2023



AP[°] **Statistics** Sample Student Responses and Scoring Commentary

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Question 4: Focus on Inference

General Scoring Notes

- This question is scored in three sections. Each section is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The first section includes statements of the null and alternative hypotheses and identification of the appropriate hypothesis test. The second section includes verifying the conditions for the test identified in the first section and calculating the value of the test statistic and the corresponding *p*-value. The third section includes the conclusion for the test identified in the first section. The response is then categorized based on the scores assigned to each section and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each section of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

	Model Solution	Scoring
Section 1	Let μ_d represent the true mean difference (placebo minus omega-3) of irritability scores for all people with this medical condition. The null hypothesis is $H_0: \mu_d = 0$ and the alternative hypothesis is $H_a: \mu_d > 0$. The appropriate inference procedure is a matched pairs <i>t</i> -test for a mean difference.	 Essentially correct (E) if the response satisfies the following four components: 1. Identifies a paired <i>t</i>-test for a population mean difference by name or by formula 2. States the hypotheses using a single mean (e.g., μ_d, μ) 3. States the correct equality for the null hypothesis (e.g., H₀: μ_d = 0) AND states the correct direction for the alternative hypothesis (e.g., H_a: μ_d > 0) 4. Provides sufficient context for the parameter, by including reference to the <i>population</i> mean difference <i>AND</i> the sampling units (people with the medical condition) <i>AND</i> the response variable (irritability score)
		Partially correct (P) if the response satisfies three of the four components.Incorrect (I) if the response does not meet the criteria for E or P.

Additional Notes:

- A response that states the null hypothesis as $H_0: \mu_d \le 0$ may satisfy component 3.
- A response that states the name of the procedure as *t*-test, or one-sample *t*-test may satisfy component 1.
- To satisfy component 2, the hypotheses must be stated in terms of a mean. If a symbol other than μ or \overline{X} is used to denote the mean, it must be clearly defined as a mean (but does not need to reflect the context of irritability score). It is acceptable to use μ_0 to denote the mean.

- A response that states the hypotheses in words (e.g., "the null hypothesis is that the mean difference is 0, and the alternative hypothesis is that the mean difference is greater than 0") may satisfy components 2 and 3. Neither context nor the concept of the *population* is required to satisfy component 2 or 3.
- A response that states the hypotheses in words [e.g., "the null hypothesis is that the mean difference in irritability score (placebo minus omega-3) for all people with this medical condition is equal to 0 and the alternative hypothesis is that the mean is greater than 0"] may satisfy components 2,3, and 4.
- The elements of component 4 do not have to be satisfied with the statement of the hypotheses. They may be satisfied by work presented anywhere in the response, most likely by the statement of the conclusion.
- If the statement of the hypotheses refers to population mean and the conclusion refers to sample mean (or vice versa), then the population aspect of component 4 is not satisfied.
- If the response clearly refers to the *sample* mean instead of the *population* mean using words or a symbol (e.g., \bar{x}), then component 4 is not satisfied unless the symbol used is defined as the *population* mean.
- A response may satisfy the population aspect of component 4 by doing the following:
 - referring to population in the statement of the conclusion of the inferential procedure.
 - o using notation such as μ when defining the hypothesis statements.
- A response may satisfy the sampling units aspect of component 4 by referring to "patients" or "people with this medical condition" or a similar statement.
- If the response identifies the correct test by name, but also states an incorrect formula, then component 1 is not satisfied.
- If the response identifies the test by formula using a *z*-percentile instead of a *t*-percentile, then component 1 is not satisfied.

Confidence Interval Approach:

- If a one-sample *t*-interval for a population mean is identified correctly by name (e.g., "one-mean *t*-interval" or "one-sample *t*-interval" or "*t*-interval") or by formula, then component 1 is satisfied.
- If a response uses a one-sample *t*-interval for a population mean, then components 2 and 3 are satisfied if the response indicates that it is a confidence interval for the mean difference in irritability score (placebo minus omega-3).

Two-Sample Approach:

• A response that clearly defines the *population* difference in means with clearly defined parameters for the means of placebo and omega-3 *AND* the sampling units (people with the medical condition or people similar to those in the study) *AND* the response variable (irritability score) may earn credit for component 4.

