2021

AP® CollegeBoard

AP® Physics C: Mechanics

Scoring Guidelines

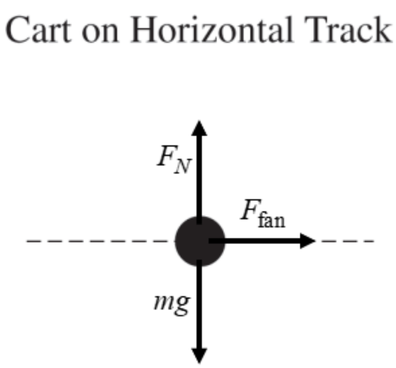
Set 1

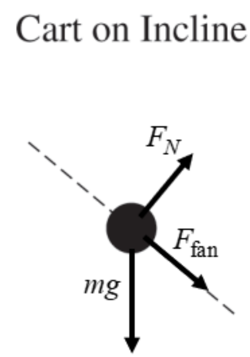
Question 1: Free-Response Question — 15 points

(a)	For correctly drawing and labeling all the forces on the cart on the flat surface	**1 point**
	For correctly drawing and labeling the weight of the cart on the incline	**1 point**
	For correctly drawing and labeling the normal force on the cart on the incline	**1 point**
	For correctly drawing and labeling the force of the fan on the cart on the incline	**1 point**

Scoring note: A maximum of three points can be earned if there are any extraneous vectors.

Example responses for part (a)





Scoring note: Examples of appropriate labels for the force due to gravity include: F_G, F_g, F_{grav}, W, mg, Mg, "grav force," "F Earth on cart," "F on cart by Earth," $F_{\text{Earth on cart}}$, $F_{\text{E,Cart}}$, $F_{\text{Cart,E}}$. The labels G or g are not appropriate labels for the force due to gravity. F_n, F_N, N, "normal force," "ground force," or similar labels may be used for the normal force.

Total for part (a) 4 points

(b)	For correctly applying Newton's second law for the cart on the flat surface	**1 point**

$$F_{\text{fan}} = m_{\text{cart}} a_1$$

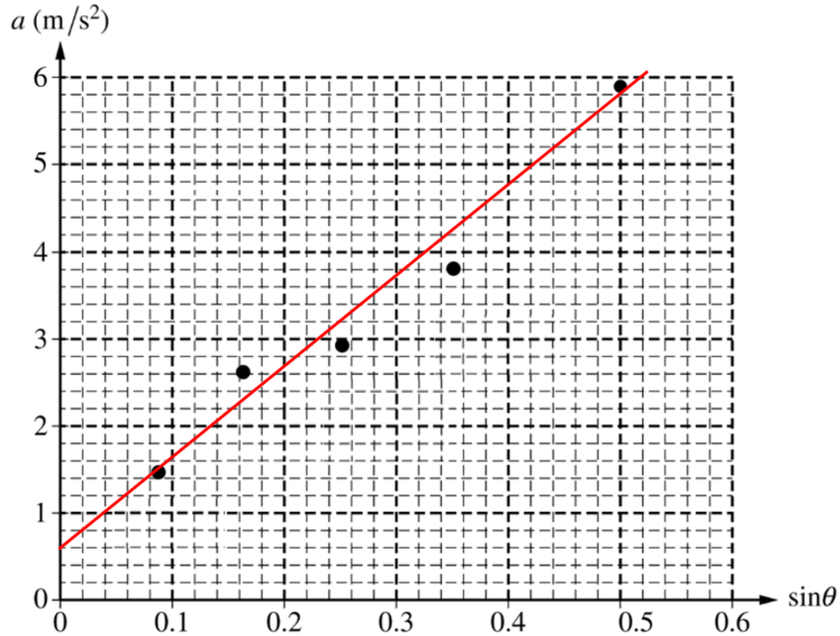
For the correct answer with units — **1 point**

$$F_{\text{fan}} = (0.50\text{ kg})\left(0.8\text{ m/s}^2\right) = 0.40\text{ N}$$

Total for part (b) 2 points

(c)	For including the correct component of weight in a Newton's second law equation for the cart $F_{\text{fan}} + mg\sin\theta = ma_2$	**1 point**
	For a correct Newton's second law equation $F_{\text{fan}} + mg\sin\theta = ma_2$	**1 point**
	For correct substitutions consistent with part (b) into the above equation $mg\sin\theta = ma_2 - F_{\text{fan}}$ $\theta = \sin^{-1}\left(\dfrac{ma_2 - F_{\text{fan}}}{mg}\right)$ $\theta = \sin^{-1}\left(\dfrac{(0.50\text{ kg})(2.4\text{ m/s}^2) - (0.50\text{ kg})(0.8\text{ m/s}^2)}{(0.50\text{ kg})(9.8\text{ m/s}^2)}\right)$ $\theta = 9.4°$	**1 point**
	Total for part (c)	**3 points**
(d)	For selecting "No" and an attempted justification	**1 point**
	For a correct justification	**1 point**
	Example response for part (d) *The mass of the cart cancels out in the equation used to find the angle of the incline.*	
	Total for part (d)	**2 points**

