# AP Physics C: Mechanics 

 Sample Student Responses and Scoring Commentary Set 2
## Inside:

Free-Response Question 1
$\checkmark$ Scoring Guidelines
$\checkmark$ Student Samples
$\checkmark$ Scoring Commentary
(a)(i) For stating that the area bounded by the curve is the displacement $\mathbf{1}$ point
(a)(ii) For indicating momentum is conserved 1 point

## Example Response

$\Sigma p_{0}=\Sigma p_{f}$
$m_{\mathrm{A}} v_{\mathrm{A} 0}+m_{\mathrm{B}} v_{\mathrm{B} 0}=m_{\mathrm{A}} v_{\mathrm{Af}}+m_{\mathrm{B}} v_{\mathrm{Bf}}$
$m_{\mathrm{A}} v_{\mathrm{A} 0}-m_{\mathrm{A}} v_{\mathrm{Af}}=m_{\mathrm{B}} v_{\mathrm{Bf}}$
For the correct answer for speed with units ( $2 \mathrm{~m} / \mathrm{s}$ )

## Example Solution

$$
\begin{aligned}
& m_{\mathrm{A}} v_{\mathrm{A} 0}+m_{\mathrm{B}} v_{\mathrm{B} 0}=m_{\mathrm{A}} v_{\mathrm{Af}}+m_{\mathrm{B}} v_{\mathrm{Bf}} \\
& m_{\mathrm{A}} v_{\mathrm{A} 0}-m_{\mathrm{A}} v_{\mathrm{Af}}=m_{\mathrm{B}} v_{\mathrm{Bf}} \\
& v_{\mathrm{Bf}}=\frac{m_{\mathrm{A}}}{m_{\mathrm{B}}}\left(v_{\mathrm{A} 0}-v_{\mathrm{Af}}\right) \\
& \rightarrow v_{\mathrm{Bf}}=\frac{1000 \mathrm{~kg}}{2000 \mathrm{~kg}}(5 \mathrm{~m} / \mathrm{s}-1 \mathrm{~m} / \mathrm{s}) \\
& \therefore v_{\mathrm{Bf}}=2 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(a)(iii) For a graph that starts at $(0,0)$ and ends at $(0.5,2)$ or value consistent with (a)(ii) $\mathbf{1}$ point

For a smooth, continuous curve that transitions from concave up to concave down

## Example Solution



## Example Response

$a(t)=\frac{d v}{d t}$
$\rightarrow a(t)=\frac{d\left(64 t^{3}-48 t^{2}+5\right)}{d t}$
$\therefore a(t)=192 t^{2}-96 t$
For setting the derivative of the previously derived expression for acceleration equal to zero $\mathbf{1}$ point

## Example Response

$\frac{d}{d t} a(t)=0$
$0=384 t-96$
For indicating the maximum acceleration is $12 \mathrm{~m} / \mathrm{s}^{2}$ or maximum acceleration occurs at time $\mathbf{1}$ point $t=0.25 \mathrm{~s}$

## Example Response

$0=384 t-96$
$\therefore t_{\text {max }}=0.25 \mathrm{~s}$
For substituting the time at which the acceleration is a maximum or the maximum
acceleration into an expression of Newton's second law to calculate the value of the maximum force

## Example Response

$F(t)=m a(t)$
$F_{\text {max }}=m a(0.25 \mathrm{~s})$

## Example Solution

$$
\begin{aligned}
& a(t)=\frac{d v}{d t} \\
& \rightarrow a(t)=\frac{d\left(64 t^{3}-48 t^{2}+5\right)}{d t} \\
& \therefore a(t)=192 t^{2}-96 t \\
& \frac{d}{d t} a(t)=0 \\
& \frac{d\left(192 t^{2}-96 t\right)}{d t}=0 \\
& \rightarrow 0=384 t-96 \\
& \therefore t_{\max }=0.25 \mathrm{~s} \\
& F(t)=m a(t) \\
& F_{\max }=m a(0.25 \mathrm{~s}) \\
& F_{\max }=(1000 \mathrm{~kg})\left(192(0.25 \mathrm{~s})^{2}-96(0.25 \mathrm{~s})\right) \\
& \therefore\left|F_{\max }\right|=12,000 \mathrm{~N}
\end{aligned}
$$

