# AP Physics C: Electricity and Magnetism 

 Sample Student Responses and Scoring Commentary Set 1
## Inside:

Free-Response Question 3
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## Question 3: Free-Response Question

(a) For a loop rule expression that includes terms for the equivalent resistance $\frac{R}{2}$ and the potential difference across the battery
For an expression that includes charge $Q$ in the term relating the potential difference across
the capacitor and includes charge per unit time $\frac{d Q}{d t}$ in the term relating the potential difference across the pair of resistors
Example Response
$\mathcal{E}-\Delta V_{C}-\Delta V_{R, e q}=0$
$\mathcal{E}-\frac{Q}{C}-\frac{R}{2} \frac{d Q}{d t}=0$

## Total for part (a) 2 points

(b) For sketching a concave down and increasing curve on the graph $\sigma$ as a function of $t \quad \mathbf{1}$ point

For sketching a curve that is concave up and decreasing on the graph of $P$ as a function of $t \quad \mathbf{1}$ point For sketching both curves that approach a slope of zero as time increases $\quad \mathbf{1}$ point
Scoring Note: The third point can be earned even if the first two points are not earned.

## Example Response



(c)(i) For a correct justification that could include one of the following:

- An indication that the current is to the right with a justification that includes a statement that indicates that positive charge has accumulated on the top plate of Capacitor 1 and/or negative charge has accumulated on the bottom plate of Capacitor 1 when the switch was closed to Position A
- An indication that the current is to the right with a justification that includes a statement that indicates that the value of the electric potential of the top plate of Capacitor 1 is larger than the electric potential of the bottom plate of Capacitor 1 when the switch was closed to Position A


## Example Responses

The current is directed towards the right because the top plate of Capacitor 1 is positively charged, meaning conventional current will flow clockwise.

## OR

Toward the right. Current flows from high to low potential so it will flow from the top plate up and right through the switch.
(c)(ii) For indicating that the total charge on the positive plate of Capacitor 2 is $\frac{2}{3} Q_{0}$

Scoring Note: This point can be earned without supporting calculations.

## Example Response

The potential difference across Capacitor 1 is equal to the potential difference across Capacitor 2. Capacitor 2 has twice the capacitance of Capacitor 1. Therefore, Capacitor 2 stores twice the charge that is stored on Capacitor 1. Due to conservation of charge,
Capacitor 2 stores an amount of charge equal to $\frac{2}{3} Q_{0}$.
(c)(iii) For an indication that the total energy dissipated by the resistors is the difference between an initial electric potential energy stored in one or both capacitors at time $t=t_{1}$ and a final
electric potential energy stored on one or both capacitors after the new steady state conditions have been reached

## Example Response

$E_{R}=U_{C}-U_{0 C}$
For indicating that only Capacitor 1 stores nonzero electric potential energy initially and both $\mathbf{1}$ point capacitors store nonzero electric potential energy after the new steady state conditions have been reached, or alternative consistent with part (c)(ii)
Example Response
$U_{0 C}=U_{01}$
$U_{C}=U_{1}+U_{2}$
For correct substitutions for the charges stored on the capacitors after the new steady state conditions have been reached consistent with part (c)(ii)

## Example Response

$\Delta E_{R}=U_{C}-U_{0 C}$
$\Delta E_{R}=\left(\frac{1}{2}\left(\frac{1}{C}\right)\left(\frac{Q_{0}}{3}\right)^{2}+\frac{1}{2}\left(\frac{1}{2 C}\right)\left(\frac{2 Q_{0}}{3}\right)^{2}\right)-\frac{1}{2} \frac{Q_{0}^{2}}{C}$

## Example Solution

$\Delta E_{R}=U_{C}-U_{0 C}$
$\Delta E_{R}=\left(\frac{1}{2}\left(\frac{1}{C}\right)\left(\frac{Q_{0}}{3}\right)^{2}+\frac{1}{2}\left(\frac{1}{2 C}\right)\left(\frac{2 Q_{0}}{3}\right)^{2}\right)-\frac{1}{2} \frac{Q_{0}^{2}}{C}$
$\Delta E_{R}=\frac{Q_{0}^{2}}{6 C}-\frac{1}{2} \frac{Q_{0}^{2}}{C}$
$\Delta E_{R}=-\frac{Q_{0}^{2}}{3 C}$
(d) For indicating that the potential difference across each capacitor is the same or that the 1 point charge stored on each capacitor in steady state is the same
Example Responses
$\Delta V_{1}=\Delta V_{2}$