(e) **i.** For drawing an appropriate best-fit line **1 point**



ii. For correctly calculating slope using two points from the best-fit line **1 point**

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{(5-1)\left(\text{m/s}^2\right)}{(0.42-0.04)} = 10.52\ \text{m/s}^2$$

For correctly using an expression that relates the slope to the acceleration due to gravity **1 point**

$$F_{\text{net}} = F_{\text{fan}} + mg\sin\theta = ma$$

$$\therefore a = g\sin\theta + \frac{F_{\text{fan}}}{m}$$

from $y = mx + b$

$a = (\text{slope})\sin\theta + (y\text{-intercept})$

$\therefore \text{slope} = g = 10.52\ \text{m/s}^2$

Total for part (e) 3 points

(f) For a correct explanation **1 point**

Example response for part (f)

The mass of the cart is in the denominator of the y-intercept, so increasing the mass decreases the y-intercept without changing the rest of the graph. So the new line of data is predicted to be parallel to and below the original line.

Total for question 1 15 points

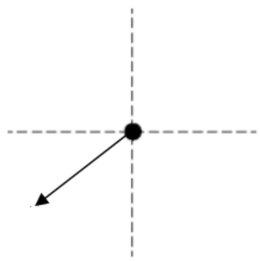
Question 2: Free-Response Question — 15 points

(a) For a single acceleration vector pointing down and to the left and attempting a justification — **1 point**

For a correct justification — **1 point**

Example response for part (a)

The block is changing direction with a centripetal component of the acceleration toward the center of the circle and a gravitational acceleration downward. Therefore, the acceleration of the block will be down and to the left.



Total for part (a) — 2 points

(b) i. For correctly using conservation of energy for the block at point B — **1 point**

$$K_A + U_{gA} = K_B + U_{gB}$$

$$\frac{1}{2}mv_A^2 + mgh_A = \frac{1}{2}mv_B^2 + mgh_B$$

For correctly substituting into the above equation — **1 point**

$$0 + mgh = \frac{1}{2}mv_B^2 + mgR$$

$$v_B = \sqrt{2g(h - R)}$$

ii. For correctly substituting the expression for v_B from part (b)(i) into an expression for centripetal force — **1 point**

$$F_c = \frac{mv^2}{r} = \frac{mv_B^2}{R} = \frac{m\left(\sqrt{2g(h-R)}\right)^2}{R}$$

For correctly substituting into an equation for vector addition to derive an expression for the net force at point B — **1 point**

$$F_{net} = \sqrt{F_c^2 + (mg)^2}$$

$$F_{net} = \sqrt{\left(\frac{mv_B^2}{R}\right)^2 + (mg)^2} = \sqrt{\left(\frac{2mg}{R}(h-R)\right)^2 + (mg)^2}$$

Total for part (b) — 4 points

(c)	For correctly using conservation of energy for the speed of the block at point C $K_A + U_{gA} = K_C + U_{gC}$ $\frac{1}{2}mv_A^2 + mgh_A = \frac{1}{2}mv_C^2 + mgh_C$ $0 + mgh = \frac{1}{2}mv_C^2 + mg(2R)$ $v_C = \sqrt{2g(h-2R)}$	**1 point**
	For correctly applying Newton's second law at point C $F_c = \frac{mv_C^2}{r}$ $F_N + mg = \frac{mv_C^2}{r}$ Set the normal force equal to zero $0 + mg = \frac{mv_C^2}{R} \therefore v_C = \sqrt{Rg}$	**1 point**
	For combining the two equations above $\sqrt{Rg} = \sqrt{2g(h-2R)} \therefore R = 2(h-2R) \therefore R/2 = h - 2R$ $h = 2.5R$	**1 point**
	Total for part (c)	**3 points**
(d)	For correctly using conservation of energy for the block compressing the spring	**1 point**
	For correctly substituting the gravitational and elastic potential energies into an equation for conservation of energy $mgh = \frac{1}{2}kx_{MAX}^2$	**1 point**
	For correctly substituting for the spring constant k into the equation above $x_{MAX} = \sqrt{\frac{2mgh}{k}} = \sqrt{\frac{2mgh}{mg/(2R)}} = \sqrt{4hR} = \sqrt{(4)(0.30\text{ m})(0.10\text{ m})} = 0.35\text{ m}$	**1 point**
	Total for part (d)	**3 points**

(e)	**i.**	For a correct justification	**1 point**
		Example response for part (e)(i) *Because the block does not make it through the loop at this height, it will not compress the spring.*	
	ii.	For indicating that as the height increases, the compression of the spring increases	**1 point**
		For indicating that the height is proportional to the square of the compression of the spring	**1 point**
		Example response for part (e)(ii) *From the equation in part (c), the compression of the spring is directly proportional to the square root of the height that the block is released. Thus, the graph would be an x-axis parabola, as shown.*	
		Total for part (e)	**3 points**
		Total for question 2	**15 points**