## Alternate Solution

For indicating the derivative of the momentum function is the force exerted on the cart
Alternate Example Response
$F(t)=\frac{d p}{d t}$
$F(t)=\frac{d}{d t}\left(64000 t^{3}-48000 t^{2}+5000\right)$
For setting the derivative of the previously derived expression for force equal to zero 1 point
Alternate Example Response
$\frac{d}{d t} F(t)=0$
$\frac{d}{d t}\left(192000 t^{2}-96000 t\right)=0$
For indicating the maximum force occurs at time $t=0.25 \mathrm{~s}$

## Alternate Example Response

$0=384000 t-96000$
$\therefore t_{\text {max }}=0.25 \mathrm{~s}$
For substituting the time at which the force is at a maximum into an expression for momentum to calculate the value of the maximum force
Alternate Example Response
$F(t)=\frac{d}{d t} p(t)$
$F_{\max }=\frac{d}{d t} p(0.25 \mathrm{~s})$

## Alternate Example Solution

$$
\begin{aligned}
& p(t)=m v(t) \\
& \rightarrow p(t)=(1000 \mathrm{~kg})\left(64 t^{3}-48 t^{2}+5\right) \\
& \therefore p(t)=64000 t^{3}-48000 t^{2}+5000 \\
& F(t)=\frac{d p}{d t} \\
& \rightarrow F(t)=\frac{d}{d t}\left(64000 t^{3}-48000 t^{2}+5000\right) \\
& \therefore F(t)=192000 t^{2}-96000 t \\
& \frac{d}{d t} F(t)=0 \\
& \frac{d}{d t}\left(192000 t^{2}-96000 t\right)=0 \\
& \rightarrow 0=384000 t-96000 \\
& \therefore t_{\max }=0.25 \mathrm{~s} \\
& F(t)=\frac{d}{d t} p(t) \\
& F_{\max }=\frac{d}{d t} p(0.25 \mathrm{~s}) \\
& F_{\max }=192000(0.25 \mathrm{~s})^{2}-96000(0.25 \mathrm{~s}) \\
& \therefore\left|F_{\max }\right|=12,000 \mathrm{~N} \\
& \hline
\end{aligned}
$$

(b)(ii) For a curve that increases and then decreases in value that is only concave down

For a labeled maximum value consistent with the calculated value in part (b)(i)
For the graph having values of 0 N at $t=0 \mathrm{~s}$ and $t=0.5 \mathrm{~s}$

## Example Solution



|  | Total for part (b) | $\mathbf{7}$ points |
| :--- | ---: | ---: |
| (c) | For selecting $F_{1}<F_{2}$ with an attempt at a relevant justification <br> For indicating that the impulse or change in momentum of each cart in both collisions is the <br> same | $\mathbf{1}$ point |
| For indicating that decreasing the time of collision means the average force must be greater | $\mathbf{1}$ point |  |
| Example Solution |  |  |
| Since the initial and final velocities are the same for both collisions, $\Delta p$ is the same for both <br> collisions; as a result, the impulse is the same for both collisions. So, if $\Delta t$ is smaller, $F_{\text {avg }}$ <br> is larger. |  |  |
|  | Total for part (c) | $\mathbf{3}$ points |

Segin your response to QUESTION 1 on this page.