OR
$Q_{1}=Q_{2}$
For recognizing that the capacitance of each capacitor is now the same in the new $\mathbf{1}$ point configuration

## Example Response

After steady state conditions are reached, both capacitors have the same potential difference. The new capacitance of Capacitor 2 is equal to the capacitance of Capacitor 1 because the capacitance of a capacitor is inversely related to the distance between the plates of a capacitor. Therefore, since $U_{C}=\frac{1}{2} C(\Delta V)^{2}, \frac{U_{2}}{U_{1}}=1$.
(e)(i) For a loop rule that includes the terms for the emf of the battery, the potential difference $\mathbf{1}$ point across the pair of resistors, and the potential difference across Capacitor 1
Example Response
$\varepsilon-\Delta V_{R}-\Delta V_{C}=0$
For a correct answer
Example Response
$I=\frac{Q_{0}}{R C}$
Example Solution
$\mathcal{E}-\Delta V_{R}-\Delta V_{C}=0$
$\varepsilon-I\left(\frac{R}{2}\right)-\left(\frac{Q_{0}}{2 C}\right)=0$
$\frac{Q_{0}}{C}-I\left(\frac{R}{2}\right)-\left(\frac{Q_{0}}{2 C}\right)=0$
$\frac{Q_{0}}{2 C}-I\left(\frac{R}{2}\right)=0$
$\frac{Q_{0}}{2 C}=I\left(\frac{R}{2}\right)$
$\frac{Q_{0}}{C}=I R$
$I=\frac{Q_{0}}{R C}$
(e)(ii) For indicating that the current is zero

## Question 3

## Continue your response to QUESTION 3 on this page.

(b) On the axes shown, sketch graphs of the surface charge density $\sigma$ on the positive plate of Capacitor 1 and the total power $P$ dissipated by the resistors as functions of time $t$ from time $t=0$ until steady-state conditions are nearly reached.



A long time after the switch is closed to Position A, the charge on the positive plate of Capacitor 1 is $Q_{0}$ and Capacitor 2 is uncharged.
(c) At time $t_{1}$, the switch is closed to Position B.
i. Immediately after time $t_{1}$, is the direction of the current in the switch directed toward the left, directed toward the right, or is there no current? Briefly justify your answer.

Toward the right. The top plate of $C_{1}$ is positive, and the bottom is negative. Charge will flow in this coop in the $C \omega$ direction until equilibrium is reached.
ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after $t_{1}$. Express your answer in terms of $Q_{0}$ and fundamental constants, as appropriate.

$$
\begin{array}{lll}
\Delta U_{1}=\Delta V_{2} & 24 Q_{1}=C Q_{2} & Q_{1}+Q_{2}=Q_{0} \\
\frac{Q_{1}}{C_{1}}=\frac{Q_{2}}{C_{2}} & 2 Q_{1}=Q_{2} & \frac{Q_{2}}{2}+Q_{2}=Q_{0} \\
\frac{Q_{1}}{C}=\frac{Q_{2}}{2 C} & Q_{1}=\frac{Q_{2}}{2} & \frac{3 Q_{2}}{2}=Q_{0} \\
& & Q_{2}=\frac{2}{3} Q_{0}
\end{array}
$$

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Continue your response to QUESTION 3 on this page.
iii. Derive an expression for the total energy $E_{R}$ dissipated by reason 1 and 2 from immediately after time $I_{1}$ until new steady-state conditions have been reached. Express your answer in terms of $C, Q_{0}$ and fundamental constants, as appropriate.

$$
\begin{aligned}
& E_{R}=U_{f}-U_{i} \quad U_{G}=\frac{1}{2} \frac{Q^{2}}{C} \\
& E_{R}=\left(\frac{1}{2} \frac{\left(\frac{2}{3} Q_{0}\right)^{2}}{2 C}+\frac{1}{2} \frac{\left(\frac{1}{3} Q_{0}\right)^{2}}{C}\right)-\left(\frac{1}{2} \frac{Q_{0}^{2}}{C}\right) \\
& E_{R}=\frac{4}{36} \frac{Q_{0}^{2}}{L}+\frac{1 Q_{0}^{2}}{36 C}-\frac{18 Q_{0}^{2}}{36 L} \\
& E Q=-\frac{13}{36} \frac{Q_{0}^{2}}{2} \quad E_{R}=\frac{13}{36} \frac{Q_{0}^{2}}{C}
\end{aligned}
$$

With the switch still closed to Position B, the parallel plates of Capacitor 2 are moved so that the separation distance increases by a factor of 2 .
(d) Determine the ratio $\frac{U_{2}}{U_{1}}$ of the energy $U_{2}$ stored in Capacitor 2 to the energy $U_{1}$ stored in Capacitor 1 a long time after the plates of Capacitor 2 have been moved. Briefly justify your answer.
Capacitance of $C_{2} \quad \mathrm{zC} \rightarrow L$. Therefore charge will redistribute equally no the capacitors will have equal capacitances as well es charges and the ratio $\frac{\theta_{2}}{U_{1}}=1$.

