{\exp } h=\frac{1}{2} m v^{2}$
$g_{\exp } h=\frac{1}{2} v^{2}$
$v=\sqrt{2 g_{\exp } h}$
$v=a t$
$a t=\sqrt{2 g_{\exp } h}$
$g_{\exp }=\frac{a^{2} t^{2}}{2 h}$
(c)(i) For identifying a physical factor that could have affected the result

Accept one of the following:

- A physical factor in the materials used (e.g., the wheels have nonnegligible rotational inertia, the ramp was bumpy, the wheels were wobbly or not perfectly round, the base of the ramp was not level, the floor was not level.)
- A physical factor in the environment (e.g., the room was being accelerated, elevator, the experiment was performed at high elevation or on a different planet.)
- A physical error in measurement collection (e.g., time, position, or angle was measured incorrectly.)

Scoring Note: A statement of "Human error" does not earn this point.
(c)(ii) For correctly indicating the functional dependence between the reason listed in part (c)(i) $\mathbf{1}$ point and $g_{\text {exp }}$

Accept one of the following:

- Correctly indicating the functional dependence between the physical factor in the materials used and $g_{\exp }$ (e.g., if the rotational inertia of the rotating wheels is nonnegligible, the cart will have a smaller acceleration and $g_{\text {exp }}$ will be smaller.)
- Correctly indicating the functional dependence between the physical factor in the environment and $g_{\exp }$ (e.g., if the experiment was performed at a high elevation, the acceleration will be smaller and $g_{\exp }$ will be smaller.)
- Correctly indicating the functional dependence between the physical error in the measurement collection and $g_{\text {exp }}$ (e.g., if the angle of the ramp is smaller than the measured value, the cart will have a smaller acceleration and $g_{\exp }$ will be smaller.)


## Example Response

The expression I derived for the value for $g_{\exp }$ did not take into consideration that the wheels had any rotational inertia. If the wheels have rotational inertia and are rotating, the acceleration of the cart would be less than $g \sin \theta$, so the value of $g_{\exp }$ would be less than $9.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$.
(d) For sketching a concave up curve with an initially negative slope for the graph of position 1 point as a function of time

## For one of the following:

- Drawing a line with a positive slope and a negative vertical intercept for the $v$ vs. $t$ graph
- Drawing a $v$ vs. $t$ graph that is consistent with the $x$ vs. $t$ graph that shows acceleration


## Example Response




Scoring Note: The following are alternate example graphs with the points the response would earn.


Total for part (d) 2 points
Total for question 212 points

For the length of the arrow at $t=t_{2}$ being longer than the arrow at $t=t_{1}$

## Scoring Notes:

- A maximum of 1 point can be earned if extraneous unlabeled arrows are drawn.
- A maximum of 1 point can be earned if incorrect labeled forces are drawn.


## Example Response


(a)(ii) For an explanation that refers to the difference in the stretch length and indicates that the magnitude of the spring force is (or is not) related to the stretch length, consistent with the force diagram drawn in part (a)(i)

## Example Response

The spring force arrow drawn at $t=t_{2}$ is longer because the spring is stretched a greater distance at that time and the spring force is related to the stretch distance.
(a)(iii) For a correct selection with an attempt at a relevant justification, or a selection and
justification consistent with the response in part (a)(ii)
For indicating that the spring force is the net force $\mathbf{1}$ point
Scoring Note: Stating $F=k x$ earns this point.
For indicating that the net force is related to the speed (or acceleration) $\mathbf{1}$ point
Scoring Note: The relationship does not need to be defined to earn this point.

## Example Response

$$
\ldots v_{1}>v_{2} \quad \mathrm{X} v_{1}<v_{2} \quad v_{1}=v_{2}
$$

The net force is the spring force. When the spring is stretched a greater length, the spring force is greater, so the net force is greater, and therefore the tangential speed is greater at $t=t_{2}$.

Total for part (a) $\mathbf{6}$ points
(b)(i) For the correct answer: $F_{\text {net }}=k_{0} d \quad \mathbf{1}$ point

## Scoring Notes:

- An answer of $k x$ does not earn this point.
- Points for part (b)(i) may be earned if correct in (b)(ii).


## Example Response

$$
\begin{aligned}
& F_{\text {net }}=\Sigma F=F_{s} \\
& F_{\text {net }}=\Sigma F=k_{0} d
\end{aligned}
$$

(b)(ii) For a multistep derivation that begins with Newton's second law: $\Sigma F=m a \quad \mathbf{1}$ point

For one of the following:
1 point

- Substituting $k x$ for force into Newton's second law
- Substituting $\frac{v^{2}}{r}$ for acceleration into Newton's second law
- Substituting $(L+d)$ for the radius

For the consistent answer in terms of the given variables: $v=\sqrt{\frac{k_{0} d(L+d)}{m_{0}}}$

## Scoring Notes:

- Subscripts for $m$ and $k$ are not required to earn this point.
- Points in (b)(ii) can be earned if correct in (b)(i).


## Example Response

$$
\begin{aligned}
& \Sigma F=m a_{c} \\
& k x=\frac{m v^{2}}{r} \\
& k_{0} d=\frac{m_{0} v^{2}}{L+d} \\
& v=\sqrt{\frac{k_{0} d(L+d)}{m_{0}}}
\end{aligned}
$$

(c) For an answer that attempts to use functional dependence to relate the tangential speed with $\mathbf{1}$ point

Scoring Note: It is not necessary to use the functional dependence correctly to earn this point.