## Question 1

## Continue your response to QUESTION 1 on this page.


(a) The graph shows the velocity $v$ of Cart A as a function of time $t$ for the time interval when the foam and Cart B are in contact.
i. What feature(s) of the graph could be used to estimate the displacement of Cart A during the collision? The area under the velocity curve
ii. Using the information shown in the graph, determine the speed of Cart B at $t=0.5 \mathrm{~s}$.

$$
\begin{aligned}
& m_{A} V_{A_{0}}=m_{A} V_{A_{f}}+m_{B} V_{B} \\
& N_{B}=\frac{m_{A}\left(V_{a_{0}}-V_{A f}\right)}{m_{B}}=\frac{1000 \mathrm{~kg}(5 \mathrm{mls}-1 \mathrm{~m} / \mathrm{s})}{2000 \mathrm{~kg}}=2 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

iii. On the following grid, draw a smooth curve of the velocity of Cart B as a function of time.


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Page 3
GO ON TO THE NEXT PAGE.
Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.


PC M Q1 Sample 1A Page 4 of 4
Question 1

Continue your response to QUESTION 1 on this page.
The foam is removed from the front of Cart A and the experiment is repeated. The carts collide, with both Cart A and Cart B having the same initial and final velocities as in the original collision. The time intervals during which the carts are in contact are different in the collision with the foam and the collision without the foam. In the collision without the foam, Cart A is in contact with Cart B for a shorter duration than in the original collision, when the foam was present.

For the original collision when the foam is present, the magnitude of the average net force exerted on Cart B is $F_{1}$. For the collision without the foam, the magnitude of the average net force exerted on Cart B is $F_{2}$.
(c) What is the relationship between the magnitude of the average net force $F_{1}$ exerted on Cart B for the collision with the foam and the magnitude of the average net force $F_{2}$ exerted on Cart B for the collision without the foam?
$\qquad$ $F_{1}>F_{2}$ $\qquad$ $F_{1}<F_{2}$ $\qquad$ $F_{1}=F_{2}$
Justify your answer.
The change in momentum between the carts is the same which means the impulse is the same. The same impulse over a shorter amount of time indicates a larger force. Hence, the shorter time frame of the second collision means a greater force.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

## PC M Q1 Sample 1B Page 2 of 4

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## Question 1

Continue your response to QUESTION 1 on this page.

(a) The graph shows the velocity $v$ of Cart A as a function of time $t$ for the time interval when the foam and Cart B are in contact.
i. What features) of the graph could be used to estimate the displacement of Cart A during the collision? integral or area under the curve of velocity for a specific period of time
ii. Using the information shown in the graph, determine the speed of Cart B at $t=0.5 \mathrm{~s}$.

$$
2 \mathrm{~m} / \mathrm{s}
$$

iii. On the following grid, draw a smooth curve of the velocity of Cart B as a function of time.


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Page 3
GO ON TO THE NEXT PAGE.
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PC M Q1 Sample 1B Page 4 of 4
Question 1

Continue your response to QUESTION 1 on this page.
The foam is removed from the front of Cart A and the experiment is repeated. The carts collide, with both Cart A and Cart B having the same initial and final velocities as in the original collision. The time intervals during which the carts are in contact are different in the collision with the foam and the collision without the foam. In the collision without the foam, Cart A is in contact with Cart B for a shorter duration than in the original collision, when the foam was present.

For the original collision when the foam is present, the magnitude of the average net force exerted on Cart B is $F_{1}$. For the collision without the foam, the magnitude of the average net force exerted on Cart B is $F_{2}$.
(c) What is the relationship between the magnitude of the average net force $F_{1}$ exerted on Cart B for the collision with the foam and the magnitude of the average net force $F_{2}$ exerted on Cart B for the collision without the foam?
$\qquad$ $F_{1}>F_{2}$ $\qquad$ $F_{1}<F_{2}$
 $F_{1}=F_{2}$
Justify your answer.
Force is mass times acceleration and "is nut affected by time. Since the carts still have the same velocities, the force must have also remained the same.

## PC M Q1 Sample 1C Page 2 of 4

## Question 1

Continue your response to QUESTION 1 on this page.