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Question 3

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With the capacitors still charged as in part (d), the switch is now closed to Position A.
(e) Express your answers to part (e)(i) and part (e)(ii) in terms of $R, C, Q_{0}$, and fundamental constants, as appropriate.
i. Derive an expression for the current $I_{0}$ from the battery immediately after the switch is closed to Position A.

$$
\begin{aligned}
& \Sigma=\pi_{0} \frac{R}{2}+\frac{Q_{0}}{2 c} \\
& \varepsilon=\frac{Q_{0}}{C}=I_{0} \frac{R}{2}+\frac{Q_{0}}{2 c} \\
& \frac{2 Q_{0}}{2 C}-\frac{Q_{0}}{2 C}=I \frac{R}{2} \quad \frac{Q_{0}}{R C}=I_{0} \\
& \frac{Q_{0}}{2 C}=I \frac{R}{2}
\end{aligned}
$$

ii. Determine the current $I_{\infty}$ from the battery a long time after the switch is closed to Position A.

$$
I \infty=0
$$

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3. The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance $R$, capacitors 1 and 2 with capacitances $C$ and 2C, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t=0$, the switch is closed to Position A.
(a) Write, but do NOT solve, a differential equation that can be used to determine the charge $Q$ on the positive plate of Capacitor 1 as a function of time $t$ after the switch is closed to Position A. Express your answer in terms of $E, R, C, Q, f$, and fundamental constants, as appropriate.



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(b) On the axes shown, sketch graphs of the surface charge density $\sigma$ on the positive plate of Capacitor 1 and the total power $P$ dissipated by the resistors as functions of time $t$ from time $t=0$ until steady-state conditions are nearly reached.



A long time after the switch is closed to Position A, the charge on the positive plate of Capacitor 1 is $Q_{0}$ and Capacitor 2 is uncharged.
(c) At time $t_{1}$, the switch is closed to Position B.
i. Immediately after time $t_{1}$, is the direction of the current in the switch directed toward the left, directed toward the right, or is there no current? Briefly justify your answer.

$$
\begin{aligned}
& \text { Right, beaus is there no current? Briefly justify your answer. } \\
& \text { so is on the to current will be CWiate, } \\
& \text { conventional cmiren t }
\end{aligned}
$$

ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after $t_{1}$. Express your answer in terms of $\boldsymbol{Q}_{0}$ and fundamental constants, as appropriate.




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Use a pencil or a pen with black or dark blue ink. Do"NOT write your name. Do NOT write outside the box.
With the switch still closed to Position B, the parallel plates of Capacitor 2 are moved so that the separation distance increases by a factor of 2 .
(d) Determine the ratio $\frac{U_{2}}{U_{1}}$ of the energy $U_{2}$ stored in Capacitor 2 to the energy $U_{1}$ stored in Capacitor 1 a long time after the plates of Capacitor 2 have been moved. Briefly justify your answer.
iii. Derive an expression for the total energy $E_{R}$ dissipated by resistors 1 and 2 from immediately after time $f_{1}$ until new steady-state conditions have been reached. Express your answer in terms of $C, Q_{0}$, and fundamental constants, as appropriate.






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Question 3

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With the capacitors still charged as in part (d), the switch is now closed to Position A.
(e) Express your answers to part (e)(i) and part (e)(ii) in terms of $R, C, Q_{0}$, and fundamental constants, as appropriate.
i. Derive an expression for the current $I_{0}$ from the battery immediately after the switch is closed to Position A.


$$
\begin{aligned}
& \text { capacitor } e t=0 \text { acts } \\
& \text { like a wire }
\end{aligned}
$$

ii. Determine the current $I_{\infty}$ from the battery a long time after the switch is closed to Position A.

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## Question 3

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3. The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance $R$, capacitors 1 and 2 with capacitances $C$ and $2 C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t=0$, the switch is closed to Position A.
(a) Write, but do NOT solve, a differential equation that can be used to determine the charge $Q$ on the positive plate of Capacitor 1 as a function of time $t$ after the $s w i t c h$ is closed to Position A. Express your answer in terms of $\mathcal{E}, R, C, Q, t$, and fundamental constants, as appropriate.



Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.
(b) On the axes shown, sketch graphs of the surface charge density $\sigma$ on the positive plate of Capacitor 1 and the total power $P$ dissipated by the resistors as functions of time $t$ from time $t=0$ until steady-state conditions are nearly reached.

## Continue your response to QUESTION 3 on this page.



A long time after the switch is closed to Position A, the charge on the positive plate of Capacitor 1 is $Q_{0}$ and Capacitor 2 is uncharged.
(c) At time $t_{1}$, the switch is closed to Position B.
i. Immediately after time $t_{1}$, is the direction of the current in the switch directed toward the left, directed toward the right, or is there no current? Briefly justify your answer.

$$
\begin{aligned}
& \text { pirecel to the left,becuuse there } \\
& \text { are restseors on the other side then tues } \\
& \text { erection of original carew and }
\end{aligned}
$$

ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after $t_{1}$. Express your answer in terms of $Q_{0}$ and fundamental constants, ass appropriate.


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iii. Derive an expression for the total energy $E_{R}$ dissipated by resistors 1 and 2 from immedinelly after lime $t_{1}$ until new steady-state conditions have been reached. Express your answer in terms of $C, Q_{0}$, and fundamental constants, as appropriate.


With the switch still closed to Position B, the parallel plates of Capacitor 2 are moved so that the separation distance increases by a factor of 2 .
(d) Determine the ratio $\frac{U_{2}}{U_{1}}$ of the energy $U_{2}$ stored in Capacitor 2 to the energy $U_{1}$ stored in Capacitor 1 a long time after the plates of Capacitor 2 have been moved. Briefly justify your answer.
$C=\frac{G d}{A}$

Sine the



2 double d the ratio raise doodled.
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With the capacitors still charged as in part (d), the switch is now closed to Position A.
(e) Express your answers to part (e)(i) and part (e)(ii) in terms of $R, C, Q_{0}$, and fundamental constants, as appropriate.
i. Derive an expression for the current $I_{0}$ from the battery immediately after the switch is closed to Position A.

ii. Determine the current $I_{\infty}$ from the battery a long time after the switch is closed to Position A.

$$
\begin{aligned}
& I_{\infty}=0 \quad I=I_{0} e^{-\left(\frac{t}{E K}\right)} \\
& \text { After reaching full }
\end{aligned}
$$

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## Question 3

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

## Overview

The responses were expected to demonstrate the ability to:

- Apply Kirchhoff's Loop Rule (conservation of energy) in a complex RC circuit.
- Use Ohm's law and the definition of capacitance for individual circuit elements.
- Graphically represent the time dependence of quantities in an RC circuit.
- Predict short-term and long-term behavior of an RC circuit while capacitors are charging and discharging.
- Apply conservation of energy to a circuit problem to determine energy or power used by a circuit element.
- Calculate how changing the physical attributes of a capacitor (e.g., plate separation distance or presence or absence of a dielectric) would affect the electric potential energy of the capacitor.


## Sample: 3A

## Score: 14

Part (a) earned 2 points. The first point was earned for correctly writing a loop rule that includes terms for the potential difference across the equivalent resistance $\frac{R}{2}$ and across the battery, E. The second point was earned for correctly substituting $\frac{Q}{C}$ for the potential difference across the capacitor and $\frac{d Q}{d t}$ for current in the term for the potential difference across the pair of resistors. Part (b) earned 2 points. The first point was earned for drawing a curve for the graph of $\sigma$ as a function of $t$ that is concave down and increasing. The second point was not earned because the curve for $P$ is concave down and increasing instead of decreasing and concave up. The third point was earned for drawing both curves with slopes that approach zero as time increases. Part (c) earned 5 points. The first point was earned for stating that the current in the switch is to the right with an appropriate justification based on the fact that the top plate has an accumulation of positive charges. The second point was earned for correctly determining that the charge on Capacitor 2 is $\frac{2}{3} Q_{0}$. The third point was earned for correctly indicating that the energy dissipated by the resistors is the difference in initial and final energies stored in the capacitors. The fourth point was earned for correctly indicating that the initial energy is the electric potential energy stored in Capacitor 1, and the final energy is the electric potential energy stored in both capacitors 1 and 2. The fifth point was earned for correctly substituting the correct charges on both capacitors after steady-state conditions have been reached. Part (d) earned 2 points. The first point was earned for indicating that Capacitor 1 and Capacitor 2 have the same charge after the plate separation distance was increased. The second point was earned for indicating that Capacitor 1 and Capacitor 2 have the same capacitance after the plate separation distance was increased. Part (e) earned 3 points. The first point was earned for correctly writing a loop rule that includes terms for the potential difference across the battery, across the pair of resistors, and across Capacitor 1. The second point was earned for correctly deriving that the current from the battery immediately after the switch is moved to Position A is $\frac{Q_{0}}{R C}$. The third point was earned for correctly indicating the current from the battery is zero a long time after the switch is moved to Position A.

