2022

AP® | CollegeBoard

AP® Calculus AB

Sample Student Responses and Scoring Commentary

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Free-Response Question 6

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Part B (AB): Graphing calculator not allowed
Question 6 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Particle P moves along the x-axis such that, for time $t > 0,$ its position is given by $x_P(t) = 6 - 4e^{-t}.$

Particle Q moves along the y-axis such that, for time $t > 0,$ its velocity is given by $v_Q(t) = \frac{1}{t^2}.$ At time $t = 1,$ the position of particle Q is $y_Q(1) = 2.$

	Model Solution	Scoring	
(a)	Find $v_P(t),$ the velocity of particle P at time $t.$		
	$v_P(t) = x_P'(t) = 4e^{-t}$	Answer	**1 point**

Scoring notes:

- A response that equates $x_P(t)$ with $v_P(t)$ does not earn the point.
- An unlabeled response earns the point.

Total for part (a) — 1 point

(b) Find $a_Q(t)$, the acceleration of particle Q at time t. Find all times t, for $t > 0$, when the speed of particle Q is decreasing. Justify your answer.

$a_Q(t) = v_Q{}'(t) = \frac{-2}{t^3}$	$a_Q(t)$	**1 point**
For $t > 0$, $a_Q(t) < 0$ and $v_Q(t) > 0$.	Considers signs of $a_Q(t)$ and $v_Q(t)$	**1 point**
Because the velocity and acceleration have opposite signs, the speed of particle Q is decreasing for all $t > 0$.	Answer with justification	**1 point**

Scoring notes:

- Earning the first point is not necessary for a response to be eligible to earn the second or third points; however, the response must present an expression for $a_Q(t)$ to be eligible for third point.
- A response earns the second point with either of the following statements: "$v_Q(t)$ and $a_Q(t)$ have opposite signs" or "$v_Q(t)$ and $a_Q(t)$ have the same sign." This statement, however, must be consistent with $v_Q(t)$ and the presented expression for $a_Q(t)$.
- A response must earn the second point to be eligible for the third point. The answer must be consistent with the presented justification. Furthermore, responses for which $a_Q(t) > 0$ for $t > 0$ must conclude that there is no time at which the speed of the particle is decreasing.
- A response that indicates $v_Q(t) < 0$ does not earn the third point, even if the answer and justification are consistent with a reported sign of $a_Q(t)$.

Total for part (b) 3 points

(c) Find $y_Q(t)$, the position of particle Q at time t.

$y_Q(t) = y_Q(1) + \int_1^t \frac{1}{s^2}\,ds$	Integral	**1 point**
	Uses initial condition	**1 point**
$= 2 - \left(\frac{1}{s}\Big\vert_1^t\right) = 2 - \frac{1}{t} + 1 = 3 - \frac{1}{t}$	Answer	**1 point**

Scoring notes:

- A response that presents $\int_1^t \frac{1}{t^2}\,dt$ (using the same variable as a limit and integrand function) does not earn the first point unless it is followed by an attempt at integration.
- A response that presents either $\int \frac{1}{t^2}\,dt$ or $-\frac{1}{t}$ (with no integral) earns the first point. If the response continues and presents $2 = -1 + C$, then the response earns the second point.
- A response that presents only $y_Q(t) = -\frac{1}{t} + 3$ will earn all 3 points. Note that the right side of this equation suffices to earn all points. A response of $y_Q(t) = -\frac{1}{t} + C$, where $C \neq 3$, (with no additional supporting work) earns only the first point.

Total for part (c) 3 points

(d) As $t \to \infty$, which particle will eventually be farther from the origin? Give a reason for your answer.