(a) The graph shows the velocity $v$ of Cart A as a function of time $t$ for the time interval when the foam and Cart B are in contact.
i. What features) of the graph could be used to estimate the displacement of Cart A during the collision? area under the curve could be used
ii. Using the information shown in the graph, determine the speed of Cart B at $t=0.5 \mathrm{~s}$.

$$
A M y / 2 \mathrm{~m} / \mathrm{s}
$$

sort A deacesed

cont $B$ th is rice re moss
iii. On the following grid, draw a smooth curve of the velocity of Cart B as a function of time.


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Page 3
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## PC M Q1 Sample 1C Page 3 of 4

= Continue your response to QUESTION 1 on this page.

## Question 1

$$
\text { Continue your response to QUESTION } 1 \text { on this page. }
$$

The foam is removed from the front of Cart A and the experiment is repeated. The carts collide, with both Cart A and Cart B having the same initial and final velocities as in the original collision. The time intervals during which the carts are in contact are different in the collision with the foam and the collision without the foam. In the collision without the foam, Cart A is in contact with Cart B for a shorter duration than in the original collision, when the foam was present.

For the original collision when the foam is present, the magnitude of the average net force exerted on Cart B is $F_{1}$. For the collision without the foam, the magnitude of the average net force exerted on Cart B is $F_{2}$.
(c) What is the relationship between the magnitude of the average net force $F_{1}$ exerted on Cart B for the collision with the foam and the magnitude of the average net force $F_{2}$ exerted on Cart B for the collision without the foam?


Justify your answer.


## Question 1

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

## Overview

The responses were expected to demonstrate the ability to:

- Identify properties of a graph related to kinematics.
- Use conservation of momentum to solve for the post-collision speed of an object.
- Graph the change in speed versus time of an object during a collision.
- Use calculus to determine the time at which the maximum acceleration occurs during a collision and calculate the maximum force during the collision.
- Sketch the change in force versus time applied to an object during a collision.
- Relate change in momentum and impulse and address the functional dependence between force applied to an object and time.


## Sample: 1A

## Score: 15

Part (a)(i) earned 1 point because the response clearly states that the area under the curve could be used to estimate the displacement. Part (a)(ii) earned 2 points. The first point was earned because the response indicates that linear momentum is conserved. The second point was earned because the response calculates the correct answer with units ( $2 \mathrm{~m} / \mathrm{s}$ ). Part (a)(iii) earned 2 points. The first point was earned because the response includes a graph that begins at $(0,0)$ and ends at $(0.5,2)$. The second point was earned because the graph drawn in the response is a smooth, continuous curve that transitions from concave up to concave down. Part (b)(i) earned 4 points. The first point was earned because the response correctly indicates that the derivative of the velocity function is the acceleration function: " $a(t)=v^{\prime}(t)=192 t^{2}-96 t$ ". The second point was earned because the response clearly sets the derivative of the acceleration function equal to zero: " $a^{\prime}(t)=384 t-96=0$ ". The third point was earned because the response clearly indicates the maximum acceleration occurs at $t=0.25 \mathrm{~s}$. The fourth point was earned because the response correctly determines the maximum acceleration and substitutes the value into a statement of Newton's second law: " $F=1000 \mathrm{~kg}\left(-12 \mathrm{~m} / \mathrm{s}^{2}\right)$ ". Part (b)(ii) earned 3 points. The first point was earned because the graph in the response has the correct shape: the curve increases then decreases in value and is only concave down. The second point was earned because the graph in the response includes a labeled maximum value on the curve that is consistent with part (b)(i). The third point was earned because the graph in the response has a value of 0 N at $t=0 \mathrm{~s}$ and at $t=0.5 \mathrm{~s}$. Part (c) earned 3 points. The first point was earned because the response correctly selects $F_{1}<F_{2}$ and includes an attempt at a relevant justification. The second point was earned because the response correctly states the change in momentum and the impulse of each cart is the same: "The change in momentum between the carts is the same which means the impulse is the same." The third point was earned because the response correctly states that a shorter collision time means a greater force: "The same impulse over a shorter amount of time indicates a larger force. Hence, the shorter time frame of the second collision means a greater force."

