Chief Reader Report on Student Responses:

2019 AP® Physics C Mechanics Free-Response Questions

Set 1

• Number of Students Scored	57,131		
• Number of Readers	377 (for all Physics exams)		
• Score Distribution	Exam Score	N	%At
	5	21,517	37.7
	4	15,268	26.7
	3	9,924	17.4
	2	5,710	10.0
	1	4,712	8.2
• Global Mean	3.76		

The following comments on the 2019 free-response questions for AP® Physics C Mechanics were written by the Chief Reader, Shannon Willoughby, Montana State University. They give an overview of each free-response question and of how students performed on the question, including typical student errors. General comments regarding the skills and content that students frequently have the most problems with are included. Some suggestions for improving student preparation in these areas are also provided. Teachers are encouraged to attend a College Board workshop to learn strategies for improving student performance in specific areas.

Question #1 **Task:** Kinematics, Newton's laws **Topic:** Terminal velocity

Max. Points: 15 **Mean Score:** 7.84

What were the responses to this question expected to demonstrate?

The responses to this question were expected to demonstrate the following:

- An understanding and interpretation of a nonlinear velocity vs. time graph, determining features that demonstrate the speed and acceleration at various times for an object subject to a resistive force
- Determination of displacement and acceleration functions from a given velocity function for an object subject to a resistive force
- Description of the net force on an object subject to a resistive force
- Determination of terminal velocity from a given velocity function for an object subject to a resistive force

How well did the responses address the course content related to this question? How well did the responses integrate the skills required on this question?

- Students that understood the content were able to read the v vs. t graph and interpret the change in speed of the object over time. These students were able to:
  - interpret the direction of the acceleration of the object and its change in magnitude.
  - calculate the acceleration of the object at a specific time by finding the slope of the line tangent to the curve at that time.
- Given a velocity function for an object subject to a resistive force, the students that understood the content were able to:
  - derive an expression for the magnitude of the vertical displacement of the falling object as a function of time by integrating the velocity function with appropriate limits.
  - derive an expression for the acceleration and the expression for the magnitude of the net force exerted on the object as it falls as a function of time by taking the derivative of the velocity function with respect to time and then multiplying this equation by the mass of the object.
- The students were then given conditions at which the object reaches a constant speed. The students that understood the content were able to:
  - determine the terminal velocity of an object subject to a resistive force from the velocity by finding the limit of the function as time approaches infinity.
  - determine that the force exerted by the fluid on the object is equal to the weight of the object at this time since the net force is equal to zero when the object moves with constant speed.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

- Students stated that since the slope of the v vs. t graph is decreasing, the acceleration of the object must be in the opposite direction to its motion.
- Students stated that since the velocity is increasing in the v vs. t graph, the acceleration must also be increasing.
- Students did not use a tangent line to determine the acceleration from the v vs. t graph. Instead these students choose two points on the given curve, typically at a significant distance from the desired time.
- Students did not use limits of integration or used the limits incorrectly in the derivation.
- When finding the terminal velocity, students did not use the given velocity function; instead they referred to the graph which does not show the asymptote of the function. These students assumed that the top of the graph is the asymptote of the function and/or assume the time limit is the end of the graph (therefore

not associating the asymptote to time approaching infinity).

- Students stated that since the net force on the object is zero, the resistive force is zero or used the equation for force determined in an earlier section to incorrectly determine the resistive force when terminal velocity is reached.

