2019

AP® CollegeBoard

AP® Physics C: Mechanics

Sample Student Responses and Scoring Commentary Set 2

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General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 2

15 points

A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration a for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0\ \text{m/s}^2$, $L = 0.25\ \text{m/s}^4$, and t is the time in seconds. At $t = 6.0$ s, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.

(a) LO INT-5.E, SP 6.B, 6.C
2 points

Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0$ s.

For an expression for calculating impulse and correct substitution of $a(t)$ and m into the correct expression.		1 point
$J = \int F(t)\,dt$		
$J = \int ma(t)\,dt = (0.50)\int\left(9.0 - 0.25t^2\right)dt$		
For integrating with correct limits or including a constant of integration		1 point
$J = (0.50)\int_{t=0}^{t=6}\left(9.0 - 0.25t^2\right)dt = (0.50)\left[9.0t - \frac{1}{12}t^3\right]_{t=0}^{t=6}$		
$J = (0.50)\left[\left((9.0)(6) - \left(\frac{1}{12}\right)(6)^3\right)\right] - 0 = 18\ \text{N•s}$		
Alternate Solution (using an alternate solution from part (b))		*Alternate Points*
For correctly relating impulse to the speed of the rocket		1 point
$J = \Delta p = m(v_2 - v_1)$ OR $J = m\Delta v = mv_f$		
For correctly substituting answer from part (b) into equation above		1 point
$J = (0.50\ \text{kg})(36 - 0)\ \therefore\ J = 18 N•s$		

Question 2 (continued)

(b) LO INT-5.A.a, SP 6.A, 6.C
2 points

Calculate the speed of the rocket at $t = 6.0$ s.

For correctly relating impulse to the speed of the rocket		1 point
$J = \Delta p = m(v_2 - v_1)$ OR $J = m\Delta v = mv_f$		
For correctly substituting answer from part (a) into equation above		1 point
$(18\ \text{N}\bullet\text{s}) = (0.50\ \text{kg})(v_2 - 0) \therefore v_2 = 36\ \text{m/s}$		
Alternate Solution		*Alternate Points*
Integrate expression for a(t) (This may have already been done in solving part (a).)		1 point
$v = \int a\,dt = \int (9.0 - 0.25t^2)dt$		
For integrating with correct limits or including a constant of integration		1 point
$v = \int_{t=0}^{t=6} (9.0 - 0.25t^2)dt = \left[9.0t - \frac{1}{12}t^3\right]_{t=0}^{t=6} = 36\ \text{m/s}$		

(c)

i. LO INT-4.C.c, SP 6.B, 6.C
2 points

Calculate the kinetic energy of the rocket at $t = 6.0$ s.

For substituting the mass of the rocket into the equation for kinetic energy		1 point
For substituting the answer from part (b) into the equation for kinetic energy		1 point
$K = \frac{1}{2}mv^2 = \left(\frac{1}{2}\right)(0.50\ \text{kg})(36\ \text{m/s})^2 = 324\ \text{J}$		

ii. LO CHA-1.B, CON-1.E, SP 6.A, 6.C
3 points

Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0$ s.

For integrating the acceleration twice to derive an expression for position		1 point
For integrating with correct limits or including a constant of integration		1 point
$v(t) = \int a(t)dt = \int (9.0 - 0.25t^2)dt = 9.0t - \frac{1}{12}t^3 + v_0 = 9.0t - \frac{1}{12}t^3$		
$\Delta y = \int v(t)dt = \int_{t=0}^{t=6} \left(9.0t - \frac{1}{12}t^3\right)dt = \left[\frac{9}{2}t^2 - \frac{1}{48}t^4\right]_{t=0}^{t=6}$		
$\Delta y = \left[\left(\frac{9}{2}\right)(6)^2 - \left(\frac{1}{48}\right)(6)^4\right] - 0 = 135\ \text{m}$		
For substituting into equation for potential energy		1 point
$\Delta U_g = mg\Delta y = (0.50\ \text{kg})(9.8\ \text{m/s}^2)(135\ \text{m}) = 660\ \text{J}$		

Question 2 (continued)