For particle P, $\lim_{t\to\infty}\left(6 - 4e^{-t}\right) = 6$. For particle Q, $\lim_{t\to\infty}\left(3 - \frac{1}{t}\right) = 3$.	One correct limit	**1 point**
Because $6 > 3$, particle P will eventually be farther from the origin.	Answer with reason	**1 point**

Scoring notes:

- A response with an incorrect $y_Q(t)$ from part (c) is eligible for both points in part (d) provided $y_Q(t)$ is a non-constant function. The second point is earned for a consistent answer with reason, and limits correct for particle P and the presented $y_Q(t)$.
- Responses that present statements such as "$6 - 4e^{-t}$ approaches 6" or "Q goes to 3" earn the first point and are eligible for the second point.
- A response that treats ∞ as an input for $x_P(t)$ or $y_Q(t)$, such as "$6 - 4e^{-\infty}$" or "$3 - \frac{1}{\infty}$" is not eligible for the second point.

Total for part (d) 2 points

Total for question 6 9 points

6 6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6 6

Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)

$x_p(t) = 6 - 4e^{-t}$

$V_p(t) = 4e^{-t}$

Response for question 6(b)

$V_q(t) = \frac{1}{t^2}$

$A_q(t) = \frac{-2}{t^3}$

$V_q(t)$ is positive on the interval $(0, \infty)$

$A_q(t)$ is negative on the interval $(0, \infty)$

so the speed is decreasing on the interval $(0, \infty)$

6 6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6 6

Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

~~Calculate~~

$v_q(t) = \frac{1}{t^2}$

$y_q(t) = \int \frac{1}{t^2}\,dt$

$y_q(t) = -(t)^{-1} + C$

$2 = -(1) + C$

$C = 3$

$y_q(t) = -\frac{1}{t} + 3$

Response for question 6(d)

$\lim_{t\to\infty} y_q(t) = 3$

$\lim_{t\to\infty} x_p(t) = 6 - 4(0)$
$= 6$

Particle P, because its end behavior is farther away from the origin than particle Q

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5106/15

6 6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6 6

Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)

$x_p(t) = 6 - 4e^{-1}$

$v_p(t) = 4e^{-t}$ $\quad x'(t) = v(t)$

Response for question 6(b)

$v_q(t) = t^{-2}$ or $\frac{1}{t^2}$

$a_q(t) = -2t^{-3}$ or $\frac{-2}{t^3}$

6 6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6 6

Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$$\int v_a(t)\,dt$$

$$\int t^{-2}\,dt$$

$$y_a(t) = \frac{t^{-1}}{-1} + c \text{ or } -\frac{1}{t} + c$$

$$y(t) = -\frac{1}{t} + 3$$

$$y(1) = -\frac{1}{1} + c$$

$$2 = -1 + c$$

$$+1 \quad +1 \qquad c = 3$$

Response for question 6(d)

$$\lim_{t \to \infty} x_p(t)$$

$$\lim_{t \to \infty} y_a(t)$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5106/15

6 6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6 6

Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)

$V_p(t) = \frac{d}{dt}(6 - 4e^{-t})$

$V_p(t) = 4e^{-t}$

Response for question 6(b)

$a_Q(t) = \frac{d}{dt}\left(\frac{1}{t^2}\right)$

$a_Q(t) = \frac{2t}{t^4}$

the speed of particle Q is always increasing for all $t > 0$

~~$t > 0$~~

for all $t > 0$ $a_Q(t) =$ positive.

Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$$Y_Q(t) = 2 + \int_1^t (V_Q(b))\,db$$

Response for question 6(d)

$$\lim_{t \to \infty} V_P(t) = 4e^{-\infty}$$

$$\lim_{t \to \infty} V_Q(t) = \frac{1}{\infty^2}$$

$\therefore$ particle P will eventually be further from the origin because particle P's speed is infinatly growing to where as particle Q's Speed is infinatley decreasing

Question 6

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

In this problem, for time $t > 0$, particle P is moving along the x-axis with position $x_P(t) = 6 - 4e^{-t}$. A second particle, Q, is moving along the y-axis with velocity $v_Q(t) = \frac{1}{t^2}$ and position $y_Q(1) = 2$ at time $t = 1$.

In part (a) students were asked to find the velocity of particle P at time t. A correct response would find the derivative of the given position function, $v_P(t) = 4e^{-t}$.