## Question 3 (continued)

## Sample: 3B

## Score: 7

Part (a) earned 1 point for correctly substituting $\frac{Q}{C}$ for the potential difference across the capacitor and $\frac{d Q}{d t}$ for current in the term for the potential difference across the pair of resistors. The second point was not earned because the response writes a loop rule that does not includes a correct term for the potential difference across the battery, E . Part (b) earned 3 points. The first point was earned for drawing a curve for the graph of $\sigma$ as a function of $t$ that is concave down and increasing. The second point was earned for drawing a curve for $P$ is decreasing and concave up. The third point was earned for drawing both curves with slopes that approach zero as time increases. Part (c) earned 1 point for stating that the current in the switch is to the right with an appropriate justification based on the fact that the top plate has an accumulation of positive charges. The second point was not earned because the charge on Capacitor 2 is incorrect. The third point was not earned because the response does not indicate that the energy dissipated by the pair of resistors is the difference between the initial electric potential energy stored in the capacitor(s) at time $t=t_{1}$ and the final electric potential energy stored in the capacitor(s) after steady-state conditions have been reached. The fourth point was not earned because the response does not indicate that the only nonzero contribution to the initial electric potential energy is from Capacitor 1 AND that the final electric potential energy has nonzero contributions from both Capacitor 1 and Capacitor 2. The fifth point was not earned because the response does not make substitutions for the charges stored on the capacitors in steady state. Part (d) earned 1 point for indicating that Capacitor 1 and Capacitor 2 have the same charge after the plate separation distance was increased. The second point was not earned because the response does not indicate that Capacitor 1 and Capacitor 2 have the same capacitance after the plate separation distance was increased. Part (e) earned 1 point for correctly indicating the current from the battery is zero a long time after the switch is moved to Position A. The second point was not earned because the response does not write a loop rule that includes terms for the potential difference across the battery, across the pair of resistors, and across Capacitor 1. The third point was not earned because the response does not correctly derive that the current from the battery immediately after the switch is moved to Position A is $\frac{Q_{0}}{R C}$.

## Question 3 (continued)

## Sample: 3C <br> Score: 2

Part (a) earned no points. The first point was not earned because the response does not write a loop rule. The second point was not earned because the response does not use $\frac{Q}{C}$ for the potential difference across the capacitor or substitute $\frac{d Q}{d t}$ for current into the term for the potential difference across the pair of resistors. Part (b) earned 1 point for drawing a curve for the graph of $\sigma$ as a function of $t$ that is concave down and increasing. The second point was not earned because the curve for $P$ is increasing and concave up instead of decreasing and concave up. The third point was not earned because the curve for $P$ does not approach a slope of zero as time increases. Part (c) earned no points. The first point was not earned because the response states that the current in the switch is to the left. The second point was not earned because the charge on Capacitor 2 is incorrect. The third point was not earned because the response does not indicate that the energy dissipated by the pair of resistors is the difference between the initial electric potential energy stored in the capacitor(s) at time $t=t_{1}$ and the final electric potential energy stored in the capacitor(s) after steady-state conditions have been reached. The fourth point was not earned because the response does not indicate that the only nonzero contribution to the initial electric potential energy is from Capacitor 1 AND that the final electric potential energy has nonzero contributions from both Capacitor 1 and Capacitor 2. The fifth point was not earned because the response does not make substitutions for the charges stored on the capacitors in steady state. Part (d) earned no points. The first point was not earned because the response does not indicate that Capacitor 1 and Capacitor 2 have the same potential difference OR the same charge after the plate separation distance was increased. The second point was not earned because the response does not indicate that Capacitor 1 and Capacitor 2 have the same capacitance after the plate separation distance was increased. Part (e) earned 1 point for correctly indicating the current from the battery is zero a long time after the switch is moved to Position A. The second point was not earned because the response does not write a loop rule that includes terms for the potential difference across the battery, across the pair of resistors, and across Capacitor 1. The third point was not earned because the response does not correctly derive that the current from the battery immediately after the switch is moved to Position A is $\frac{Q_{0}}{R C}$.