(d) LO CHA-1.B, CON-2.B, SP 6.B, 6.C
3 points

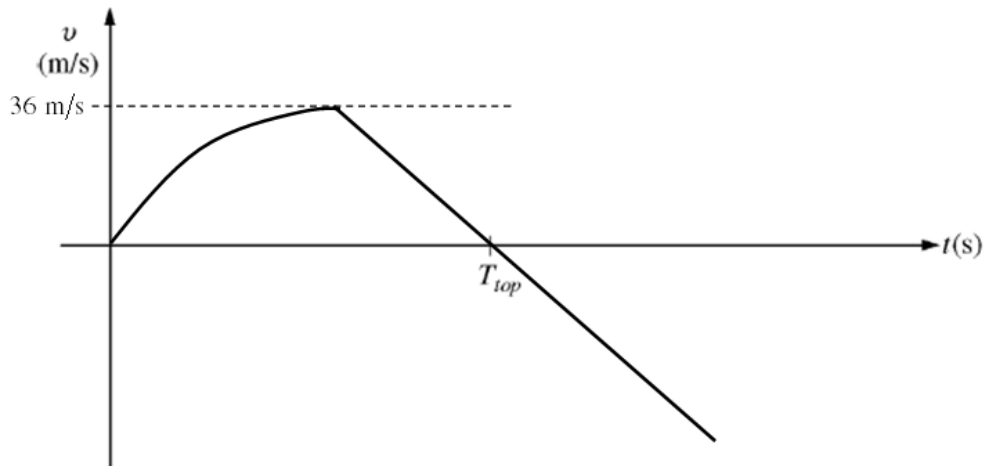
Calculate the maximum height reached by the rocket relative to its launching point.

For using $a = g$ in a correct kinematics equation to solve for height		1 point
$v_2^2 = v_1^2 + 2a(y_2 - y_1) \therefore 0 = v_1^2 + 2a(y_2 - y_1)$		
$y_2 - y_1 = -\dfrac{v_1^2}{2a} \therefore y_2 = -\dfrac{v_1^2}{2a} + y_1$		
For substituting the speed from part (b)		1 point
$y_2 = -\dfrac{(36\ \text{m/s})^2}{2(-9.8\ \text{m/s}^2)} + y_1$		
For substituting the height from part (c)		1 point
$y_2 = 66\ \text{m} + 135\ \text{m} = 201\ \text{m}$ (199.8 m if $g = 10\ \text{m/s}^2$)		
Alternate Solution 1		*Alternate Points*
For using energy conservation to find maximum height, consistent with the speed found in part (b).		1 point
$mg\Delta y = \dfrac{1}{2}mv^2 \therefore \Delta y = \dfrac{v^2}{2g}$		
For substituting the speed from part (b)		1 point
$\Delta y = \dfrac{(36\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)} = 66\ \text{m}$		
For substituting the height from part (c)		1 point
$y_2 = 66\ \text{m} + 135\ \text{m} = 201\ \text{m}$ (199.8 m if $g = 10\ \text{m/s}^2$)		
Alternate Solution 2		*Alternate Points*
For using energy conservation to find maximum height, from kinetic and potential energies found in part (c)		1 point
$K_1 + U_1 = 0 + U_{top}$		
For substituting the kinetic energy from part (c)(i) into the equation above		1 point
For substituting the potential energy from part (c)(ii) into the equation above		1 point
$324 + 660 = (0.5)(9.8)\Delta y$		
$\Delta y = 201$ m (199.8 m if g = 10m/s^2)		

Question 2 (continued)

(e) LO CHA-1.C, SP 3.C
3 points

On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground. T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



For an initial concave down curve that starts at the origin.		1 point
For a transition that occurs before T_{top} into a straight line with a negative slope.		1 point
For labeling the maximum value of the velocity, consistent with part (b), and a line that crosses the x-axis at T_{top}		1 point

Question 2 (continued)