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• Since the slope of the graph of speed as a function of time is decreasing, the acceleration must be upward.	• Since the slope of the graph of speed as a function of time is decreasing, the magnitude of the acceleration must be decreasing.
• Since the speed of the object is increasing, the acceleration of the object is increasing.	• Since the object is moving downward and speeding up, the acceleration must be downward; however, since the slope of the graph of speed as a function of time is decreasing, the magnitude of the acceleration must be decreasing.
• Calculating the slope between two points on the curve: $\text{slope} = a = \frac{\Delta v}{\Delta t} = \frac{0.91 - 0.27}{0.30 - 0.05} = 2.56 \text{ m/s}^2$	• Calculating the slope using points from a tangent line: $\text{slope} = a = \frac{\Delta v}{\Delta t} = \frac{0.8 - 0.6}{0.226 - 0.136} = 2.22 \text{ m/s}^2$
$\Delta y = \int A\left(1 - e^{-Bt}\right)dt = A\left[t - \frac{1}{-B}e^{-Bt}\right]$ $= A\left(t + \frac{1}{B}e^{-Bt}\right)$	$\Delta y = \int_{t'=0}^{t'=t} A\left(1 - e^{-Bt'}\right)dt' = A\left[t' - \frac{1}{-B}e^{-Bt'}\right]_{t'=0}^{t'=t}$ $= A\left(t + \frac{1}{B}\left(e^{-Bt} - 1\right)\right)$
• The constant speed is 1 m/s because the graph levels off at this value.	• After a long time, the falling object will reach a terminal constant speed in the fluid. This can be determined by setting the time t in the equation for speed equal to infinity. By doing this, the constant speed is determined to be υ = A = 1.18 m/s.
• After a long time, the object will reach terminal velocity. At this point the acceleration of the object will be 0 m/s^2 since the object is moving at a constant speed. Therefore, the force exerted by the fluid on the object at this time must be equal to zero. • $F(t) = mABe^{-Bt}$ so as $t \to \infty$ the force exerted by the fluid approaches zero.	• When the falling object reaches a constant speed in the fluid, the net force must be zero. Since the only vertical forces acting on the object are Earth's gravitational pull and the resistive force of the fluid, these two forces must be equal. So, the resistive force must be equal to the weight of the object or 0.12 N.

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

- AP Physics C teachers can find useful resources on the Course Audit webpage and the AP Central home page for AP Physics C. In addition, topic questions that are tied to specific learning objectives and science practices can be found on the new AP Classroom.
- The new AP Physics 1 Student Workbook contains many helpful scenarios which address topics and skills also covered in AP Physics C. These scenarios can be modified or scaffolded as needed for Physics C students.
- The AP Physics Online Teacher Community is active, and there are many discussions concerning teaching tips, techniques, and activities that AP Physics teachers have found helpful. It is easy to sign up, and you can search topics of discussions from all previous years.
- New teachers (and career changers) might want to consider signing up for an AP Summer Institute (APSI). An APSI is a great way to get in-depth teaching knowledge on the AP Physics curriculum and exam, as well as network with colleagues from around the country.

Question #2 **Task:** Collisions, energy **Topic:** Momentum, energy conservation

Max. Points: 15 **Mean Score:** 6.94

What were the responses to this question expected to demonstrate?

The responses to this question were expected to demonstrate the following:

- Determination of the speed of a block attached to the end of a string, when the block is released from rest
- Creation of a free-body diagram to represent the forces that are exerted on an object undergoing vertical circular motion
- Determination of a specific force exerted on an object undergoing vertical circular motion
- Calculation of the time of fall of an object in projectile motion from kinematic equations, given a horizontal launch from a specific height
- Calculation of the horizontal speed of the object from kinematic equations given a horizontal launch from a specific height
- Calculation of the final speed of the block attached to the string after a collision with a second block from conservation of momentum in a one-dimensional collision
- Calculation of the maximum angle a pendulum will reach using conservation of mechanical energy

How well did the responses address the course content related to this question? How well did the responses integrate the skills required on this question?