	Model Solution	Scoring
Section 2	The independence condition for performing a paired <i>t</i> -test for a mean difference is satisfied because the data were obtained from a randomized experiment where the week in which the patient received the treatment was randomly assigned. The sampling distribution of the mean difference must be approximately normal. Although the sample size is less than 30 (n = 19), this is satisfied because the boxplot for the sample differences shows an approximately symmetric distribution with no outliers.	 Essentially correct (E) if the response satisfies the following four components: 1. Checks the independence condition by referring to the random assignment 2. Indicates that the distribution of irritability scores for the differences (placebo minus omega-3) is not badly skewed supports the assumption that the sampling distribution of x and is approximately normal 3. Correctly reports the value of the <i>t</i>-statistic consistent with the named test 4. Correctly reports the <i>p</i>-value, consistent with the stated alternative hypothesis and reported test statistic
	The value of the test statistic is:	Partially correct (P) if the response satisfies only

$$t = \frac{\overline{x_d - \mu_0}}{\frac{s}{\sqrt{n}}} = \frac{1.789 - 0}{\frac{2.485}{\sqrt{19}}} \approx 3.138$$

Using 18 degrees of freedom, the corresponding *p*-value is $P(t > 3.138) \approx 0.0028.$

Partially correct (P) if the response satisfies only two or three of the four components.

Incorrect (I) if the response does not meet the criteria for E or P.

Additional Notes:

- Component 1 is not satisfied if the response indicates that the independence condition is met because the sample was selected at random from all patients with the medical condition.
- Component 1 is satisfied if the response indicates that the treatment was randomly assigned even if the terms "independent" or "independence" are not used.
- Discussion of degrees of freedom will be treated as extraneous in scoring for section 2.
- If the response incorrectly identifies the test as a *z*-test in section 1, the correct *z*-statistic of 3.138 satisfies component 3 and a *p*-value of 0.00085 satisfies component 4. However, if the test statistic is not reported, component 3 is not satisfied, but if the correct *p*-value (consistent with the alternative hypothesis) is reported then component 4 is satisfied.
- If the response does not identify a z-test in section 1 but reports the test statistic as z = 3.138 (instead of *t*) then component 3 is not satisfied; however, component 4 may still be satisfied if the correct *p*-value of 0.00085 is reported with a z-statistic of 3.138.
- If the response compares the value of the test statistic to a critical value instead of computing a *p*-value, then a comparison consistent with the stated alternative hypothesis, satisfies component 4.
- If a two-tailed alternative hypothesis is stated, then the *p*-value must be consistent with the stated alternative hypothesis to satisfy component 4.
- A response that reports the correct value for the *t*-statistic but contains errors in supporting work may still satisfy component 3.
- If the response satisfies component 4, any supporting work for the *p*-value may be treated as extraneous.

If no test statistic is reported, and the *p*-value is consistent with the stated alternative hypothesis or equal to 0.0028 component 4 is satisfied.

Confidence Interval Approach:

- If the stated alternative hypothesis is correct or no alternative hypothesis is provided:
 - If either a one-sided 95 percent confidence interval for μ_d is correctly calculated as $(0.8004, \infty)$ or a two-sided 90 percent confidence interval for μ_d is correctly calculated as (0.8004, 2.7776) then component 3 is satisfied.
 - If only the lower end of the confidence interval for μ_d is used to reach a conclusion, then component 0 4 is satisfied.
- If the stated alternative hypothesis is incorrect (two-sided or reversed direction), the confidence interval approach must be consistent with the stated alternative to satisfy components 3 and 4:
 - o An interval consistent with the stated two-sided alternative will satisfy component 3. A two-sided 95 percent confidence interval for μ_d is (0.591, 2.987). If the two-sided confidence interval is correctly interpreted based on whether zero is in the interval, then component 4 is satisfied.
 - An interval consistent with the incorrect reversed alternative will satisfy component 3. If either a two-0 sided 90 percent confidence interval for μ_d is correctly calculated as (0.8004, 2.7776), or a onesided 95 percent confidence interval for μ_d is correctly calculated as ($-\infty$, 2.7776), then component 3 is satisfied. If the two-sided confidence interval is correctly interpreted based on whether zero is in the interval, then component 4 is satisfied; if only the upper end of the lower one-sided confidence interval is used to reach a conclusion, then component 4 is satisfied.
- If the difference calculated is μ_d , omega-3 minus placebo, the intervals will be the negative of those ٠ provided above.