Question 3: Free-Response Question — 15 points

(a) For using integral calculus to calculate the rotational inertia of the rod — **1 point**

$$I = \int r^2 dm$$

$$dm = \lambda dr = \gamma x^2 dx$$

For correctly substituting γx^2 into the above equation — **1 point**

$$I = \int x^2\left(\gamma x^2 dx\right) = \int_{x=0}^{x=L} \gamma x^4 dx = \gamma\left[\frac{x^5}{5}\right]_{x=0}^{x=L} = \left(\frac{3M}{L^3}\right)\left(\frac{L^5}{5} - 0\right) = \frac{3}{5}ML^2$$

Total for part (a) — 2 points

(b) For using integral calculus to determine the center of mass of the rod — **1 point**

$$X_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{\int x dm}{\int dm}$$

For correctly substituting γx^2 into the numerator of the above equation — **1 point**

For correctly substituting M into the denominator of the above equation OR evaluating the integral $\int dm$ to find the mass of the rod — **1 point**

$$X_{CM} = \frac{\int x\lambda dx}{M} = \frac{\int x\left(\gamma x^2\right)dx}{M} = \frac{\int_{x=0}^{x=L} \gamma x^3 dx}{M} = \frac{\left[\frac{\gamma x^4}{4}\right]_{x=0}^{x=L}}{M} = \frac{\left(\frac{3M}{L^3}\right)\frac{L^4}{4}}{M} = \frac{3}{4}L$$

OR

$$X_{CM} = \frac{\int x\lambda dx}{\int \lambda dx} = \frac{\int x\left(\gamma x^2\right)dx}{\int_{x=0}^{x=L} \gamma x^2 dx} = \frac{\int_{x=0}^{x=L} \gamma x^3 dx}{\left[\frac{\gamma x^2}{3}\right]_{x=0}^{x=L}} = \frac{\left[\frac{\gamma x^4}{4}\right]_{x=0}^{x=L}}{\frac{\gamma L^2}{3}} = \frac{\frac{\gamma L^4}{4}}{\frac{\gamma L^2}{3}} = \frac{3}{4}L$$

Total for part (b) — 3 points

(c) For selecting "Greater than" with an attempted justification **1 point**

For a correct justification **1 point**

Example responses for part (c)

Because more of the mass of the rod is at the end of the rod opposite point P, more mass is concentrated away from the axis of rotation; thus, the rotational inertia of the rod would be greater around point P than around its center of mass.

OR

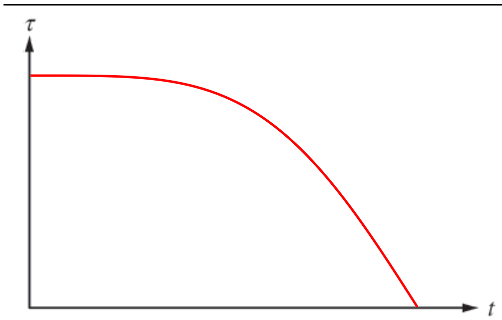
According to the parallel axis theorem, $I = I_{cm} + md^2$*, if the axis is at a position away from the center of mass, the rotational inertia is larger than if the axis were at the center of mass.*

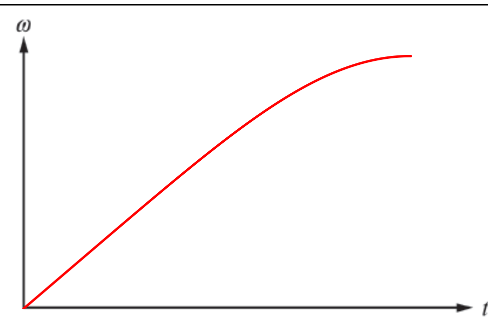
Total for part (c) 2 points

(d) For a concave down curve that decreases to zero for the graph of τ as a function of t **1 point**

For a concave down curve that approaches horizontal for the graph of ω as a function of t **1 point**

For consistency between the two graphs **1 point**





Total for part (d) 3 points

(e) For selecting "Decreases" with an attempted justification **1 point**

For a correct justification **1 point**

Example responses for part (e)

As the rod rotates downward, the angle θ *in the torque equation* $\tau = rF\sin\theta$ *decreases. Thus, the torque on the rod decreases.*

OR

As the rod rotates downward, the lever arm between point P and the rod's center of mass continues to decrease; thus, the torque on the rod decreases.

Total for part (e) 2 points

(f)	For using conservation of energy to calculate the speed of the rotating rod $U_i + K_i = U_f + K_f$ $U_i + 0 = 0 + K_f$ $U_i = K_f$	**1 point**
	For correctly substituting into the above equation $mgh_i = \frac{1}{2}I\omega_f^2$	**1 point**
	For correctly solving for the linear speed of point S $Mg\left(\frac{3}{4}L\right) = \frac{1}{2}\left(\frac{3}{5}ML^2\right)\left(\frac{v}{L}\right)^2$ $\frac{3}{4}MgL = \frac{3}{10}Mv^2 \therefore v = \sqrt{\frac{5}{2}gL} = \sqrt{\frac{5}{2}\left(9.8 \text{ m/s}^2\right)(1.0 \text{ m})} = 4.9 \text{ m/s}$	**1 point**
	Total for part (f)	**3 points**
	Total for question 3	**15 points**