- Students were generally able to determine the speed of a block on the end of a pendulum string when the object is released from rest by using conservation of mechanical energy.
- Students understood that the free body diagram included only two forces (tension in the string up and the weight of the block down) with the tension longer than the weight of the block. The most common misconception here was drawing the two vectors as having equal magnitude.
- Students were able to use Newton's second law to calculate the tension in the string. Most students used an appropriate form of Newton's second law to calculate the tension in the string with appropriate variables.
- The students with understanding of the content were able to use appropriate kinematic equations to calculate the time for a projectile to fall a particular height as well as the horizontal speed of the projectile as it left the table and landed at a particular horizontal distance.
- The students with understanding of the content were able to use conservation of momentum to calculate the speed of the block attached to the string after a collision with a second block.
- The students with understanding were able to use conservation of mechanical energy to find the maximum angle that the pendulum would reach after collision with another object.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

- When drawing the free body diagrams, students would assume the block was hanging at the bottom of the string and not moving in an arc; thus, drawing the tension in the string and the weight of the block with the same magnitude.
- Students labeled the tension in the string as centripetal force.
- When using Newton's Second Law for the object undergoing vertical circular motion, many students set the net acceleration equal to zero.
- Students did not use appropriate units.
- Students mixed horizontal and vertical dimensions in one-dimensional kinematics.
- Students assumed a perfectly elastic collision in an inelastic collision problem to find the velocity of the object.
- Students used rotational kinematics to find the maximum angle the pendulum string made with the vertical after the collision.

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• Students drawing tension in the string up and the weight of the block down, each with the same magnitude.	• Students drawing tension in the string up and the weight of the block down, with the tension longer than the weight of the block.
• Students drawing centripetal force up and the weight of the block down.	• Students drawing tension in the string up and the weight of the block down.
• $F_T = ma_c + mg = 0 + mg$	• $F_T = ma_c + mg = m\frac{v^2}{r} + mg$
• $t = \sqrt{\frac{2\Delta y}{g}} = \sqrt{\frac{2(2L)}{g}} = \sqrt{\frac{(4)(75\text{ cm})}{(9.8\text{ m/s}^2)}} = 5.53\text{ s}$	• $t = \sqrt{\frac{2\Delta y}{g}} = \sqrt{\frac{2(2L)}{g}} = \sqrt{\frac{(4)(75\text{ m})}{(9.8\text{ m/s}^2)}} = 5.53\text{ s}$
• Using horizontal displacement in a vertical expression for time. $t = \sqrt{\frac{2\Delta y}{g}} = \sqrt{\frac{2(4L)}{g}}$	• Using vertical displacement in a vertical expression for time. $t = \sqrt{\frac{2\Delta y}{g}} = \sqrt{\frac{2(2L)}{g}}$
• Assuming a perfectly inelastic collision in applying Conservation of Momentum to calculate the speed of the second block as it leaves the table: $v_{2f} = \frac{2(3M)}{(3M+M)}\sqrt{2gL}$ • Assuming a perfectly inelastic collision in applying Conservation of Momentum to calculate the speed of the first block after an elastic collision: $v_{1f} = \frac{3M-M}{4M}\sqrt{2gL}$	• Kinematics: $v = \frac{\Delta x}{t}$ • Conservation of Momentum to calculate the speed of the first block after an elastic collision: $v_{1f} = \frac{(3M)v_{1i} - (M)v_{2f}}{(3M)}$
• Use of angular kinematics:	• Use of conservation of energy:

$\omega^2 = \omega_0^2 + 2\alpha\theta$	$\frac{1}{2}mv^2 = mgh$ $\frac{1}{2}mv^2 = mgL(1-\cos\theta)$ $\theta = \cos^{-1}\left(1-\frac{v^2}{2gL}\right)$

What resources would you recommend to teachers to better prepare their students for the content and skill(s) required on this question?

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- The new AP Physics 1 Student Workbook contains many helpful scenarios, which address topics and skills also covered in AP Physics C. These scenarios can be modified or scaffolded as needed for Physics C students.
- The AP Physics Online Teacher Community is active, and there are many discussions concerning teaching tips, techniques, and activities that AP Physics teachers have found helpful. It is easy to sign up, and you can search topics of discussions from all previous years.
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Question 3 **Task:** Calculate the moment of inertia **Topic:** Rotational Motion

Max. Points: 15 **Mean Score:** 4.74

What were the responses to this question expected to demonstrate?