Learning Objectives

CHA-1.B: Determine functions of position, velocity, and acceleration that are consistent with each other, for the motion of an object with a nonuniform acceleration.
CHA-1.C: Describe the motion of an object in terms of the consistency that exists between position and time, velocity and time, and acceleration and time.
CON-1.E: Calculate the potential energy of a system consisting of an object in a uniform gravitational field.
CON-2.B: Describe kinetic energy, potential energy, and total energy in relation to time (or position) for a "conservative" mechanical system.
INT-4.C.c: Calculate changes in an object's kinetic energy or changes in speed that result from the application of specified forces.
INT-5.A.a: Calculate the total momentum of an object or system of objects.
INT-5.E: Calculate the change in momentum of an object given a nonlinear function, $F(t)$, for a net force acting on the object.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.
6.A: Extract quantities from narratives or mathematical relationships to solve problems.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

2. A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration a for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0\ \text{m/s}^2$, $L = 0.25\ \text{m/s}^4$, and t is the time in seconds. At $t = 6.0$ s, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.

(a) Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0$ s.

$\Delta j = \int F\,dt \qquad \int_0^6 4.5 - 0.125t^2\,dt$

$a = 9 - 0.25t^2$

$F = ma = 4.5 - 0.125t^2$

$18\ \text{N}\cdot\text{s}$

(b) Calculate the speed of the rocket at $t = 6.0$ s.

$a = 9 - 0.25t^2$

$\frac{dv}{dt} = 9 - 0.25t^2$

$\int 1\,dv = \int (9 - 0.25t^2)\,dt$

$V(t) = 9t - \frac{0.25}{3}t^3 + C$ 0

$t = 6$

$V(6) = 36\ \text{m/s}$

GO ON TO THE NEXT PAGE.

(c)

i. Calculate the kinetic energy of the rocket at $t = 6.0$ s.

$\frac{1}{2}(0.5)(36^2) = KE$

$KE = 324 J$

ii. Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0$ s.

$\int V(t) = \int 9t - \frac{0.25}{3}t^3$

$X(t) = 4.5t^2 - \frac{0.25}{12}t^4 + \overset{0}{C}$

$t = 6$

$x = 135$ meter

ΔPE

$\Delta PE = mg\Delta h = (0.5)(9.8)(135) = 661.5 J$

(d) Calculate the maximum height reached by the rocket relative to its launching point.

$V_i = 36 m/s$

$V_f = 0 m/s$

$a = -9.8 m/s^2$

$\Delta X = \underline{66.12}$ meters

$V_f^2 = V_i^2 + 2a\Delta X$

$0 = 1296 + 2(-9.8)\Delta X$

$\Delta X = 66.12$

$135 + 66.12$

201.12 meters

Question 2 continues on the next page.

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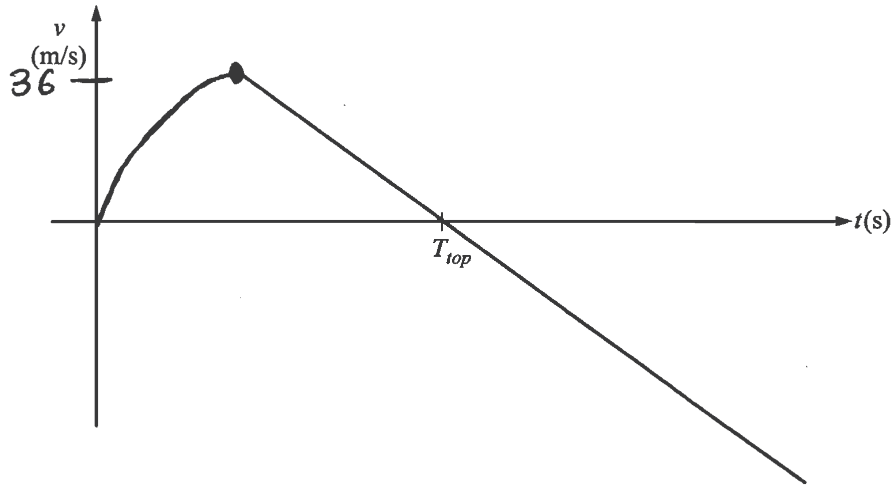
(e) On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground. T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



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(a) Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0$ s.