## Question 1 (continued)

## Sample: 1B Score: 8

Part (a)(i) earned 1 point because the response clearly states that the area under the curve could be used to estimate the displacement: "integral or area under the curve of velocity for a specific period of time." Part (a)(ii) earned 1 point. The first point was not earned because the response does not indicate that linear momentum is conserved. The second point was earned because the response states the correct answer with units ( $2 \mathrm{~m} / \mathrm{s}$ ). Part (a)(iii) earned 2 points. The first point was earned because the response includes a graph that begins at $(0,0)$ and ends at $(0.5,2)$. The second point was earned because the graph drawn in the response is a smooth, continuous curve that transitions from concave up to concave down. Part (b)(i) earned 3 points. The first point was earned because the response correctly indicates that the derivative of the velocity function is the acceleration function: " $a(t)=192 t^{2}-96 t$ ". The second point was not earned because the response does not set the derivative of the acceleration function equal to zero; the response sets the acceleration equal to zero. The third point was earned because the response clearly indicates the maximum acceleration occurs at $t=0.25 \mathrm{~s}$. The fourth point was earned because the response correctly determines the maximum acceleration and substitutes the value into a statement of Newton's second law: " $F=1000(12)$ ". Part (b)(ii) earned 1 point. The first point was not earned because the graph in the response does increase then decrease in value, but the graph in the response has portions that are not concave down. The second point was not earned because the labeled maximum value on the curve is not consistent with the maximum value determined in part (b)(i). The third point was earned because the graph in the response has a value of 0 N at $t=0 \mathrm{~s}$ and at $t=0.5 \mathrm{~s}$. Part (c) earned no points. The first point was not earned because the response does not select $F_{1}<F_{2}$. The second point was not earned because the response does not state the change in momentum and the impulse of each cart is the same. The third point was not earned because the response does not relate a shorter collision time with a greater force.

## Sample: 1C

## Score: 4

Part (a)(i) earned 1 point because the response clearly states that the area under the curve could be used to estimate the displacement: "area under the curve could be used." Part (a)(ii) earned 2 points. The first point was earned because the response indicates that linear momentum is conserved by reasoning ratio of changes in mass and changes in velocity. The second point was earned because the response calculates the correct answer with units ( $2 \mathrm{~m} / \mathrm{s}$ ). Part (a)(iii) earned no points. The first point was not earned because the response does not include a graph starting at $(0,0)$ and ending at $(0.5,2)$. The second point was not earned because the graph drawn in the response is a smooth, continuous curve but the curve drawn does not transition from concave up to concave down. Part (b)(i) earned 1 point. The first point was earned because the response correctly indicates that the derivative of the velocity function is the acceleration function: " $F_{\text {net } \max }=1000 \times\left(192 t^{2}-96 t\right)$ ". The second point was not earned because the response does not clearly set the derivative of the acceleration function equal to zero. The third point was not earned because the response does not indicate that the maximum acceleration occurs at $t=0.25 \mathrm{~s}$. The fourth point was not earned because the response does not substitute the value of maximum acceleration into a statement of Newton's second law. Part (b)(ii) earned no points. The first point was not earned because the graph in the response is not concave down. The second point was not earned because the graph in the response does not include a labeled maximum value. The third point was not earned because the graph in the response does not have a value of 0 N at $t=0 \mathrm{~s}$ and at $t=0.5 \mathrm{~s}$. Part (c) earned no points. The first point was not earned because the response does not select $F_{1}<F_{2}$. The second point was not earned because the response does not state the change in momentum and the impulse of each cart is the same. The third point was not earned because the response does not state that a shorter collision time leads to a greater force.