The responses to this question were expected to demonstrate the following:

- The ability to recognize that an unbalanced force exerted on an object some distance from a perpendicular axis gives the object a rotational acceleration.
- The ability to recognize Newton's second law of motion for rotational motion and its correlation to rotational kinematics through a derived equation.
- Recognition between linear motion and rotational motion for a nonslip scenario where two different discs were rotating together.
- Recognition that angular momentum is conserved during a collision between the platforms in the absence of external forces/torques.
- Given a plot of data, students should be able to determine the relationship between two quantities and draw a graph that best represents the data plotted.
- Analysis of the graphed data in order to form a relationship between the data and develop an expression from the graph in order to answer a prescribed objective.
- Recognition that an object's rotational inertia is dependent on how the mass of the object is distributed relative to an axis of rotation.
- How to use a graph to determine a quantity and describe the effect of a potential error in lab results, requiring application of the parallel axis theorem.

How well did the responses address the course content related to this question? How well did the responses integrate the skills required on this question?

- Students typically recognized the relationship between Newton's second law of motion and rotational kinematics.
- Students made appropriate substitutions of kinematics quantities for rotational kinetic energy and angular momentum.
- Students generally recognized the plotted data on the graph demonstrated a linear relationship.
- Students recognized that the interaction between the two platforms resulted in a reduced rotational velocity and rotational kinetic energy.
- Student responses show a lack of understanding of the term "derive" in context of an AP Physics question. Many student responses showed a complete understanding of the topics, but fell short of the "derive" requirements.
- Many students showed a good understanding of angular momentum, but some confused angular momentum with rotational energy.
- Students' ability to justify a choice (check box) is highly variable. Some simply restated the information that was given, others wrote far more than necessary.

What common student misconceptions or gaps in knowledge were seen in the responses to this question?

- Student responses show that many have no experience at graphing by hand, specifically drawing a best-fit line.
- Students struggled with equating the rotating platform's linear velocity to the linear velocity of the wheel. This led students to linking the angular velocities of the platform to the wheel incorrectly or incompletely.

- Students incorrectly used conservation of rotational mechanical energy and conservation of angular momentum interchangeably.
- Students struggled to provide a fully developed reasoning as to why the rotational velocity and rotational kinetic energy were less post collision.
- Students did not differentiate between the center of mass and the axis of rotation when it was recognized that the center of mass of the two platforms were not aligned during the collision.
- Students did not begin the "derive" portions of the question by stating the fundamental physics principle followed by pairing it with an equation from the equation sheet.

Common Misconceptions/Knowledge Gaps	*Responses that Demonstrate Understanding*
• Students did not recognize the relationship between the linear velocities and angular velocities of the platform and wheel.	• The platform and wheel have the same linear velocities since they are in contact and the surfaces did not slip. Therefore, $\upsilon = r\omega$ of the platform is equal to $\upsilon = r\omega$ for the wheel. ($D\omega_p = r\omega_w$) ** Noting the students must start from the fundamental principle above or the equation $\upsilon = r\omega$ from the equation sheet.
• Students commonly used conservation of mechnical energy or conservation of energy in place of conservation of momentum to describe or determine quantities post collision.	• For part (d): Starting with $L_o = L_f$ to derive $I_p\omega_p = (I_p + I_p)\omega_f = I_p\omega_p = 2I_p\omega_f$ • For part (e): $I_p\omega_o = (I_p + I_u)\omega_f$ to relate the graphed data to conservation of momentum. • For part (f): angular momentum is conserved when the object is dropped on the platform; however, mechanical energy is transformed to non-mechanical forms of energy such as sound or thermal energy due to the frictional force between the plates.
• Students had trouble recognizing that while there were no external nonconservative forces acting on the system, dropping the unknown object onto the platform did result in a nonconservative force between the objects that caused the reduction of mechanical energy of the system.	• $K_f < K_i$: The collision between the object and the platform is inelastic in nature. The frictional force between the plates results in a reduction of mechanical energy in the system.
• Students did not realize that a rotating object may rotate about a pivot point that is not directed through the object's center of mass. • Students did not clearly differentiate among the axis of rotation, the	• Since the center of mass is off the axis of the platform, the moment of inertia would be greater due to more mass being further from the axis of rotation. • Due to the center of mass being some distance (x) from the axis of rotation the experimental value I_u is larger and can be