$a = 9 - 0.25t^2$ $\quad F = ma = 0.50(9 - 0.25t^2)$

$F = 4.5 - 0.125t^2$

$J = \int_0^6 (4.5 - 0.125t^2)\,dt$

$J = 18\ \text{kg}\cdot\text{m/s}$

(b) Calculate the speed of the rocket at $t = 6.0$ s.

$J = \Delta p = p_f - p_i = mv_f - mv_i$

$18\ \text{kg}\cdot\text{m/s} = (0.50\ \text{kg})(v_f) - (0.50\ \text{kg})(0)$

$v_f = 36\ \text{m/s}$

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(c)

i. Calculate the kinetic energy of the rocket at $t = 6.0$ s.

$KE = \frac{1}{2}mv^2$

$= \frac{1}{2}(0.50\text{ kg})(36\text{ m/s})^2$

$= 324\text{ J}$

ii. Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0$ s.

$\Delta KE = \Delta PE$

$\Delta PE = 324\text{ J}$

(d) Calculate the maximum height reached by the rocket relative to its launching point.

$\Delta PE = mgh = 324\text{ J}$

$= (0.5\text{ kg})(10\text{ m/s}^2)(h) = 324\text{ J}$

$h = 66\text{ m}$

Question 2 continues on the next page.

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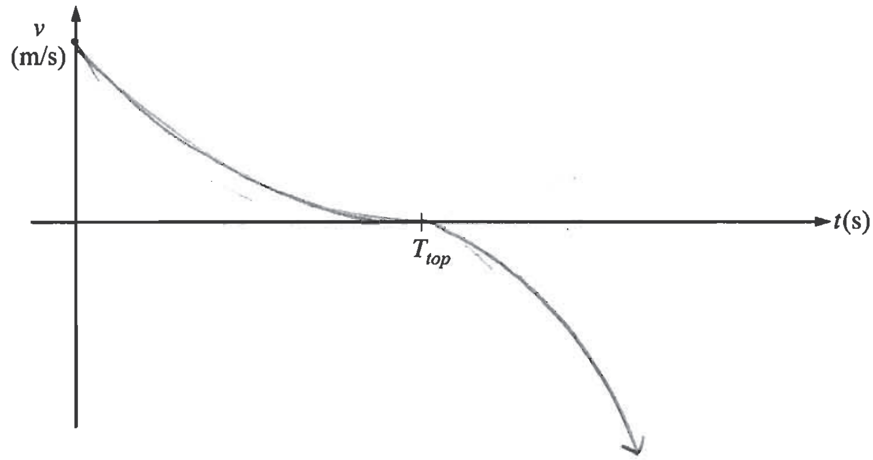
(e) On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground. T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



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(a) Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0$ s.

$J = \int F\,dt = \Delta p$ $\qquad F = ma$

$J = \int ma\,dt$

$J = \int_0^6 ((0.5)(K - Lt^2))\,dt$

$J = 18$

(b) Calculate the speed of the rocket at $t = 6.0$ s.

$\frac{d}{dt}\left(|K - Lt^2|\right)_{a=6} = 2.5 \cdot 10^{-4}$ m/s

GO ON TO THE NEXT PAGE.

(c)

i. Calculate the kinetic energy of the rocket at $t = 6.0$ s.

$U_i + K_i = U_f + K_f$

0

$mgh = \frac{1}{2}mv^2$

$mgh = mgh + \frac{1}{2}mv^2$

$0 = mgh + \frac{1}{2}mv^2$

ii. Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0$ s.

(d) Calculate the maximum height reached by the rocket relative to its launching point.