Two-Sample Approach:

- A response that indicates that the distribution of irritability scores for each of the two samples, placebo AND omega-3, are not badly skewed may satisfy component 2.
- If the response correctly reports the value of the *t*-statistic as 2.256, then the response may satisfy • component 3.
- Values for the *p*-value depend on how the degrees of freedom were determined. The following *p*-values all • assume the correct alternative hypothesis and satisfy component 4:
 - The degrees of freedom for $t \approx 2.256$ on a two-sample solution computed with the Satterthwaite 0 approximation used by many calculators is:

$$df = \frac{\left(\frac{s_{placebo}}{n_{placebo}} + \frac{s_{omega-3}^{2}}{n_{omega-3}}\right)^{2}}{\frac{1}{n_{placebo}} - 1\left(\frac{s_{placebo}}{n_{placebo}}\right)^{2} + \frac{1}{n_{omega-3}} - 1\left(\frac{s_{omega-3}^{2}}{n_{omega-3}}\right)^{2}} = \frac{\left(\frac{(2.987)^{2}}{19} + \frac{(1.739)^{2}}{19}\right)^{2}}{\frac{1}{19 - 1}\left(\frac{(2.987)^{2}}{19}\right)^{2} + \frac{1}{19 - 1}\left(\frac{(1.739)^{2}}{19}\right)^{2}} \approx 28.94$$

≈ 28.94,

resulting in a *p*-value of 0.0159 or a *p*-value of 0.0318 if a two-sided alternative hypothesis was stated. Using $t \approx 2.256$ and (19 + 19 - 2) = 36 degrees of freedom, the resulting *p*-value is 0.0151 or a

p-value of 0.0302 if a two-sided alternative hypothesis was stated.

- Using $t \approx 2.256$ and (19 1) = 18 degrees of freedom, the resulting *p*-value is 0.0184 or a *p*-value of 0.0368 if a two-sided alternative hypothesis was stated.
- If no test statistic is reported, and the *p*-value is consistent with the stated alternative hypothesis or equal to 0.0159 or 0.0151 or 0.0184, component 4 is satisfied.

	Model Solution	Scoring
Section 3	Because the <i>p</i> -value ≈ 0.0028 is less than the significance level, $\alpha = 0.05$, the null hypothesis should be rejected. The data provide convincing statistical evidence that for patients similar to those in the study, the true mean difference (placebo minus omega-3) in irritability scores for people with this medical condition is greater than zero. This suggests the omega-3 fatty acids are helpful in reducing irritability scores in people with this medical condition.	 Essentially correct (E) if the response satisfies the following two components: Provides correct comparison of the <i>p</i>-value to alpha (<i>p</i>-value is less than/greater than alpha) <i>AND</i> provides a correct decision about the null and/or alternative hypothesis States a conclusion in context, consistent with, and in terms of, the alternative hypothesis using non-deterministic language Partially correct (P) if the response satisfies only one of the two components. Incorrect (I) if the response does not meet the criteria for E or P

Additional Notes:

- In order to satisfy component 1, the response must clearly identify the number that is compared to alpha as a *p*-value.
- If the response provides an unreasonable *p*-value (that is clearly identified as the *p*-value) and correctly compares it to alpha, component 1 may be satisfied.
- To satisfy the *p*-value comparison in component 1, the response can compare the value of the test statistic to an appropriate critical value, e.g., $t_{\alpha} = 1.734$, and 3.13 > 1.734.
- An explicit decision is not required to satisfy component 1.
- If an explicit decision is stated and the conclusion is inconsistent with the decision, component 1 is not satisfied.
- The decision part of component 1 may be satisfied by implying the decision within the conclusion statement (sufficient evidence/insufficient evidence for the alternative hypothesis).
- To satisfy the context in component 2, the response must include the population parameter, sampling/experimental units, and the response variable.
- If the response omits hypotheses, assume the correct alternative hypothesis is provided when scoring component 2.
- If the response states incorrect hypotheses, component 2 may be satisfied by either stating a conclusion in terms of the stated alternative hypothesis or by answering the inference question.
- Examples of non-deterministic language in component 2 include "evidence to accept the alternative," "there is evidence for the alternative," "there is not sufficient evidence for the alternative."
- Examples of deterministic language in component 2 include "proves the null," "proves the alternative," "accepts the alternative," "there is not evidence for the alternative," and "no evidence for the alternative."
- If the comparison and decision are consistent with an incorrect *p*-value (or an incorrect value of the test statistic, or an incorrect confidence interval), the response may satisfy component 1.
- If components 1 and/or 2 are satisfied and the response provides an incorrect interpretation of the *p*-value, the score is lowered from E to P or P to I.
- In section 1, if component 4 is not satisfied only because of the omission of sampling units (patients), in section 3 component 2 may be satisfied by an appropriate conclusion that is only missing sampling units.

Confidence Interval Approach:

- Component 2 should be scored according to the rubric and component 1 should be scored with regard to a comparison of zero to the appropriate end of the reported confidence interval.
- If no alternative hypothesis is specified in the response, then assume the correct alternative hypothesis is provided when scoring component 2.
- If an incorrect two-sided alternative hypothesis is specified, then component 2 is satisfied if the justification is based on whether zero is included in the confidence interval.
- If the response includes an incorrect interpretation of the confidence interval, then the score for section 3 is lowered from E to P or from P to I.

Two-Sample Approach:

• Components should be scored according to the rubric using the reported *p*-value, or by comparing the *t*-statistic to an appropriate critical value.

AP® Statistics 2023 Scoring Guidelines

Scoring for Question 4	Score
Complete Response Three sections essentially correct	4
Substantial Response Two sections essentially correct and one section partially correct	3
Developing Response Two sections essentially correct and no section partially correct OR One section essentially correct and one or two sections partially correct OR Three sections partially correct	2
Minimal Response One section essentially correct and no section partially correct OR No section essentially correct and two sections partially correct	1

Sample 4A

Question 4

Begin your response to QUESTION 4 on this page.

4. A medical researcher completed a study comparing an omega-3 fatty acids supplement to a placebo in the treatment of irritability in patients with a certain medical condition. Nineteen patients with the medical condition volunteered to participate in the study. The study was conducted using the following weekly schedule.

- Week 1: Each patient took a randomly assigned treatment, omega-3 supplement or placebo.
- <u>Week 2</u>: The patients did not take either the omega-3 supplement or the placebo. This was necessary to reduce the possibility of any carryover effect from the assigned treatment taken during week 1.
- Week 3: Each patient took the treatment, omega-3 supplement or placebo, that they did not take during week 1.

At the end of week 1 and week 3, each patient's irritability was given a score on a scale of 0 to 10, with 0 representing no irritability and 10 representing the highest level of irritability.

For each patient, the two irritability scores and the difference in their scores (placebo minus omega-3) were recorded. The results are summarized in the table and boxplots.



Sample 4A



Question 4

Begin your response to **QUESTION 4** on this page.

4. A medical researcher completed a study comparing an omega-3 fatty acids supplement to a placebo in the treatment of irritability in patients with a certain medical condition. Nineteen patients with the medical condition volunteered to participate in the study. The study was conducted using the following weekly schedule.

- Week 1: Each patient took a randomly assigned treatment, omega-3 supplement or placebo.
- Week 2: The patients did not take either the omega-3 supplement or the placebo. This was necessary to reduce the possibility of any carryover effect from the assigned treatment taken during week 1.
- Week 3: Each patient took the treatment, omega-3 supplement or placebo, that they did not take during week 1.

At the end of week 1 and week 3, each patient's irritability was given a score on a scale of 0 to 10, with 0 representing no irritability and 10 representing the highest level of irritability.