For a correct explanation for why the derived equation in part (b)(ii) does or does not $\mathbf{1}$ point support the reasoning in part (a)

## Example Response

My equation from part (b) (ii) agrees with my reasoning in part (a). The tangential speed of the block as it travels in a horizontal circle is related to the distance the spring is stretched. The greater the tangential speed of the block, the greater distance the spring is stretched. The equation shows this because the $d$ is in the numerator.

## Question 4: Short Answer Paragraph Argument

7 points
(a) For a correct expression for the angular acceleration of the pulley in terms of the appropriate $\mathbf{1}$ point quantities: $\alpha_{\mathrm{D}}=\frac{2 F_{T}}{M R}$

## Example Response

$$
\alpha_{\mathrm{D}}=\frac{R F_{T}}{\frac{1}{2} M R^{2}} \quad \text { OR } \quad \alpha_{\mathrm{D}}=\frac{2 F_{T}}{M R}
$$

(b) For indicating that the torque, $\tau$, is the same for both pulleys $\mathbf{1}$ point

For indicating that the impulse, $\tau \Delta t$, (or change in momentum $\Delta L$ ) is the same for both
1 point pulleys because $\tau$ and $\Delta t$ are the same

For indicating that the rotational inertia, $I$, of the disk and hoop are different $\mathbf{1}$ point
For providing reasoning that because the rotational inertia, $I$, are different for the disk and $\mathbf{1}$ point hoop, the kinematic quantities $(\Delta \theta, \omega, \alpha)$ are also different for the disk and hoop

For one of the following:
1 point

- Relating $I$ and $\omega$ to reason that $\Delta K$ is greater for the disk
- Indicating that because $\Delta \theta$ is greater for the disk the work done on the disk is greater

For a logical, relevant, and internally consistent argument that follows the guidelines described in the published requirements for the paragraph-length response

## Example Response

The rotational inertia, $I$, of the hoop is larger than the rotational inertia of the disk because the hoop's mass is all on the outside instead of distributed throughout like the disk. Equal forces are applied to both pulleys at the same distance, which means that the torques exerted on the pulleys will also be equal. Since the same torque is applied to both pulleys for the same time period, the change in angular momentum will be the same for the disk and hoop. The magnitude of the angular velocity for the hoop will be smaller than that of the disk since angular velocity is inversely proportional to the rotational inertia $\left(\omega=\frac{L}{I}\right)$. Since kinetic energy is proportional to rotational inertia and the square of angular velocity $\left(K_{R}=\frac{1}{2} I \omega^{2}\right)$, the difference in angular velocity more greatly affects the rotational kinetic energy. That means the disk will have a greater rotational kinetic energy than the hoop.
(a)(i) For indicating "Frame C" with correct reasoning about the magnitude of the torque being 1 point the greatest

Accept one of the following:

- This is the instant when the lever arm is greatest.
- This is when the angle between radius vector and weight force vector is most perpendicular.

For correctly relating torque and angular acceleration: $\alpha \propto \tau \quad \mathbf{1}$ point

## Example Response

The angular acceleration is greatest in Frame C because angular acceleration is proportional to torque, and in Frame C the gravitational force vector is directed perpendicular to the rod (lever arm) which means this is where the torque will be the greatest.
(a)(ii) For indicating "Frame E" with correct reasoning $\mathbf{1}$ point

Accept one of the following:

- Work or energy (e.g., this is when the maximum work has been done on the system by gravity.)
- Angular momentum (e.g., the torque due to gravity is clockwise the entire time, causing the rod to gain angular momentum.)
- Kinematics (e.g., the rod speeds up the entire time.)


## Example Response

The rotational kinetic energy is greatest in Frame E because this is where the rod-sphere system has the greatest rotational speed since the torque has been in the same direction as the motion the entire time.

## Total for part (a) $\mathbf{3}$ points

(b)(i) For a multistep derivation that begins with conservation of energy

1 point

$$
E_{i}=E_{f} \quad \text { OR } \quad \Delta E=0 \quad \text { OR } \quad U_{g i}+K_{i}=U_{g f}+K_{f}
$$

For indicating the change in height is equal to $\frac{3}{2} L$
1 point
$\Delta y=\frac{3}{2} L$

For an answer consistent with the height change indicated previously in the response
$K_{f}=\frac{3}{2} M g L$

Scoring Note: A correct answer of $K_{f}=\frac{3}{2} M g L$ with no supporting work can earn only this point.

## Example Response

$$
\begin{aligned}
& E_{i}=E_{f} \\
& U_{g i}+K_{i}=U_{g f}+K_{f} \\
& \Delta K=U_{g i}-U_{g f} \\
& \Delta K=M g \Delta y \\
& \Delta y=\frac{3 L}{4}+\frac{3 L}{4}=\frac{3}{2} L \\
& \Delta K=\frac{3}{2} M g L
\end{aligned}
$$

(b)(ii) For indicating that the gravitational force is the external force that does work on the 1 point rod-sphere system

## Example Response

The rod and sphere gain kinetic energy due to the positive work done by the gravitational force, which is an external force for the rod-sphere system.