$\cancel{x = x_0 + v_0 t + \frac{1}{2}at^2}$

$v_x = v_0 + at$

$= 0 + (K - Lt^2)(t)$

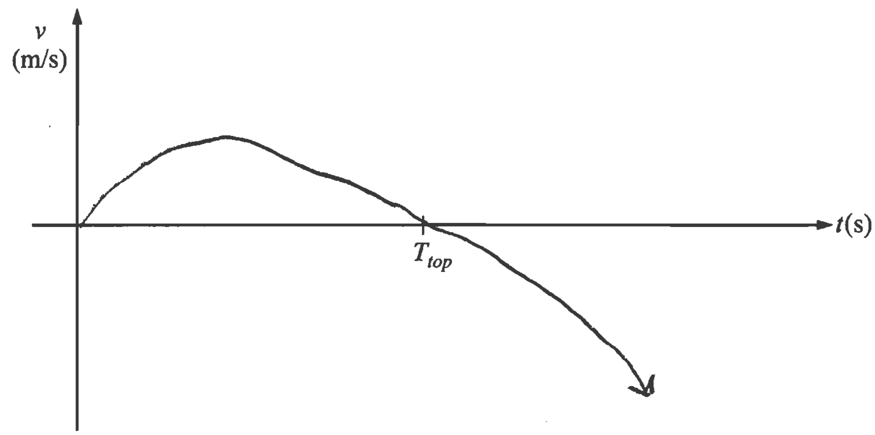
$36^2 = 0 +$

36 m/s

Question 2 continues on the next page.

GO ON TO THE NEXT PAGE.

(e) On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground. T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



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Question 2

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses to this question were expected to demonstrate the following:

- How to perform velocity and position calculations for both constant and nonconstant acceleration:
  - Working with an object that is undergoing nonconstant accelerating motion requires primarily the understanding that the kinematics formulas are no longer applicable.
  - Calculus is necessary to obtain the necessary results.
- How to graph velocity of an object over time throughout the situation given in the problem. This initially involves a nonlinear motion where the speed of the rocket is increasing but at a slower rate, followed by free-fall motion where the rocket continues to move upward for a short period of time until the speed of the object reaches zero (and thus maximum height) and comes back down with the same acceleration.
- Students were also required to demonstrate the fundamental understanding that while the engine is being fired, the energy of the rocket itself is not conserved. In fact, it is gaining energy due to the work done by the engine on the rocket. However, once the fuel is exhausted, the Law of Conservation of Energy applies.

Sample: M Q2 A
Score: 15

This paper earned full credit. In part (a) 2 points were earned for an expression to calculate impulse with a correct substitution of $a(t)$ and a correct integration clearly showing the limits of integration. In part (b) 2 points were earned for integrating the expression for $a(t)$ and using the correct limits of integration to arrive at the correct velocity. In part (c)(i) 2 points were earned for substituting mass and velocity into the equation for kinetic energy. In part (c)(ii) 3 points were earned for integrating the velocity expression from part (b) to derive an expression for position, integrating using the correct limits, and then for correctly substituting into the equation for potential energy. In part (d) 3 points were earned for using a correct kinematics equation with "a = g" to calculate the vertical displacement from the moment when the fuel is exhausted to its maximum height and then adding the value to the height obtained in part (c)(i) to obtain the height relative to its launching point. In part (e) 3 points were earned for drawing a graph initially concave down, transitioning into a straight line before T_{top}, and labeling the maximum value of velocity and a line that crosses T_{top}.

Sample: M Q2 B
Score: 8

Parts (a), (b), and (c)(i) earned full credit, 2 points each. In part (c)(ii) no points were earned because conservation of energy cannot be applied since acceleration is not constant. In part (d) 2 points were earned for using energy conservation to find the vertical displacement from the moment the fuel is exhausted to the point it reaches maximum height and for substituting the kinetic energy from part (c)(i). In part (e) no points were earned because the curve has incorrect concavity, does not start at the origin, does not transition into a straight line with negative slope, and there is also no label for v_{max}.

Question 2 (continued)

Sample: M Q1 C
Score: 3

Part (a) earned full credit. In part (b) the method shown is inconsistent with the question, so no points were earned. In part (c)(i) no points were earned because there is no indication of substituting mass of the rocket and velocity consistent with part (b) into the equation for kinetic energy. Part (c)(ii) is left blank, so no points were earned. In part (d) no points were earned because there is no indication that an appropriate kinematics equation is being used. In part (e) there is an initial concave down curve that starts at the origin, but the curve after the transition is not linear, and there is no indication that a label for maxima is shown, so 1 point was earned.