For each patient, the two irritability scores and the difference in their scores (placebo minus omega-3) were recorded. The results are summarized in the table and boxplots. mean difference -



Sample 4B



Continue your response to QUESTION 4 on this page.

The researcher claims the omega-3 supplement will decrease the mean irritability score of all patients with the medical condition similar to the volunteers who participated in the study. Is there convincing statistical evidence to support the researcher's claim at a significance level of $\alpha = 0.05$? Complete the appropriate inference procedure to support your answer. let lo be the true mean difference in irratibility score for the placebo minus the omega-3 treatment for all patients with a certain medical condition similar to the volunteers in this study We will conduct as t-matched poir test for one sample at the & = . 05 significance level Normal ! the inrability score for each treatment is approximately normal and symmetric with no apparent outliers. Independent : each individual can be assumed independent Rondom ! the treatments were randomly assigned to the subjects. Ho: 10=0 0(F = 18. p-value = .003. Ha: 10>0 $\frac{\bar{x} - M_0}{\frac{\bar{x}}{\bar{x}}} = 3.139$ test t = We Reject the Null hypothesis. Be cause the p-value .003 < Our alpha confidence level .05 there is significant evidence to support that the Omega - 3 treatment decreases The mean imphility score for patients with the medical condition similar to the volunteers, whe study, Unauthorized copying or reuse of this page is illegal. Page 9 GO ON TO THE NEXT PAGE Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box. O6214/0

Question 4

Begin your response to QUESTION 4 on this page.

4. A medical researcher completed a study comparing an omega-3 fatty acids supplement to a placebo in the treatment of irritability in patients with a certain medical condition. Nineteen patients with the medical condition volunteered to participate in the study. The study was conducted using the following weekly schedule.

- Week 1: Each patient took a randomly assigned treatment, omega-3 supplement or placebo.
- Week 2: The patients did not take either the omega-3 supplement or the placebo. This was necessary to reduce the possibility of any carryover effect from the assigned treatment taken during week 1.
- Week 3: Each patient took the treatment, omega-3 supplement or placebo, that they did not take during week 1.

At the end of week 1 and week 3, each patient's irritability was given a score on a scale of 0 to 10, with 0 representing no irritability and 10 representing the highest level of irritability.

For each patient, the two irritability scores and the difference in their scores (placebo minus omega-3) were recorded. The results are summarized in the table and boxplots.



Sample 4C

Question 4

Continue your response to QUESTION 4 on this page.

The researcher claims the omega-3 supplement will decrease the mean irritability score of all patients with the medical condition similar to the volunteers who participated in the study. Is there convincing statistical evidence to support the researcher's claim at a significance level of $\alpha = 0.05$? Complete the appropriate inference procedure to support your answer.

I) Md = the true difference of menses between the irratibility score of the placebor and Omega - 3 taken by the patients (P-0) Matched poirs & test for (P-0) H1: M1=0 Ha: MI #0 () Random independent sample 19 < 107, of all patients 19 x 30, data already plotted F) I sample t test () t= 3.1381 p=,0057 I) since the p value of .0057 is less than d = .05 we reject the pull. There is sufficient evidence to conclude the omegan 3 supplement will decrease the mean irritability score of all porthents with the medical condition similar to the volunteers who participated in the study Unauthorized copying or reuse of this page is illegal. Page 9 GO ON TO THE NEXT PAGE. Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box. 05214/9

Question 4

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The primary goals of the question were to assess a student's ability to (1) identify an appropriate procedure for conducting a hypothesis test for paired data; (2) identify the correct hypotheses for conducting a paired *t*-test of a mean difference; (3) check the conditions for the hypothesis test for paired data; (4) calculate the test statistic and *p*-value for a paired *t*-test of a mean difference; (5) compare the *p*-value to a significance level to make a decision regarding the hypotheses; and (6) determine an appropriate conclusion for a hypothesis test for paired data.

This question primarily assesses skills in skill category 1: Selecting Statistical Methods, skill category 3: Using Probability and Simulation, and skill category 4: Statistical Argumentation. Skills required for responding to this question include (1.E) Identify an appropriate inference method for significance tests, (1.F) Identify null and alternative hypotheses, (3.E) Calculate a test statistic and find a *p*-value, provided conditions for inference are met, (4.C) Verify that inference procedures apply in a given situation, and (4.E) Justify a claim using a decision based on significance tests.