object's center of mass, and the center of the platform. • Students recognized the offset had a distance dependent change for the system's moment of inertia, but did not clearly reference the distance that impacted the moment of inertia.	determined using the parallel axis theorem, $I_u = I_{cm} + mx^2$, where x is the distance between the object's center of mass and the axis of rotation.
• Many students do not understand the expectations of a "derive" question, and jump into a solution without showing the fundamental equation/s.	• Work should begin with a fundamental equation, principle, or law. Stating "Conservation of Angular Momentum" or "$L_i = L_f$" both show that the student understands the underlying principle of the derivation.
• Students confuse rotational kinetic energy with linear kinetic energy.	• Use of $K = ½ I\omega^2$ • Clearly using angular quantities in a written explanation (e.g., angular velocity instead of velocity; rotational inertia instead of mass)
• Students confuse angular momentum with rotational kinetic energy.	• Correct application of angular momentum as a conserved quantity in a collision. • Recognize that conservation of angular momentum does not require conservation of energy
• Students' written responses were too vauge to receive full credit, e.g. not distinguishing between two similar quanties that were to be compared: "the inertia would be greater"; "$I>I_{u.}$" • Students' written responses were overly wordy. Some students wrote lengthy paragraphs when a single sentence would have sufficed.	• Clearly stating which quantity they were describing: ○ "The experimental inertia would be greater than the actual inertia." ○ Using subscripts in an equation that clearly indicate which quantity: $I_{exp} > I_{actual.}$ • Responses that were brief and to the point demonstrate much stronger understanding of the physics than those that ramble.
• Students using data points to calculate a slope. • Students drawing poor best fit lines (curves, or lines not falling between the data points).	• Students should use points from a best-fit line to determine the slope of a graph. • Students should use a straightedge, and draw a line that splits the middle of the points, with a balance of points above and below the line.

Based on your experience at the AP® Reading with student responses, what advice would you offer teachers to help them improve the student performance on the exam?

- Be sure to discuss the root words of the prompts with students. “Derive” portions of the prompts require students to begin by stating a fundamental physics principle or begin work with an equation from the equation sheet. Students should not substitute into the equations until the fundamental principle has been stated.
- Have students practice questions that incorporate the use of conservation of energy and conservation of momentum in the same question. This will allow students to develop their skills and recognize scenarios where the use of conservation of energy and conservation of momentum are appropriate.
- Do not train students to calculate the slope for every linear graphing question as a default move aimed at getting points. Instead, ask them to consider how the relationship between the variables being graphed relates to a physically relevant equation, so that the meaning of the slope follows the physics rather than the other way around.
- Teachers must be sure that they teach students the AP Physics exam meaning of the terms “derive”; “determine”; and “calculate.” Model the expectations for students, especially for “derive” questions, and give students practice at deriving equations in class, and have the students score each other’s derivations using an AP rubric.
- Teachers should specifically teach students how to write clear, concise explanations. Begin with examples drawn from the student samples posted on AP Central, have students evaluate the clarity and correctness of the response. Then have students practice writing justifications in AP exam problems, and peer review them for clarity, brevity, and correct physics. Repeating this activity near the end of each unit, as a review prior to the unit test, can help students build skills throughout the school year.
- Teachers must provide lab experiences for students. Have students graph data by hand, draw best-fit lines, and calculate slope. In homework and practice problems, giving students data that has a significant y-intercept can show students that using one data point to determine slope does not yield accurate results.
- Lab experience with rotational motion and rotational inertia can help students’ kinesthetic understanding of the difference between rotational energy and angular momentum: a small electric motor spinning at 1200 rpm (such as a power drill) has more energy but less angular momentum than a bicycle wheel spinning 120 rpm.

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