In part (b) students were asked to find the acceleration of particle Q at time t and then to find all times ($t > 0$) when the speed of particle Q is decreasing. A correct response should recognize that the acceleration of the particle is the derivative of the velocity, $a_Q(t) = v_Q'(t) = \frac{-2}{t^3}$, then observe that for all times $t > 0$ this acceleration is negative and the given velocity $\frac{1}{t^2}$ is positive. Therefore, the acceleration and velocity of particle Q have opposite signs and thus the speed of this particle is decreasing for all $t > 0$.

In part (c) students were asked to find the position of particle Q at time t. A correct response should integrate the given velocity function, $\int_1^t v_Q(s)\, ds = \int_1^t \frac{1}{s^2}\, ds$, and add the given initial position, $y_Q(1) = 2$, to obtain a position function of $y_Q(t) = 3 - \frac{1}{t}$.

Lastly, in part (d) students were asked to reason which particle would eventually be farther from the origin as the time t approaches infinity. A correct response should evalute the limits, as $t \to \infty$, of the position functions of both particles, $\lim_{t\to\infty} x_P(t) = 6$, and $\lim_{t\to\infty} y_Q(t) = 3$. Because $6 > 3$, particle P would eventually be farther from the origin than would be particle Q.

Sample: 6A
Score: 9

The response earned 9 points: 1 point in part (a), 3 points in part (b), 3 points in part (c), and 2 points in part (d).

In part (a) the response earned the first point in the second line with the correct velocity for particle P.

In part (b) the response earned the first point in the second line on the left with the correct acceleration of particle Q. The response earned the second point in the first two lines on the right with the correct behaviors of $v_Q(t)$ and $a_Q(t)$. The response earned the third point in the last line with the conclusion that "the speed is decreasing on the interval $(0, \infty)$."

Question 6 (continued)

In part (c) the response earned the first point in the second line on the left with the integral $\int \frac{1}{t^2}\,dt$. Note that the correct antiderivative is presented in the line below. The equation $2 = -(1) + C$ in the fourth line on the left earned the second point. The equation on the right earned the third point.

In part (d) the response earned the first point with the correct evaluation of the limit for $y_Q(t)$ in the first line on the left. The correct limit values for the positions of particles P and Q in the first line together with the conclusion in the last two lines that "Particle P, because its end behavior is farther away from the origin than particle Q," earned the second point.

Sample: 6B
Score: 5

The response earned 5 points: 1 point in part (a), 1 point in part (b), 3 points in part (c), and no points in part (d).

In part (a) the response earned the point with the correct expression for the velocity of particle P in the second line on the left.

In part (b) the response earned the first point in the second line with the correct acceleration of particle Q. Because no further work is presented, the response earned neither the second nor the third point.

In part (c) the response earned the first point with the integral expression in the first line. Note that either the equivalent expression in the second line or the correct antiderivative on the third line would have also earned the point. The response earned the second point in the fifth line with the equation $2 = -1 + C$. The response earned the third point with the correct circled answer on the right.

In part (d) the response earned neither point as neither limit has been evaluated, nor has a conclusion been drawn.

Sample: 6C
Score: 3

The response earned 3 points: 1 point in part (a), no points in part (b), 2 points in part (c), and no points in part (d).

In part (a) the response earned the point in the second line with the correct expression for the velocity of particle P.

In part (b) the response did not earn the first point because the presented acceleration of particle Q is missing the negative sign. The response did not earn the second point because no consideration of the sign of $v_Q(t)$ is presented. While the statement presented that "the speed of particle Q is always increasing for all $t > 0$" is consistent with the $a_Q(t)$ presented, the response is not eligible for the third point because the second point was not earned.

In part (c) the response earned the first two points with the presented equation. Because no integration has been done, the response did not earn the third point.

In part (d) the response did not earn the first point because no limit values are presented. Because the expressions on the left treat ∞ as a function input, the response is not eligible for the second point.