This question covers content from Unit 7: Inference for Quantitative Data: Means of the course framework in the AP Statistics Course and Exam Description. Refer to topics 7.4 and 7.5 and learning objectives VAR-7.B, VAR-7.C, VAR-7.D, VAR-7.E, and DAT-3.F.

Sample: 4A Score: 4

The response earned the following: Section 1 - E; Section 2 - E; Section 3 - E.

In section 1 the response identifies the "*t*-test for paired data" as the appropriate procedure, satisfying component 1. The response states the hypotheses using a single mean, satisfying component 2. The response correctly states the correct equality in the null hypothesis and states the correct direction for the alternative hypothesis, satisfying component 3. The response provides all required parts of context in the definition of " μ_{diff} ," satisfying component 4. All four components are satisfied. Section 1 was scored essentially correct (E). In section 2 the response indicates that treatments were randomly assigned, satisfying component 1. The response refers to the boxplot of the differences and states that the boxplot shows no strong skew or outliers, satisfying component 2. The response reports the correct test statistic value and *p*-value, satisfying components 3 and 4. All four components are satisfied. Section 2 was scored essentially correct (E). In section 3 the response correctly compares the *p*-value to α and provides the correct decision, satisfying component 1. The response states a conclusion in terms of, and consistent with, the alternative hypothesis, with non-deterministic language, in context, satisfying component 2. Both components are satisfied. Section 3 was scored essentially correct (E).

Sample: 4B Score: 3

The response earned the following: Section 1 - E; Section 2 - P; Section 3 - E.

In section 1 the response identifies the "*t*-matched pair test for one sample" as the appropriate procedure, satisfying component 1. The response states the hypotheses using a single mean, satisfying component 2. The response states the correct equality for the null hypothesis and states the correct direction for the alternative hypothesis, satisfying component 3. The response provides all required parts of context in the definition of μ_0 , satisfying component 4. All four components are satisfied. Section 1 was scored essentially correct (E). In section 2 the response indicates that treatments were randomly assigned, satisfying component 1. The response addresses the symmetry of the scores for each treatment rather than the symmetry of the difference in scores, additionally, it

Question 4 (continued)

states that the distributions of scores are approximately normal, which cannot be determined from the information provided; thus, component 2 is not satisfied. The response reports the correct test statistic value and *p*-value, satisfying components 3 and 4. Three of four components are satisfied. Section 2 was scored partially correct (P). In section 3 the response correctly compares the *p*-value to α and provides the correct decision, satisfying component 1. The response states a conclusion in terms of, and consistent with, the alternative hypothesis, with non-deterministic language, in context, satisfying component 2. Both components are satisfied. Section 3 was scored essentially correct (E).

Sample: 4C Score: 2

The response earned the following: Section 1 - I; Section 2 - P; Section 3 - E.

In section 1 the response identifies the "matched pairs t test" as the appropriate procedure, satisfying component 1. The response states the hypotheses using a single mean, satisfying component 2. The response states the correct equality for the null hypothesis but states a two-sided alternative hypothesis; thus, component 3 is not satisfied. The response defines μ_d as the true difference in means rather than the true mean difference; thus, component 4 is not satisfied. Two of the four components are satisfied. Section 1 was scored incorrect (I). In section 2 the response does not report that treatments were randomly assigned; thus, component 1 is not satisfied. Near the boxplot of differences, the response notes that there is no severe skew or outliers, satisfying component 2. The response reports the correct test statistic value and the *p*-value consistent with a two-sided alternative hypothesis, satisfying components 3 and 4. Three of the four components are satisfied. Section 2 was scored partially correct (P). In Section 3 the response correctly compares the *p*-value to α and provides the correct decision, satisfying component 1. The response states a conclusion in terms of, and consistent with, the alternative hypothesis with non-deterministic language, in context, satisfying component 2. Both components are satisfied. Section 3 was scored essentially correct (E).